2. The applicability of the M&M's leverage irrelevance theorem to banks

2.1 M&M assumptions

Before analysing how the capital structure decisions affect the value of the banking firm and its cost of financing it is important to recap the conditions under which the M&M model can be effectively used. As already pointed out by Rubinstein (2003), by looking backward to the M&M world from the perspective of modern finance a minimal set of four axioms are required for M&M to hold (Giesecke *et al.*, 2004). These are:

1. There are no riskless arbitrage opportunities and information asymmetries.
2. Operating income (from assets) and its volatility are not affected by capital structure.
3. The proportion of operating income that is jointly allocated to stocks and bonds is not affected by the firm's capital structure.
4. The present value function (the economy wide state prices) is not affected by capital structure.

The four axioms cited above define an idealized economy. Rubinstein points out that the M&M theorem serves less as a statement that the leverage ratio is irrelevant to firm value than as a benchmark from which to measure the ways in which leverage ratio affects firm value. He adds that each of the axioms mentioned above has generated "its own virtual cottage industry of academic research." Furthermore, among academics and researchers there has been disagreement about whether or not M&M holds if bankruptcy is allowed. The main source of the debate is related to the existence of bankruptcy costs. In fact, there is a significant chance that a levered firm can face bankruptcy and, therefore, incurs charges that an unlevered firm does not have to worry about. Therefore, while the M&M might apply to an abstract (non-costly) bankruptcy, it does not hold with more realistic bankruptcy assumptions (see for instance Vernimmen *et al.*, 2005).

Before analysing the M&M's propositions for the equilibrium cost of equity, some additional remarks on the fourth axiom are needed. From finance theory we know that, all other things being equal, most investors prefer less risk to more (i.e. they are risk-adverse). Consequently, market prices include compensation for the risk that cannot be diversified away. This risk premium is realized as an increase in expected return on investment. It generates a marketimplied (or martingale) probability model in which the likelihood of an event is the "actual" likelihood, adjusted by the risk premium. In the M&M idealized world it is assumed that the leverage decisions of the firm cannot affect risk premiums. Market economy wide's discounting functions are assumed exogenous (Giesecke *et al.*, 2004). This can be a strong assumption, if we are considering the financing decisions of the banking industry. Higher capital requirements determine a simultaneous increase in the demand of capital, which in turn affects the market price of risk. The aggregate capital supply function determines the magnitude of these changes in the market price of risk. Ultimately, the market supply of equity can be constrained. This is true especially during a financial crisis when the market price of risk increases (the investors' risk appetite declines). The supply curve for equity shifts downward and flattens (i.e. for a given level of ROE, investors will offer less capital than before because of their increased reluctance towards risk). In this situation, higher capital requirements imposed by regulators determine an exogenous floor on equity demanded by banks. This floor produces an upward shift of demand (for a given level of ROE, stringent capital requirements impose an extra-burden in terms of capital which is not optimal on a risk-adjusted basis). The consequence might be a market failure.

2.2 The cost of equity

The WACC is the average rate of return required by all the bank's investors – equity, debt, deposit holders. Formally it can be defined as:

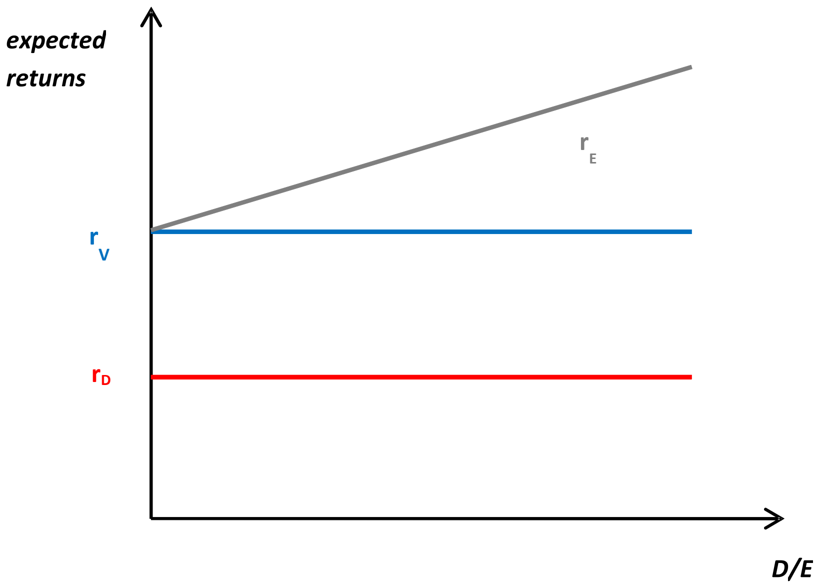
$$r_V = r_E\left(\frac{E}{E+B}\right) + r_B\left(\frac{B}{E+B}\right) = r_E\left(\frac{E}{V}\right) + r_B\left(\frac{B}{V}\right) \quad [8]$$

