RISK MANAGEMENT MAGAZINE

Vol. 16, Issue 2
May – August 2021

EXCERPT

https://www.aifirm.it/newsletter/progetto-editoriale/

Certificate pricing using Discrete Event Simulations and System Dynamics theory

Pier Giuseppe Giribone, Roberto Revetria

Certificate pricing using Discrete Event Simulations and System Dynamics theory

Pier Giuseppe Giribone (University of Genoa, Banca Carige), Roberto Revetria (University of Genoa)

Article submitted to double-blind peer review, received on 1st July 2021 and accepted on 23rd July 2021

Abstract
The study proposes an innovative application of Discrete Event Simulations (DES) and System Dynamics (SD) theory to the pricing of a certain kind of certificates very popular among private investors and, more generally, in the context of wealth management.
The paper shows how numerical simulation software mainly used in traditional engineering, such as industrial and mechanical engineering, can be successfully adapted to the risk analysis of structured financial products.
The article can be divided into three macro-sections: in the first part a synthetic overview of the most widespread option pricing models in the quantitative finance branch is given to the readers together with the fundamental technical-instrumental background of the implemented DES and SD simulator.
After dealing with some of the most popular models adopted for Equity and Equity index options, which are the most common underlying assets for the certificates structuring, we move, in the second part, to describe how the mathematical models can be integrated into a general simulation environment able to provide both DES and SD extensively used in the engineering field.
The core stochastic differential equation (SDE) will therefore be translated, together with all its input parameters, into a visual block model which allows an immediate quantitative analysis of how market parameters and the other model variables can change over time.
The possibility for the structurer to observe how the variables evolve day-by-day gives a strong sensitivity to evaluate how the price and the associated risk measures can be directly affected.
The third part of the study compares the results obtained from the simulator designed by the authors with the more traditional pricing approaches, which consist in programming Matlab® codes for the numerical integration of the core stochastic dynamics through a Euler-Maruyama scheme. The comparison includes a price check using the Bloomberg® DLIB pricing module and a check directly against the valuation provided by the counterparty.
In this section, real market cases will therefore be examined with a complete quantitative analysis of two of the most widespread categories of certificates in wealth management: Multi-asset Barrier Reverse Convertible with Issuer Callability and Multi-asset Express Certificate with conditional memory fixed coupon.

Key Words:
Discrete Event Simulation (DES), System Dynamics (SD), Structured Products, Certificate pricing, Barrier Reverse Convertible, Express Certificate, Financial Engineering, Wealth Management

1) A short time-travel in the Option Pricing theory

Modern Option Pricing theory starts in 1973 when Black and Scholes (1973) and Merton (1973) proposed the well-known Nobel-prize formula, which is considered one of the founding blocks of Quantitative Finance. According to Stewart (2013), this should undoubtedly be considered one of the seventeen equations that changed the world but it is for sure not the first attempt to try to price options. Curious readers may be asking how people used to price these derivatives before the Black-Scholes-Merton (BSM) pricing framework (Haug, 2007).
As early as 1900, Luis Bachelier published the first study about this topic (Bachelier, 1900). In contrast to the BSM theory, he assumed a normal distribution for the asset price: in other words, an arithmetic Brownian motion process was adopted to modelize the underlying dynamic. This implies that there is a positive probability to observe a negative asset price: a feature that cannot be found for any limited liability assets.
With the presence of negative price levels for interest rates, the log-normal Black formula for pricing options on interest rates (caps and floors) has to be switched to a normal pricing framework model. This recent abnormal context allows to reconsider the work of Bachelier both in the academic scientific community (among others Burro *et al.*, 2017; Giribone, Ligato, and Mulas, 2017) and in specialized professional magazines (Kochkodin, 2019; Stafford, 2020). It is worth noting that negative interest rates also affect options with early-exercise features although in a less invasive way. In the case of a call option written on an underlying that has no pay-out, the traditional theory states that it is not convenient for the holder to exercise it before maturity (Hull, 2018). In other words, the early exercise feature is worth zero. This property is valid only in a standard financial context in which rates are positive. This fact has gain interest both among academics (Cafferata, Giribone, and Resta, 2017) and professionals (FINMAB, 2019).
After Bachelier, other less popular mathematical models have been proposed. In the following literary review, more relevant studies that led to the well-known BSM formula will be mentioned (Haug, 2007). Sprenkle assumed the stock price was lognormally distributed and thus the asset price followed a geometric Brownian motion, just as in the Black-Scholes analysis (Sprenkle, 1964). In this way, he ruled out the possibility of negative stock prices, consistently with limited liability. Moreover, the Sprenkle model allowed for a drift in the stochastic differential equation, thus taking into account positive interest rates and risk aversion (Smith, 1976). He also assumed today's value was equal to the expected value at maturity.
Boness (1964) and Samuelson (1965) also assumed a lognormal asset price and they were able to derive a closed formula for a call option that looks like the BS formula. In contrast to Boness, Samuelson model can take into account that the expected return from the option is larger than that of the underlying asset. These models can be considered preparatory for the mathematical proof that Black-Scholes-Merton gave eight years later for their formula.
1973 BSM works only for options on a single stock that pays no dividends, it is based on strong hypotheses and it handles only a vanilla (i.e. derivative with a standardized pay-off) European (i.e. the right to exercise the option is only at maturity) option.
It was a challenging task for both researchers and professionals to generalize the BSM formula and enhance it with all the potential fields of improvement:

