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Abstract

This study proposes an analysis of the main drawbacks emerged when adopting the traditional short rate dynamics under extreme market conditions such as under negative interest rates.
In fact, this condition has led to invalidate the use of the majority of the most popular stochastic differential equations (SDE) reported in the scientific literature.
The first part of the paper makes an overall introduction to the problem, analyzing it from different perspectives. Given that the author dealt with these topics in previous research items, the objective of the paper is to focus on only one of these aspects which was not scrutinized before and to examine it in full detail.
As a result, the most popular stochastic dynamics are shown in the second part and the problems raised by their numerical integration are then discussed.
Starting from the equations for which it is feasible to implement a numerical scheme for their solution, the problem thus becomes how to find a reasonable estimation for the SDE parameters. The third section deals with the problems occurred in the application of Maximum Likelihood Estimation (MLE) caused by the negative interest rates during the implementation of the well-established approaches. For every analyzed dynamics a real market case is provided.
The paper highlights the appropriateness of the Hull-White model which can be considered a feasible and reliable solution for simulating short rates also under extreme market conditions.

Key Words: negative interest rates, short rate models, short rate dynamics calibration, stochastic differential equation, interest rates simulation

1 Introduction

The interest in the analysis and discussion of the problems related to the advent of negative interest rates (present for more than four years in the Eurozone) was rekindled when the main info-providers officially announced that they would adopt the same conventions in estimating the fair value and risk measures even for deals denominated in the USD currency.
Editors of the most important worldwide newspapers, including the Financial Times, took an interest in the consequences that this phenomenon can entail, interviewing professionals and academics in the sector who had already dealt with this problem for the Eurozone.
As already discussed in 2017 at ABI Risk and Supervision [14] and, more recently, in the webinar "Valuation risk management in the time of the crisis" (July 14, 2020), many of the problems introduced by negative interest rates in the field of quantitative finance have already been analyzed. The main considerations are summarized below, referring for each of them to the related studies already published:

- *Anomalies in the shape of interest rates term structure*

The particularly low level of interest rates has led to anomalies in the shape of the term structure and introduced strong discontinuities mainly due to the high illiquidity recorded in the financial markets. This phenomenon has led to problems in modeling the term structure used for simulations and what-if scenarios in the field of risk management and in the actuarial context [7]. The most popular traditional regressive models used for this purpose are: Nelson-Siegel (1987) [27], Svensson (1994) [22] and de Rezende-Ferreira (2011) [12]. The anomalies in the shape of the interest rates term structures worsened the statistical fit and introduced numerical instability in the estimation of the parameters of the regressive models. In these critical cases, a solution that enables to reach high performance in terms of statistical measures can be found in the implementation of a Machine Learning methodology such as a supervised neural network: a Radial Basis Function [5] or a static feed-forward shallow network [8].

- *Interest rates options*

Before the advent of negative interest rates, the most widespread pricing framework in finance, especially among practitioners and professionals, for the pricing of vanilla options on interest rates (Caps and Floors) was undoubtedly the log-normal one.
All the main info-providers provided modules to traders for calculating the price and estimating the Greeks in accordance with Black's reference model. With the particular critical conditions of today's markets, this pricing framework cannot work due to the impossibility of calculating the terms of the formula for caplets and floorlets that involve the logarithmic function (referring to traditional notations: d_1 e d_2) [4].
Therefore, for more than four years in the Eurozone and for only a few months for derivatives denominated in USD, the default pricing model for standard options written on interest rates (i.e. Black log-normal model) has been replaced with the normal model through the Bachelier closed formula. Due to the change of model made by all the main info-providers, including Bloomberg®, a valuation and pricing risk was indoubtly introduced.

- *Hedging gap*

A logical consequence of replacing the valuation model is the discrepancy in the Greeks estimation in accordance with the new reference pricing framework. Recalling that the Greeks are computed by estimating the derivatives of the price with respect to a key parameter of the theoretical valuation model, it is clear that if the fair value estimation framework is replaced, the same change will be reflected on the sensitivity measures, regardless of whether these derivatives can be analytically calculated using a closed formula or numerically adopting an integration scheme, such as a finite difference method or a lattice approach. The study [16] shows both the analytical derivation of the Greek for the more important interest rate derivatives and how their replacement impacts from a hedging perspective.