where E is the market value of equity, B is the market value of liabilities (debt and deposits), r_E is the cost of equity required by shareholders and r_B is the rate of return required by lenders. In the simplest/textbook version of the M&M propositions default risk is not explicitly considered. In this framework, the cost of debt financing is assumed fixed and not affected by the leverage of the bank. This simplifying hypothesis determines that the financial and the book values of debt coincide ($B = D$) and r_D, which in this special case is equal to r_B, is constant. Thus, under the M&M set of assumptions, the equilibrium cost of equity, r_E, is:

$$r_E = r_V + (r_V - r_D)\frac{D}{E}. \quad [9]$$

The equilibrium cost of equity, r_E, will rise at a constant rate as the leverage increases (Figure 5) because of the increasing financial risks to shareholders.

Figure 5: Equilibrium expected returns in the standard M&M model



The essential message of the M&M model is precisely that the ex-ante cost of equity is a function that depends both on the risk/return of the banking firm's earning assets and the degree of leverage (Miller, 1995).

However, the result obtained in equation [9] is only a first approximation. As pointed out by M&M themselves (1958), if bankruptcy is factored in, the leverage changes also the cost of debt. As the leverage increases, the cost of debt, r_B [1], increases too, to account for the higher

[1] As previously explained in this general case the required return on debt, r_B does not coincide the coupon interest rate payment r_D.

risk faced by debt holders. In this case the cost of equity, r_E, will still tend to rise as the leverage increases, but at a decreasing rather than constant rate.

This means that, even accepting the fundamental proposition of the M&M theorem (i.e. the weighted average cost of capital is constant), equity' raising is not expensive for banks' shareholders only if default risk is not considered and the cost of debt financing is assumed to be constant and, most importantly, risk insensitive[2]. These are very strong assumptions, especially for the modelling of banking business. If, more realistically, default risk is considered (and consequently the cost of debt financing increases with risk) the cost of equity does not increase linearly with leverage. More capital (less leverage) does not produce a proportional effect in terms of cheaper cost of capital. A formal proof of these results can be analytically derived by using the Merton model (1974)[3] which considers default but at the same time assumes that bankruptcy is costless. This abstract bankruptcy assumption allows to derive results consistent with the M&M proposition. In the Merton model, although the values of both debt and equity are a function of leverage, their sum (i.e. the value of the assets) is independent of leverage. Thus, the Modigliani-Miller (M&M) theorem holds: the value of assets is not affected by the financing policies adopted by the firm.

In this framework the market value of debt (i.e. the value of the risky bond) is equal to:

$$B = De^{-rT} - put(V, D, \sigma_V) = B^* - put(V, D, \sigma_V) \qquad [10]$$

where $B^* = De^{-rT}$ is the present value of the risk-free debt and $put(V, D, \sigma_V)$ is the present value of the expected loss for the bond holder.

The Merton model allows to understand that, in the economic (full fair value) balance sheet, changes in assets are directly linked to changes in the market value of equity and the expected losses to creditors. In the economic (full fair value) balance sheet framework, a decline in the value of assets leads to a less than proportional decline in the market value of equity; the amount of change in the value of equity depends on the degree of financial distress affecting

[2] Furthermore, M&M (1958) pointed out that the cost of equity may even start to decrease when the leverage increases significantly. They noted that this phenomenon may characterize "*companies whose earnings prospects have fallen substantially since the time when their debts were issued*".