A) As regards the underlying assets, thanks to the studies of Merton (1973), Black (1976), Asay (1982), Lieu (1990), Garman and Kohlhagen (1983) and Grabbe (1983); Haug (2007) was able to formulate a generalized Black-Scholes-Merton (GBSM) including the cost-of-carry variable in his book for closed formulas option pricing;
B) Regarding the strong efficient market hypothesis under the 1973 BSM model, excluding those that deal with the class of underlyings on which the option is written, many improved models were proposed. These closed-form model categories take the name of "Black-Scholes-Merton Adjustments and Alternatives" (Haug, 2007, chapter 6). The proposed adjustments take into account the following aspects: Delayed settlement (Hull, 2018), trading-day volatility versus calendar day volatility (French, 1984), discrete time hedging (Derman and Kamal, 1999; Wilmott, 2000), transaction costs (Leland, 1985; Hoggard, Whalley, and Wilmott, 1994) and trending markets (Lo and Wang, 1995). The alternative models refer to different hypotheses about the stochastic processes followed by an input (typically underlying and/or volatility). In this context, the most popular variants of the original model are: constant elasticity of variance (Cox and Ross, 1976), BSM adjusted for excess skewness and kurtosis (Jarrow and Rudd, 1982; Corrado and Su, 1996), jump-diffusions (Bates, 1991) and stochastic volatility (Hull and White, 1988, Hagan *et al*., 2002);
C) The original BSM (1973) was derived by the solution of a Partial Differential Equation, called Fundamental BSM PDE (Duffie, 2006) where its boundary and initial conditions are strictly related to the pay-off of the European option. In function of the pay-off complexity, in some cases this PDE can be analytically solved and we have an exact closed-form solution. The most important results are collected by Haug in 2007 (also known as "The Collector" in the eclectic Quant Community) in the bible of pricing closed-form formulas, titled "The Complete Guide to Option Pricing Formulas". This field of research was particularly active in the 90’s during the boom of financial engineering and quantitative finance. Academics, together with professional Quants, were able to derive various ready-to-use closed formulas even for exotic (i.e. non-standard and highly non-linear) and multi-assets pay-off. Among all the formulas, the most popular ones with important practical applications are: Barrier (Reiner and Rubinstein, 1991a), Binary (Reiner and Rubinstein, 1991b), Lookback (Goldman, Sosin, and Gatto, 1979; Conze and Viswanathan, 1991) and Chooser (Rubinstein, 1991) options;
D) Another field of extension for the BSM formula has been covered by numeric methodologies (Brandimarte, 2006). These are necessary in order to fill the gap of not always managing to solve analytically the Fundamental PDE. As a result, it is reasonable to implement these approaches when the pay-off is too complex to obtain an exact closed-form solution (for instance in the case of Asian Options (Turnbull and Wakeman, 1991) where the pay-off is path-dependent and relies on an arithmetic mean calculation) or if the holder has an early-exercise right in discrete (Bermuda option) or in continuous (American option) time (Hull, 2018). The most common families of discrete techniques are: numerical schemes for the solution of PDE - Finite Difference Method (Duffie, 2006), Finite Elements Method (Chacur, Ali, and Salazar, 2011) or Radial Basis Functions (Pena, 2004), stochastic trees – binomial (Cox, Ross, and Rubinstein, 1979), trinomial (Boyle, 1986; Figlewski and Gao, 1999), multinomial lattice (Fabbri and Giribone, 2019) and Monte Carlo - numerical solution of Stochastic Differential Equation (Huynh, Lai, and Soumare, 2008). The first two methods are deterministic, while Monte Carlo is stochastic although it allows a particular flexibility in the pay-off definition.
It is worth to note that numerical techniques converge to the BSM pricing framework and they can be programmed (especially the Monte Carlo method) in order to take into consideration all the other above extensions (A-B-C) to the traditional formula. Given that there is no such thing as a free-lunch, they require a correct control of the numerical error introduced by the discrete integration schemes (Cassettari, Giribone and Mosca, 2012).
With the purpose of pricing certificates on equities or indexes, one of the most important pricing framework adopted by professionals is the so-called Local Volatility Monte Carlo model (Bloomberg, 2016).
It represents an extension of the traditional multi-assets Black-Scholes SDE because volatility is a function of time and of the current simulated spot price (Dupire, 1994). This particular volatility, called local or Dupire volatility, is no more a function of time and of a single strike price. As a result, it can be very useful with structured products that are typically characterized by more than one exercise price, considering that they are composed by option strategies.