- *Changing the implied volatility surfaces*

Implied volatility surfaces are an essential tool for the OTC interest rate options valuation in line with the future expectations of financial markets, as well as being a tool used by traders for their investment choices [15]. From a quantitative point of view, they are derived from the numerical inversion of the theoretical pricing formula using a goal seeking algorithm: starting from the premiums actively quoted on the market, the implicit volatility is obtained. It is clear that this calculation depends on the assumptions made on the underlying valuation model, therefore the reference volatility surfaces have changed, leading to new market quotes conventions (from percentage to basis points), the need for a new sensitivity to the data by the market agents, a different data pipe-line feeding the automatic pricing systems, ...

- *Surface Reconstruction*

Under the hypothesis of continuing to use the same log-normal pricing model (for example for contractual constraints related to deals signed decades ago), the problem of managing missing data within the implied volatility surfaces becomes a priority.
Since it is not possible to perform the numerical inversion of the pricing formula deriving from the log-normal framework for options with underlying negative rates, the data missing problem is not a local problem, for which there are various statistical methods that are easy and quick to implement (such as bidimensional interpolation), but a global one: it involves the reconstruction of entire sections of the surface [9]. The surface reconstruction problem can be solved, for example, by implementing machine learning techniques. Paper [10] shows an example of a surface reconstruction for the ATM implied volatilities in application of swaptions pricing through a Non linear Principal Component Analysis (NLPCA) carried out using an Artificial Neural Network (AANN) Autoencoder.

- *Violation of fundamental properties for American call options written on an equity underlying with no pay-out*

The advent of negative interest rates leads to the invalidation of many theoretical and mathematical properties related to options, including a property heavily used in the early exercise valuation. Under the condition of negative interest rates, the value of an American call, written on an underlying that does not pay dividends, is no longer equal to its European counterpart [6]. The majority of commercial calculation routines used for the evaluation of standard American options are based on a set of pricing formulas, known in the literature as quasi-closed formulas. The most accurate approximated formula is the Bjerksund-Stensland 2002 formula. These approaches carry out an a-priori check on the value used for the continuous dividend yield: if this is not positive, they return the corresponding value of the European option. This control does not take into account that the optionality due to its possible early exercise has, in interest rate negative environments, a potentially non-zero value.
Therefore, as discussed in the latest FINMAB (Finance MathWorks Advisory Board) in Frankfurt, in order to adjust the valuation taking into account this extreme condition, it is necessary to take into consideration, regardless the pay-out of the underlying, a numerical techniques such as stochastic trees or a free-boundary PDE able to correctly price the early exercise feature.

The purpose of this article is to add to the scientific contributions reported in this brief introduction by examining the problems emerged in the prospective simulations of short term rates after the advent of negative interest rates.
We will therefore analyze the most widespread stochastic differential equations in the literature that allow to perform this task and we will discuss the potential problems that make them unusable in the current anomalous financial context and, once a subgroup has been identified for which it is possible to apply a numerical integration scheme, we will proceed to analyze the process of estimating the parameters of these dynamics.
After having re-examinated the calibration techniques derived by the application of the maximum likelihood method on the observed market data, we will discover that not all the SDEs of the subgroup for which the application of a numerical integration scheme was feasible lead to consistent or admissible results.
The Hull-White model reveals to have robust dynamics both in the numerical integration part and in the tuning of its characteristic parameters during the market calibration phase.

2 Short rate dynamics

The most widespread stochastic differential equations for the simulation of the short rate are:

[V] Vasicek [23]

$$dr_t = k[\Theta - r_t]dt + \sigma dW_t \quad (1)$$

[D] Dothan [13]

$dr_t = ar_t dt + \sigma r_t dW_t$ (2)

[CIR] Cox – Ingersoll – Ross [11]

$dr_t = k[\Theta - r_t]dt + \sigma\sqrt{r_t}dW_t$ (3)

[EV] Exponential Vasicek [3]

$dr_t = r_t[\eta - a \ln r_t]dt + \sigma r_t dW_t$ (4)

[HW] Hull – White [17]

$dr_t = k[\theta_t - r_t]dt + \sigma dW_t$ (5)

[BDT] Black – Derman – Toy [1]