[3] See Appendix 3 for a brief description of the model.

the bank. The decline in bank assets values simultaneously leads to an increase in the value of expected losses to creditors. The amount of the increase can be large when banks are in severe financial distress. In the economic (fair value) balance sheet of the bank, the "expected loss to bank creditors" relates to the total debt and deposits on the full bank balance sheet[4].

It is important to point out that the economic/fair value bank balance sheet and the traditional accounting bank balance sheet are related. The accounting balance sheet can be "derived" as a special case of the economic balance sheet — the case where uncertainty is set to zero (i.e., bank's assets have no volatility and, consequently, no risk). With zero volatility (risk) on the balance sheet, the expected loss to bank creditors goes to zero and equity becomes book equity. The "risk exposure" becomes zero (Gray *et al.*, 2011). The Merton model allows to derive the analytical values of the required costs of equity (r_E) and debt (r_B) without any approximations.

As previously recalled, the second M&M proposition indicates that the weighted average cost of capital, or the expected rate of return on total assets, is independent of the capital structure. To keep the cost of capital constant as the leverage is changed requires that either the cost of debt or the cost of equity or both change with the capital structure. Using no arbitrage conditions and Ito's Lemma it is easy to show that the excess expected returns on equity, ($r_E - r$), and debt, ($r_B - r$), are given respectively by:

$$r_B - r = \eta(V, D, \sigma_V)(r_V - r) \quad [11]$$

$$r_E - r = \left(1 + \frac{B}{E}\left[1 - \eta(V, D, \sigma_V)\right]\right)(r_V - r) \quad [12]$$

where $\eta(V, D, \sigma_V) = (\partial B/\partial V) \times (V/B)$ is the elasticity of bank's debt to the underlying value of its assets.

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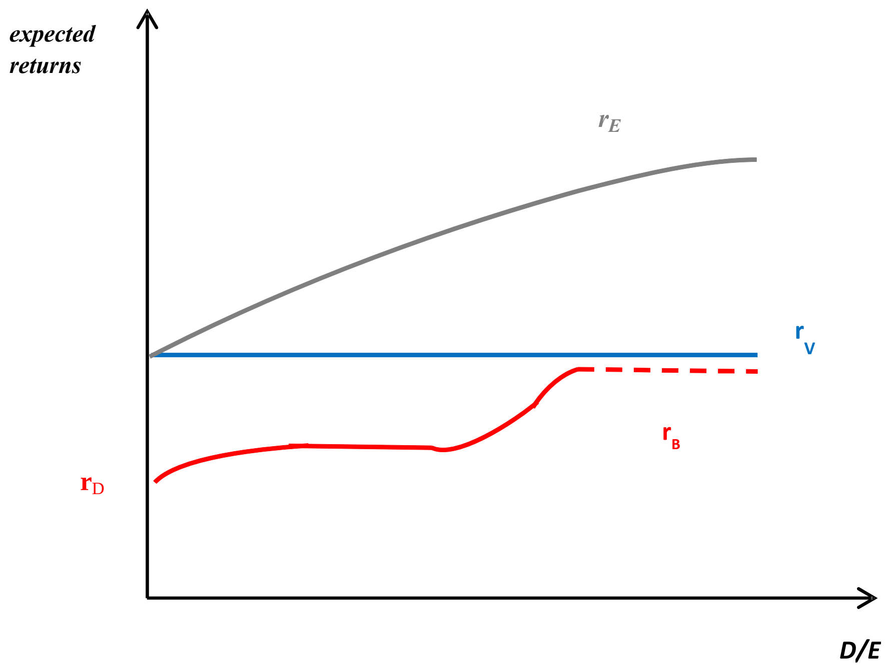
4 F This expected loss to bank creditors can be viewed as the probability of bank default (PD), times a loss given default (LGD), times an "exposure" (EAD) represented by the default free value of the bank's total debt and deposits. The expected loss to creditors is a "risk exposure" in the economic balance sheet. Formally the Expected Loss (EL) can be written as: PD*LGD*EAD.