The designed platform is able to evaluate simultaneously the same products using different pricing models. This can be considered an added-value of the software to check the different outputs in function of the adopted approach, as a result the platform is able to quantify the model risk.

It is worth to note that the model to be used could depend not only on the available market data but also on the purpose of the pricing itself: in fact during the trading sections, traders for assuming long (/short) positions on the single options used to design the products can adopt a pricing model different from the counterparties and/or from the model used for the evaluation of the overall structured products.

2) A brief introduction to the Discrete Event Simulation and System Dynamics

Simulation is the best tool to predict the behavior of real world systems. For the analysis of complex systems, simulation is often used prior to the operation of the system as a mediator for a dynamic situation.

As is well known, the two main methods for dealing with complex systems are the DES (Discrete Event Simulation) and the SD (System Dynamics) approaches. The DES approach consists in the representation of a system through the use of discrete state variables. In this case, the state variables, which describe the state of the system at any given moment, vary with discontinuities in time (i.e. coupon payment, barrier hit, etc.). Since many among such events are dependent from the evolution of the underlying, it is useful to rely on continuous variables in time instead of discrete variables, with an SD approach through which it is possible to simulate the market prices in detail, allowing to evaluate evolution over time. The mere SD analysis is however not sufficient to solve the problem, as it does not allow to simulate with a sufficient computational speed a complex system such as a logistic system, characterized by multiple variables. The use of the SD technique would allow to study the system in full detail but it would also make the equation system very complex to solve and to understand.

Market liquidity is an issue of very high concern in financial risk management. In a perfectly liquid market the option pricing model becomes the well-known Black–Scholes problem while nonlinear models are used when illiquid market effects are taken into account. The Black-Scholes pricing allows investors to calculate the “fair” price of a derivative whose value depends on an underlying. One of the major assumptions of the Black–Scholes model is that the market of the underlying asset is perfectly elastic, and this occurs for perfectly liquid markets, but the case is clearly unrealistic (Company *et al.* 2012). The presence of price impact of investors' trading has been widely documented and extensively analyzed in the literature. In the presence of asymmetric information use, an equilibrium approach is to investigate how informed traders reveal information and affect the market price through trading. The main advantages of using the SD for modeling Black-Scholes dynamics are described below. Given the non-linear, behavior driven, and interconnected characteristics of trading in illiquid markets, modeling concepts from the SD theory provide appropriate and attractive features. Furthermore the graphical based modeling approach allows the equations and the complexities to be added and integrated step-by-step, thus validated in a simpler way rather than using Matlab or spreadsheet based scripting (Cooke, 2004). Finally, SD models are much more maintainable since are easier to understand due to the explicit graphical link among variables and their derivates (Giribone, Revetria and Testa, 2013). In this paper, the authors propose a simulative approach combining SD and DES where the continuous part is treated using the SD building blocks (stocks for variables and flows for their derivatives) and the event-based payoff schedules are modeled using DES in the Auxiliary blocks. The resulting advantage of the proposed approach is the possibility to demonstrate the price forming process as a set of events easily understandable and transparent to the trader. In this way the pricing algorithm is no longer a "black box" accessible only to "quants", but it becomes instead a link of events and variables that can be reviewed by the traders to confirm better understand their market feelings and expectations even from a quantitative point of view. This approach has also been adopted by authors belonging to different economic fields and is now mature enough to be moved to the financial sector, where applicability could be easily proven (Damiani *et al.* 2016).

3) Implementing the dynamics using System Dynamics: the Black-Scholes and the Local Volatility pricing models

This section deals with the most popular methodologies used for pricing options and certificates written on Equity assets or indexes: Black-Scholes (Eq. 1) and Local Volatility (Eq. 2) pricing framework. After the description of the dynamics, the inputs of these Stochastic Differential Equations are described. In particular, a special focus on how to choose the most suitable representation for the market data on the selected model has been provided to the reader.