$d \ln r_t = \left[\theta_t + \frac{\sigma_t'}{\sigma_t} \ln r_t\right] dt + \sigma_t dW_t$ (6)

[BK] Black – Karasinski [2]

$dr_t = r_t[\eta_t - a \ln r_t]dt + \sigma r_t dW_t$ (7)

[MM] Mercurio – Moraleda [20]

$dr_t = r_t \left[\eta_t - \left(\lambda - \frac{\gamma}{1+\gamma t}\right) \ln r_t\right] dt + \sigma r_t dW_t$ (8)

The mathematical notations used in the equations are the same adopted by Brigo-Mercurio [3]. With the reference of the EURIBOR time-series, the level of the short rate, r_t is deeply negative.

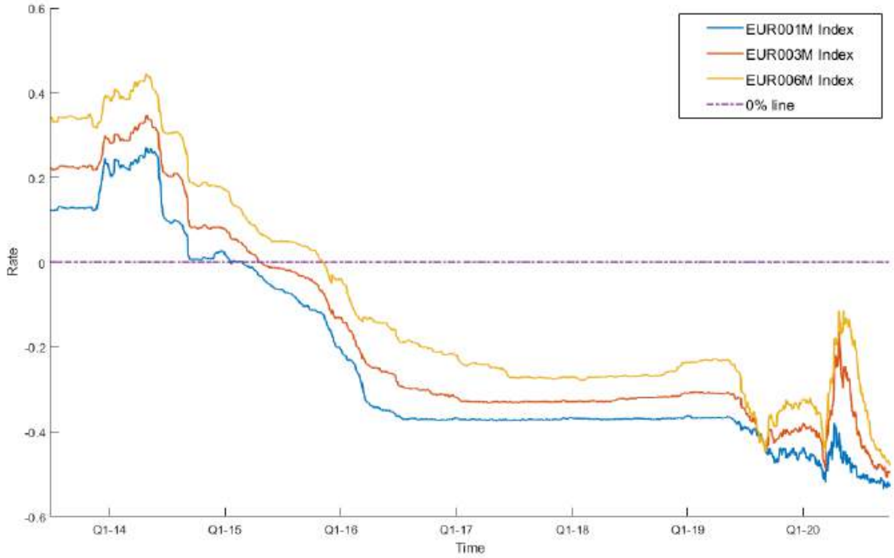

Figure 1. EURIBOR time series. Tenor: 1 month, 3 months and 6 months – Source: Bloomberg®

The starting value of the rate used for the SDE numerical integration is equal to the level recorded by the market for the initial starting time: $r_{t=0} = r_{MKT}$.
Given that the initial value is negative, the majority of the listed models cannot be used for the simulation of the short-rate. The reason is simple and clear: many dynamics involve the presence of mathematical operators which cannot work with negative real numbers (logarithm and square roots). These correspond to the red terms in the above list.
Among the traditional models, Vasicek [V], Dothan [D] and Hull-White [HW] SDEs can be integrated using a traditional numerical integration scheme (for instance Euler-Maruyama or Milstein method).

3 SDE parameters estimation

The aim of this paragraph is to analyze the most widespread calibration methodologies necessary for the determination of the characteristic parameters for the SDEs which can be numerically solved: Vasicek [V], Dothan [D] and Hull-White [HW] models. The paragraph provides the theoretical and practical evidences for which the HW model has proved to be more suitable to be implemented for conducting prospective short-rate simulations.

In particular, Vasicek calibration fails under low-level rates because the canonical tuning method leads to the estimation of two negative SDE characteristic parameters. As a result the long-run rate will not converge. In the case of the Dothan model, the two parameters which rule the dynamics cannot be found because in the zero-coupon bond closed formula used for pricing, there is a mathematical operator which does not work with negative numbers. The Hull-White model has a bigger flexibility to match the current interest rate structure and this feature allows to calibrate the parameter using a larger set of financial instruments, such as caps, floors, swaptions and zero-coupon bonds as well. As a result, after the preliminary feasibility to the SDE integration, even the parameter estimation phase proves to be robust also in an extreme market context.