As we recalled, in the standard M&M textbook case debt is assumed to be riskless, the cost of debt is constant, and the cost of equity is linear in the market debt-to-equity ratio (equation [9]). But we can see from [12] that this is not a general result. The debt is riskless only when its elasticity to the value of total assets, $\eta(V, D, \sigma_V)$, is zero. Similarly, when default risk is explicitly considered, the cost of equity capital is an increasing, concave, unbounded function of the market debt-to-equity ratio of the firm. At a debt-to equity ratio of zero, the cost of equity capital is equal to the firm's cost of capital, r_V. For a small addition of debt, the increase in the marginal default risk is negligible, so the equity risk and, therefore, the premium on the cost of equity capital will be increased in proportion to the debt-to-equity ratio. In other words, just as in the textbook case, the slope of the equity cost of capital curve at a debt-to-equity ratio of zero is $r_V - r$, the risk premium on the bank. In this framework the cost of debt capital is an increasing, *S*-shaped, function of the market debt-to-equity ratio of the firm. At a debt-to-equity ratio near zero, the cost of debt is equal to the risk-free rate and has a slope of zero. It is bounded above by the firm's cost of capital. In fact, in the special case of a full equity capital structure, equity holders earn exactly the rate of return on assets. They are the only claimants on assets.

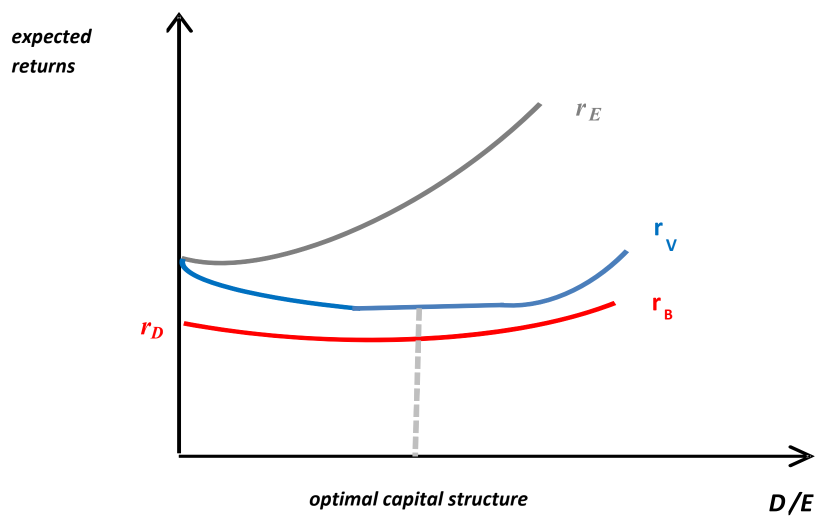
Figure 6: Equilibrium expected returns in the Merton model



Before concluding this section it is important to stress that the enterprise value of a banking firm depends primarily on future expected revenue flows and how the related, non-diversifiable, risks are assessed by market operators (see section…). But this does not necessarily imply that the enterprise value is totally divorced from the capital structure. The theoretical arguments developed to build the so-called "trade-off" model are based on the relaxation of the main simplifying assumptions behind the M&M theorems. M&M in 1963 expanded their original model to allow for corporate income tax (but no other taxes). Once taxes are factored in, there is a certain advantage to issue, within limits, more debt. Furthermore, as previously pointed out, in the case of banking firms, another important complicating factor is represented by bank deposits, which share a strict money character and therefore require in equilibrium a lower, ex-ante, cost of funding with respect to bonds.

More fundamentally, three other factors should be taken into account: information asymmetries, agency costs and incentives, financial distress. Trade-off models show that when these points are factored in, the capital structure has a, limited, role in explaining the bank's value. The message of M&M remains true: the present value of tax and other savings arising from more debt are rapidly overcome by the increase in the present value of financial distress and possible bankruptcy cost (Vernimmen , 2005). A full analytical analysis of these problems is beyond the scope of this paper.

Figure 7: Equilibrium expected returns in a "trade-off model"



2.3 The cost of a capital increase for equity holders and the risk-shifting problem: a formal proof

We address here the fundamental reason why the M&M's conclusions may not hold: risk-shifting. In the Merton model the equity value, *E*, can be defined as[5]:

$$E = call(V, D, \sigma_V) = V - B = V - De^{-rT} + put(V, D, \sigma_V) \quad [13]$$

The value of the put embedded in equity prices represents the "right to default" for the equity holders deriving from the limited liability nature of the contract.