The models implemented in the platform used for the prospective modeling of the prices of a stock or of an equity index are:

- the traditional Black-Scholes (BS) stock process in which the underlying price evolves according to the standard Black-Scholes model and where the volatility is a constant or a deterministic function of time.

The stochastic differential equation that describes the model is:

$$dS(t) = [r(t) - q(t)]S(t)dt + \sigma(t)S(t)dW_t \quad (1)$$

Where:

$S(t)$ is the stock (or index) price at time t

$r(t)$ is the short rate prevailing at time t

$q(t)$ is the continuous dividend yield, possibly time-varying, and prevailing at time t

$\sigma(t)$ is the volatility and in this pricing framework it may be constant or time-varying

dW_t is the standard Wiener process, that is a stochastic process with mean zero and variance t

The Black-Scholes model for stock price movement assumes stock price paths are continuous, changes in log-price over any time interval are normally distributed, and changes in price over one or more disjoint time periods are independent.

- the Local Volatility (LV) stock process refers to an extension of the BS model in which the volatility is a function of both time and stock price.

As a result, the dynamics which rules the underlying simulation can be expressed according to the following:

$$dS(t) = [r(t) - q(t)]S(t)dt + \sigma(t, S(t))S(t)dW_t \quad (2)$$

Where $\sigma(t, S(t))$ is the so-called "local volatility".

The advantage of using this more general functional form for σ is that the model can now be calibrated to match market option prices at multiple strikes at a single expiration. In particular, local volatility models overcome Black-Scholes' inability to model the volatility smile observed in the market.

Observing the SDE reported in Eq. 1 and in Eq. 2 it is clear that the two processes differ exclusively in the way volatility is modeled.

An explanation of the input data constituting the stochastic dynamics and how these have been represented in the simulator is provided below:

- S is the asset price, and, for $t = 0$, it is the spot price. $S(t = 0)$ is therefore the initial value for the numerical scheme.

- $r(t)$ is the zero-rates term structure for the reference currency, derived from the bootstrap of the risk-free par rates curve at the valuation date. For instance, if we have a certificate with N assets and N different currencies, we have to take into account N dynamics described by Eq. (1) or Eq. (2), therefore we have to consider N different arrays in which we stored the different forward rates to be used for calculating the future prices projections. In addition to these values of $r_{FWD}(t)$ used for the forwarding process, it is necessary to specify the product currency for determining the zero-rates term structure to be used for discounting the future cash flows. Typically, the discount factors are implied from the zero rates of the currency in which the structured product has been denominated by the issuer and it can potentially be different from the currency of the underlying assets. For this reason, it is a good practice to calculate the zero rates (possibly adjusted for the issuer's funding spread) in an array different from the matrix in which the rates used in the forwarding are stored. The discount factors are calculated using the continuous compounding formula: $\exp(-r_{DISC}(\tau_j) \cdot \tau_j)$, where τ_j is the time in which the holder may receive the expected j-th cash flow.

- The system is able to handle different approaches for the continuous dividend yield estimation, $q(t)$. In function of the available market data this can be estimated in a backward or forward looking manner. The former estimation method has been performed by calculating the continuous dividend yield starting from the ratio between the last paid cash dividend and the current stock spot value (or the average of the closing prices recorded in the last year). Obviously, since it is not a methodology based on the expectation of future values, a different method is to be preferred, when possible. Among the implemented forward-looking approaches, there is the same method described above, but using an expected pay-out estimation into the numerator of the ratio. The disadvantage is that it is heavily based on the sentiment of traders. A reasonably more quantitative method is to derive q implicitly from the prices of actively traded derivatives: for this purpose, the cost-of-carry method can be adopted if quotations of forward contracts are available or, if the market is particularly complete, even the put-call parity principle.