3.1 Vasicek calibration problem

In order to estimate the parameters of the stochastic models, we use a deterministic expression for the expected theoretical value for a financial instrument in accordance with the selected dynamics and, therefore, we find the parameters that better fit the quoted realized price for that financial instrument (or set of financial instruments).
Given that the market provides a set of quotes for these instruments, the characteristic parameters of the stochastic model can be calibrated starting from the values observed on the market using the Maximum Likelihood Principle.
In the case of the Vasicek model, there is a set of closed formulas that allow direct tuning of the parameters of the dynamics (θ, k e σ) starting from the rates observed on the market [3], [23].

$$\hat{\theta} = \frac{S_y S_{xx} - S_x S_{xy}}{n(S_{xx} - S_{xy}) - (S_x^2 - S_x S_y)} \quad (9)$$

$$\hat{k} = -\frac{1}{\delta} \ln \frac{S_{xy} - \Theta S_x - \Theta S_y + n\Theta^2}{S_{xx} - 2\Theta S_x + n\Theta^2} \quad (10)$$

$$\hat{\sigma}^2 = \frac{2k[S_{yy} - 2\alpha S_{xy} + \alpha^2 S_{xx} - 2\Theta(1-\alpha)(S_y - \alpha S_x) + n\Theta^2(1-\alpha)^2]}{n(1-\alpha^2)} \quad (11)$$

Where:

$\alpha = \exp(-k\delta)$, $S_x = \sum_{i=1}^{n} r_{i-1}$, $S_y = \sum_{i=1}^{n} r_i$, $S_{xx} = \sum_{i=1}^{n} r_{i-1}^2$, $S_{yy} = \sum_{i=1}^{n} r_i^2$, $S_{xy} = \sum_{i=1}^{n} r_{i-1} r_i$ and δ is the tenor of the rates.

The example below demonstrates how this procedure does not prove to be robust in extreme market conditions.
Consider the 3-month EURIBOR interest rates term as of 30th June 2017 shown in Table 2 and obtained from the market par rates (Table 1). By calibrating the parameters that rule the Vasicek dynamics using the set of equations (9), (10) and (11) we obtain the values: $\hat{\theta} = -0.0436$, $\hat{k} = -0.0257$ e $\hat{\sigma} = 3.1676 \cdot 10^{-4}$

The scientific literature reports that the dynamics model can be considered reliable if all three parameters are positive.
Negative interest rates have led to a negative mean-reversion rate k, which in turn leads to an asymptotic divergence of the short-term rate (Figure 2)

$$r(T) = \exp(-kT) \cdot r_0 + \Theta(1 - \exp(-k \cdot T)) = \exp(0.0257 \cdot T) \cdot (-0.33\%) - 0.0436 \cdot (1 - \exp(0.0257 \cdot T))$$
$$r(T \to \infty) \to +\infty$$

Instrument	Bloomberg® Codifier	Par Rates (Mid Value)	Day Basis
Deposit	EONIA Index	-0.350	ACT/360
Deposit	EUDR2T Index	-0.410	ACT/360
Deposit	EUR001W Index	-0.379	ACT/360
Deposit	EUR002W Index	-0.373	ACT/360
Deposit	EUR001M Index	-0.373	ACT/360
Deposit	EUR002M Index	-0.342	ACT/360
Deposit	EUR003M Index	-0.331	ACT/360
Deposit	EUR006M Index	-0.271	ACT/360
Deposit	EUR009M Index	-0.197	ACT/360
Deposit	EUR012M Index	-0.156	ACT/360
Swap	EUSW2V3 BGN Curncy	-0.202	30/360
Swap	EUSW3V3 BGN Curncy	-0.085	30/360
Swap	EUSW4V3 BGN Curncy	0.038	30/360
Swap	EUSW5V3 BGN Curncy	0.166	30/360
Swap	EUSW6V3 BGN Curncy	0.294	30/360
Swap	EUSW7V3 BGN Curncy	0.427	30/360
Swap	EUSW8V3 BGN Curncy	0.557	30/360
Swap	EUSW9V3 BGN Curncy	0.681	30/360
Swap	EUSW10V3 BGN Curncy	0.796	30/360

Table 1. Market Yield curve. Tenor: EURIBOR 3 months. Reference Date: 30th June 2017 (Source Bloomberg)