Suppose now that equity holders are obliged to reduce bank's leverage by buying back an amount of debt equal to Δ. They are worse off issuing securities to recapitalize the firm and reduce their outstanding debt. Formally their loss is equal to:

$$put(V, D - \Delta, \sigma_V) - put(V, D, \sigma_V) \leq 0 . \quad [14]$$

Thus the Merton framework allows to derive an easy proof that raising equity, particularly if the equity market signals conditions of stress (by pricing the stock well below book value), represents, in general, a transfer of wealth from existing equity holders to bondholders.

It s important to point out that equity holders can recover their losses (completely and/or in part) by increasing the volatility of their business. In response of the "forced" transfer of wealth deriving from capital injection, equity holders' best (value maximizing) strategy is to gamble bank's assets on high-risk investments. If the gamble pays off, the equity holders receive everything left after the bondholders are paid. This is the well-known risk-shifting problem, and it is more relevant when default risk is high (i.e. price-to book is low). Formally the new volatility, σ_V^*, that allows to eliminate completely the loss for the equity holders is found by solving the following equation:

$$put(V, D - \Delta, \sigma_V^*) - put(V, D, \sigma_V) = 0 , \quad [15]$$

[5] By using the put-call parity.

with $\sigma_V^* > \sigma_V$. Equity holders can "recover" their losses, vis-à-vis bondholders, by increasing the risk of banks' assets. The more the new capital injection is expensive for equity holders (this happens when price-to-book ratios are significantly below one) the more the risk-shifting phenomenon may be relevant. This means that higher capital requirements may produce an unintended consequence by increasing the overall risk of the bank[6]. Therefore, this result has important implications for the applicability of the M&M' theorem to banks. Higher capital requirements may easily induce equity holders to change the risk/return profile of their assets. These results are even more relevant if government guarantees are factored in[7] (see next sub-section) for a formal proof.

Furthermore, since higher capital requirements are imposed simultaneously on the whole banking system risk shifting behavior may have important, unintended, consequences in terms of systemic risk. Higher capital requirements are generally invoked to reduce systemic risk. Admati et al. (2012) argue that the costs of capital injections are private costs that should not be considered by the regulators. Since banks' high leverage is a source of systemic risks and imposes costs on the public, they conclude that resistance to leverage reduction leads to social inefficiencies.

In the simple framework that we use here the negative externalities posed by a systemic bank may be considered as a "digital option" which is in-the-money (i.e. produces systemic losses) when the bank defaults because of its excessive leverage, $SC(V,D,\sigma_V)$. Since the social costs increase with the bank's probability of default and, hence bank's leverage, we can formally write:

$$\frac{\partial SC(V,D,\sigma_V)}{\partial D} > 0\,. \qquad [16]$$

A reduction in the leverage reduces the value of this negative externality, by apparently confirming the "conventional wisdom" that higher leverage in the banking system increases systemic risk. But, as we previously showed, equity holders, which are value maximizing agents,

[6] These results are easily confirmed also in a more general model which includes bankruptcy costs. We do not explicatively present this generalization to keep the analysis as simple as possible.
[7] As pointed out by the "Squam Lake Group" (2013).

can "recover" their losses by increasing volatility by solving equation [15]. This means that in some extreme cases (i.e. when the magnitude of the risk-shifting phenomena is non-negligible) we could have the unintended consequence that the overall systemic cost associated to a systemic bank's default can increase when:

$$SC(V, D-\Delta, \sigma_V^*) > SC(V, D, \sigma_V)\,. \qquad [17]$$

In this extreme case the overall, endogenous, risk of the system has increased.