Start Date	End Date	Zero Rates	Discount Factors
06/30/2017	07/03/2017	-0.35423738	1.000029168
07/03/2017	07/04/2017	-0.369389988	1.000040557
07/04/2017	07/11/2017	-0.378395184	1.00011426
07/04/2017	07/18/2017	-0.375692917	1.000185639
07/04/2017	08/04/2017	-0.376596976	1.000361868
07/04/2017	09/04/2017	-0.347653817	1.000629928
07/04/2017	10/04/2017	-0.336602074	1.000887196
07/04/2017	01/04/2018	-0.276594944	1.001427645
07/04/2017	04/04/2018	-0.20213019	1.001542258
07/04/2017	07/04/2018	-0.160458749	1.001624793
07/04/2017	07/04/2019	-0.202868958	1.004092105
07/04/2017	07/06/2020	-0.08604541	1.002602363
07/04/2017	07/05/2021	0.036960075	0.998516897
07/04/2017	07/04/2022	0.165512383	0.991742847
07/04/2017	07/04/2023	0.29475367	0.982456191
07/04/2017	07/04/2024	0.429774866	0.970357931
07/04/2017	07/04/2025	0.5629423	0.955996167
07/04/2017	07/06/2026	0.69109287	0.939755663
07/04/2017	07/05/2027	0.811061357	0.92225464

Table 2. Interest rates term structure. Zero Rates are stripped using market par-rates reported in Table 1

The parameter that regulates the mean-reversion, k, is negative and constant: in the presented case it produces an asymptotic divergence of the short-term rate which cannot be reached at the long-term value θ [14].

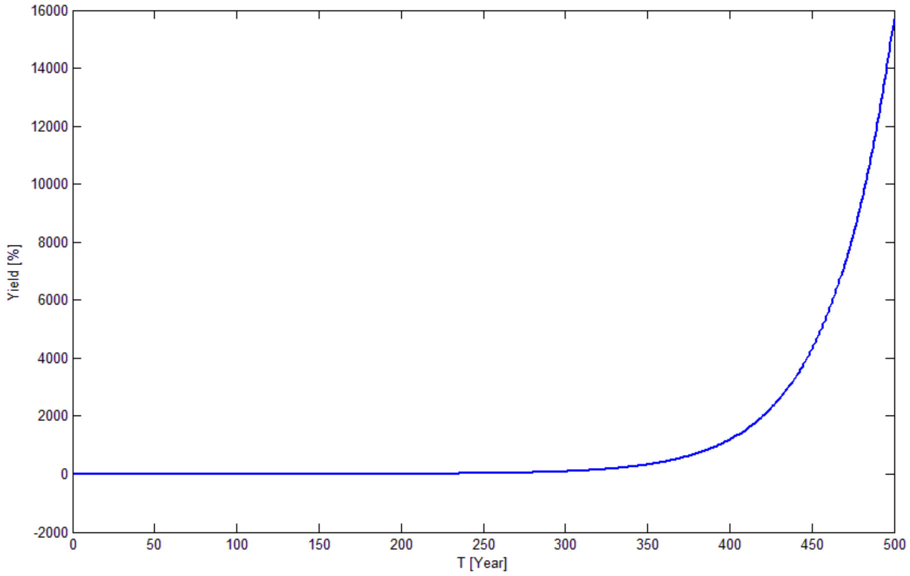

Figure 2. Divergence of the short rate in the long run caused by negative interest rates

3.2 Dothan calibration problem

Dothan's model is the only log-normal short rate model to have an analytical formula for pure zero-coupon bonds, $P(t,T)$ [3], [13].

$$P(t,T) = \frac{\overline{r^P}}{\pi^2}\int_0^\infty \sin\left(2\sqrt{\bar{r}}\sinh(y)\right)\int_0^\infty \left(f(z)\sin(yz)dzdy\right) + \frac{2}{\Gamma(2p)}\overline{r^P}K_{2p}\left(2\sqrt{\bar{r}}\right) \quad (12)$$

Where:

$f(z) = \exp\left[\frac{-\sigma^2(4p^2+z^2)(T-t)}{8}\right] z \left|\Gamma\left(-p+i\frac{z}{2}\right)\right|^2 \cosh\left(\frac{\pi z}{2}\right)$, $\bar{r} = \frac{2r(t)}{\sigma^2}$, $p = \frac{1}{2} - a$ and K_q denotes the modified Bessel function of the second kind of order q.