A strict and prompt supervision may mitigate, at least in part, the threat of these unintended consequences associated to new capital injections. However, in case of severe stress it seems difficult to eliminate completely this risk shifting incentive

2.4 Bank's debt and government guarantees

Governments have assumed the responsibility of explicitly protecting banks' depositors and, implicitly for systemic banks, banks' bond holders. There are four types of guarantees: the first three supplied by the government and the fourth supplied by the central bank: a) guarantees on deposits, b) guarantees on bond debt (term debt), c) guarantees on equity and d) the discount facilities and credit lines of last resort at subsidized interest rates of the Central Bank (see TARP and LTRO). These guarantees increase the value of bank and hence reduce the overall cost of debt. They must be considered both in accounting[8] and value-based balance sheets even if their accounting representation is difficult.

Figure 8: Accounting Balance sheet with Government Guarantees (GG)

ASSETS	LIABILITIES
A	***K***
GG(V,D, σ)	***D***

[8] Note that the value of the guarantees should depend from the market/fair value of bank' also for accounting purposes.

Figure 9: Financial value Balance sheet with Government Guarantees (GG)

ASSETS	LIABILITIES
V	E
$GG(V,D,\sigma)$	B

The empirical relevance and the distortions created by these guarantees are well-documented[9]. Higher capital requirements - and hence lower debt - reduce the value of public guarantees that can be exploited by banks. These contingent guarantees affect the whole value of the bank and violate the M&M proposition on the neutrality of the capital structure. This means that the M&M proposition on the irrelevance of the capital structure must be relaxed, at least in part. As already indicated, these points had been recognized by Miller (1991[10] and 1995) himself, but have been overlooked by many recent followers of his approach.

The government guarantees on banks' liabilities (explicit, but not fully priced, on deposits, implicit on bonds and in some extreme cases on equity) allow to obtain debt financing advantages (i.e. banks pay lower costs on liabilities)[11]. This "cheap funding" determines an immediate transfer of resources from governments to banks' equity holders that can be also analytically represented as a put on the bank's assets. The put option is written by governments without receiving any fee in exchange from banks' equity holders[12]. A wrong incentive/moral

[9] The evidence accumulated on these points is overwhelming and comes from academic and official sources. Reference is made to Haldane (2010), Baglioni and Cherubini (2010, Estrella e Schich (2011), Moody's (2011), Noss and Sowerbutts (2012), Schich and Lindh (2012), Ueda and Weder (2012), Gray (2012), Tsesmelidakis and Merton (2013).

[10] In his Nobel lecture Miller pointed out that: "*Because the deposit guarantees gave the owners of the S&L's what amounted to put options against the government, they actually encouraged the undertaking of uneconomic long-odds projects, some of which made junk bonds look safe by comparison. The successes went to the owners; the failures, to the insurance fund*".

[11] The importance of the government guarantees (explicit/implicit) in cost of funding for large European universal banks has been recently stressed by the Liikanen report (2012).

[12] Governments do not explicitly report the value of the contingent liabilities on their balance sheets, which are not based on fair value principles. In case of a negative shock affecting banks' assets the governments' implicit guarantees increase. This, in turn, makes governments' financial positions worse, creating a compounding effect, which may result in the governments' failure to honor their guarantee obligations and cause a collapse of the

hazard problem inevitably arises. In fact, by considering the value of the governments' guarantees, the total value of the bank is given by:

$$V + GG(V, D, \sigma_V), \quad [18]$$

where $GG(V, D, \sigma_V)$ is the free government put on the bank's assets. We note that:

$$\frac{\partial GG(V, D, \sigma_V)}{\partial D} > 0. \quad [19]$$

The higher is the value of the debt (assuming that the government supplies an implicit full guarantee on the whole face value of bank's debt, *D*[13]) the higher is the value of the free put, the higher is the value of equity, the higher is the transfer of resources from government to equity holders.

By imposing a reduction of the leverage with higher capital requirements this guarantee may be reduced. But, as indicated, a wrong incentive problem arises: risk shifting behavior by the equity holders can amplify the overall risk of the bank by increasing again the value of the government guarantees /contingent liability.

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banking system. The analytical framework proposed here can be used to understand the risk transmission channels and the vicious circle that we are experimenting in Europe between sovereigns and financial industry.

[13] In many cases, notably in Europe, governments have guaranteed the full value of banks' assets by covering banks' losses through expensive recapitalizations (not priced at fair value), which have contributed to worsen their fiscal positions and have determined a net transfer of resources from tax payers to banks' equity holders and bondholders.