Its calibration therefore takes place using the maximum likelihood method starting from the set of zero-rates observed by the term structure of the reference rate.
As already observed in the term structures shown in Table 2, a significant sample of rates that can be used for calibration is negative, therefore the closed formula from which it is possible to calculate the value of a zero-coupon in accordance with the Dothan model is not applicable [14].

In fact, if $r(t) < 0$ then $\bar{r} = \frac{2r(t)}{\sigma^2} < 0$ and, consequently, the amount in the squared root cannot be computed.

3.3 Hull – White calibration

Compared to Vasicek's model, the dynamics proposed by Hull-White sets Θ as a function of time.
This assumption allows on the one hand to significantly improve the fit of the interest rates term structure and on the other hand to have a greater range of financial instruments that can be used for the tuning of the stochastic differential equation parameters [18], [19].
In fact, the parameters of the model can be calibrated starting from the premiums of the most popular and listed options linked to the interest rate such as caps and floors.

According to this dynamics, the short-rate can be represented as follows [3]:

$$\begin{cases} dr = [\Theta(t) + \alpha r]dt + \sigma dW_t \\ \Theta(t) = \frac{\partial f^M(0,t)}{\partial T} + \alpha f^M(0,t) + \frac{\sigma^2}{2\alpha}(1 - \exp(-2\alpha t)) \end{cases} \quad (13)$$

Where α and σ are the mean reversion and the volatility of the model, respectively, $\Theta(t)$ is chosen in order to exactly adapt the interest rate term structure on the evaluation date and $f^M(0,t)$ are the instantaneous market forward rates at time 0 for the maturity t.

Eq. (13) can be discretized using a trinomial stochastic tree, which is commonly used for pricing exotic instruments: this integration scheme has the parameters α e σ and have to be tuned in accordance with the market prices for cap and floor [3].

The analytical tractability of the model allows to derive a closed formula that expresses the theoretical value of the caps and floors: this is an interesting property as it allows greater speed in parameter tuning without necessarily having to code a recursive routine call of the stochastic tree itself.

$$Cap(t,\tau,N,X) = N\sum_{i=1}^{n} P(t,t_{i-1})\Phi\left(-h_i + \sigma_P^i\right) - (1 + X\tau_i)P(t,t_i)\Phi(-h_i) \quad (14)$$

$$Floor(t,\tau,N,X) = N\sum_{i=1}^{n}(1 + X\tau_i)P(t,t_i)\Phi(h_i) - P(t,t_{i-1})\Phi\left(h_i - \sigma_P^i\right) \quad (15)$$

With:

$$\begin{cases} \sigma_P^i = \sigma\sqrt{\frac{1-\exp[-2\alpha(t_{i-1}-t)]}{2\alpha}}B(t_{i-1},t_i) = \sigma\sqrt{\frac{1-\exp[-2\alpha(t_{i-1}-t_i)]}{2\alpha}}\frac{1}{\alpha}\{1-\exp[-\alpha(t_i-t_{i-1})]\} \\ h_i = \frac{1}{\sigma_P^i}\ln\left[\frac{P(t,t_i)(1+X\tau_i)}{P(t,t_{i-1})}\right] + \frac{\sigma_P^i}{2} \end{cases}$$

Where:

X is the strike of the option
N is the nominal value
τ is the year fraction between each caplet (/floorlet)
$P(t,t_i)$ is the discount factor
Φ is the standardized normal cumulative distribution function
α and σ are the mean reversion and the volatility of the Hull-White tree to find

This model allows to deal with negative interest rates both in the integration phase of the stochastic differential equation and in the parameter estimation phase.
A calibration is carried out starting from the normal implied volatility of caps quoted by the market on 30th June 2017 (Figure 3).

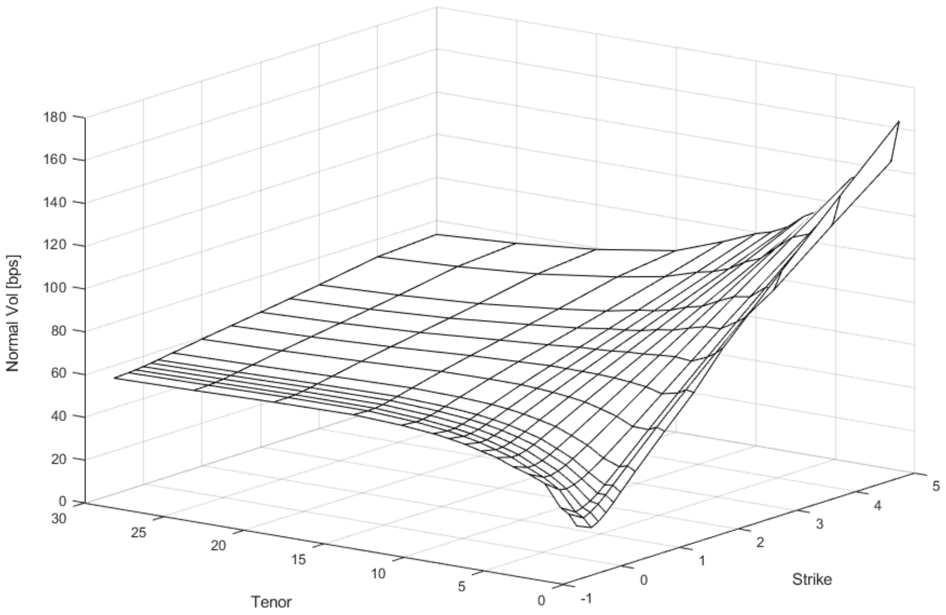

Figure 3. Cap Normal Implied Volatility. Reference Date: 30th June 2017. Source: Bloomberg®

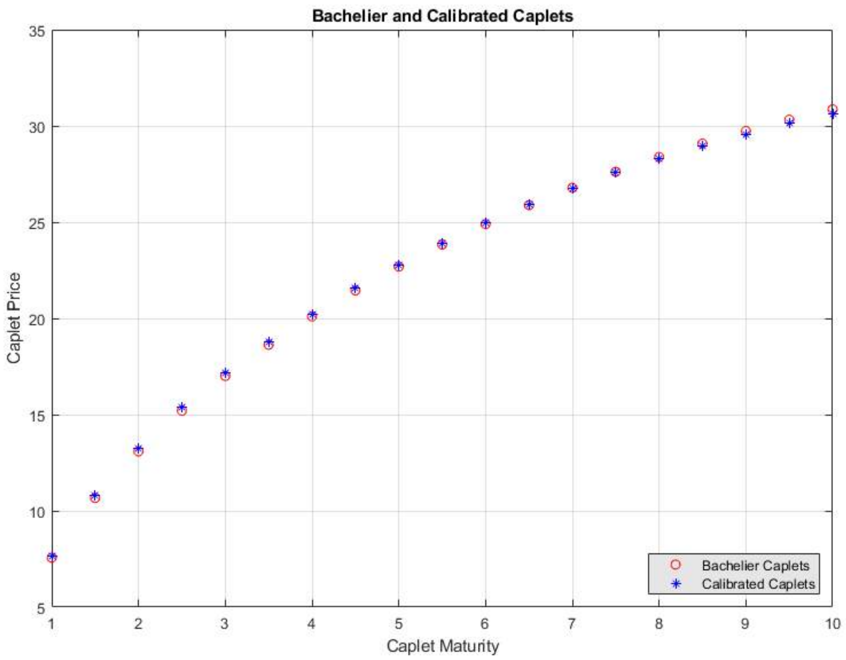

Figure 4. Hull White parameter estimation

In the case of the Hull-White model, no anomalies were found in the set of pricing formulas used to calibrate the dynamics. These considerations make the HW a robust and reliable model even in a context of negative rates [14].

4 Conclusions

The article highlighted some critical issues introduced by negative interest rates in the most popular models for representing the short rates.
The problems caused by these were found not only in the definition of the descriptive dynamics of motion, but also in the calibration procedure of the SDE characteristic parameters.
The Hull-White dynamics proved to be robust and reliable: although the analysts in the 90's made several criticisms about this model, since it did not guarantee the positivity of rates in future simulations, in the anomalous current market context it must be strongly taken into consideration precisely for this aspect.

Pier Giuseppe Giribone

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