- Volatility plays a central role in the projection of the simulated values and therefore it is possible to model it in different ways, especially according to the available market data. The simplest and most adopted approach if the market does not quote an implied volatility is the historical method. It is a completely backward-looking approach and given that it typically provides only one volatility value associated with the underlying, it has to be kept constant in the implemented SDE. In this case, the estimation has been made using the close-to-close method, which is essentially based on the calculation of the standard deviation of the daily returns, thus converting them on an annual basis by multiplying the appropriate annuity factor (usually set to $\sqrt{260}$). If an implied volatility surface is available, it is reasonable to use this information in order to better reflect the future expectations linked with the asset price level. The quantitative analyst can implement the SDE (2), only if the contributions are complete along the two dimensions of the surface (i.e. tenor and moneyness), otherwise the volatility along the surface section used for the calibration of the stochastic dynamics can be used for pricing the structured product. In the case of SDE (1), once the strike price, considered particularly significant for tuning, has been fixed, σ remains only in function of time. If there are more prevailing strike prices, the approach described may not be adequate since too much weight would be given to a section of the implied volatility surface corresponding to a single strike price rather than to other portions characterized by values that are still important and significant for an adequate calibration of the model parameters. Taking this fundamental aspect into account, especially in the presence of potential highly structured products such as certificates that can incorporate dozens of exotic options at the same time, it is reasonable, if the market provides to the Quant complete and consistent portions of the volatility surface, to increase by one dimension the representation of σ which goes from being a function of the single time variable, $\sigma(t)$, to a surface, which is in function of both time t and the prospective underlying value S_t, $\sigma(t, S_t)$. This extension of the model is called Local Volatility and is currently the model implemented by default by the calculation modules for the most popular structured products in the financial field, including Bloomberg® - DLIB (Bloomberg, 2016). Bruno Dupire proposed a methodology capable of converting the surface of the market implied volatilities, which have the time and strike price dimensions, into one of equivalent dimensions, suitable to be used for the Local Volatility model that has the characteristic to not use fixed strike prices, but use projected spot values instead, hence the "local" term (Dupire, 1994). The proposed pricing platform is able to create a Dupire volatility surface starting from a generic implied volatility surface using the conversion formula:

$$\sigma(S_t, t) = \left. \sqrt{\frac{\frac{1}{2}\frac{\sigma_{IMPL}}{T} + \frac{\partial \sigma_{IMPL}}{\partial T} + K(r-q)\frac{\partial \sigma_{IMPL}}{\partial K}}{\frac{1}{2}K^2\left[\frac{1}{\sigma_{IMPL}K^2T} + 2\frac{\partial \sigma_{IMPL}}{\partial K}\frac{d_1}{\sigma_{IMPL}K\sqrt{T}} + \frac{\partial^2 \sigma_{IMPL}}{\partial K^2} + \left(\frac{\partial \sigma_{IMPL}}{\sigma K}\right)^2 \frac{d_1 d_2}{\sigma_{IMPL}}\right]}} \right|_{K=S_t, T=t} \quad (3)$$

with K being the strike price of the implied volatility surface (Franco, Polimeni, and Proietti, 2002).

- the Wiener process, dW_t has been implemented using the traditional numerical discretization: $\varepsilon\sqrt{dt}$, where ε is a random draw from a standard normal distribution. This contribution, which makes the dynamics stochastic, plays a key role in the modelling of the correlation between the underlyings in case of multi-assets certificates (Bagnato and Giribone, 2021). In order to incorporate the correlation matrix for the assets in the Monte Carlo simulation, the Cholesky decomposition can be used. Assuming you have a set of unrelated random numbers $\vec{\varepsilon} = \varepsilon_1, \varepsilon_2, \varepsilon_3, \dots\dots \varepsilon_T$, the Cholesky decomposition allows you to transform them into a set of correlated variables $\vec{a} = a_1, a_2, a_3, \dots\dots a_T$. If $\vec{a}$ and $\vec{\varepsilon}$ are column vectors with N rows, and R is the correlation matrix, it is possible to apply the following transformation:

$$\vec{a} = M\vec{\varepsilon} \quad (4)$$

Where M is a matrix that must satisfy the condition $MM^T = R$. The matrix M can be obtained by applying the Cholesky decomposition to R. Subsequently, the correlated shocks ($\vec{a}$) are substituted to the innovations (ε).

Another aspect that must be taken into consideration when implementing stochastic dynamics is the potential coexistence of different currencies between basket assets and/or discount currencies. This characteristic can be implemented through the so-called "Quanto effect", which provides an adjustment to the core SDE drift adding to the canonical term $[r(t) - q(t)]$ the corrective factor equal to $-\rho_{S,FX}\sigma_S\sigma_{FX}$, where $\rho_{S,FX}$ represents the correlation between the equity underlying S and the exchange rate, σ_S is the underlying volatility and σ_{FX} is the forex volatility (Hull, 2018).

These elements make it therefore possible to carry out simulations of the underlying prices (typically equity and equity index) till the expiry of the structured product to be valued and analyzed. Considering that the single numerical solution of the SDEs obviously leads to the estimation of a single price, it is necessary to conduct more integrations of Eq. (1) and (2) in order to have more prices for the structured instrument and, in line with the founding principles of the Monte Carlo pricing methodology, the average of these prices, discounted back at the valuation time, provides the expected theoretical value for the certificate (Glasserman, 2003). The proposed simulation approach allows to carry out multiple concurrent numerical integrations for each time-step (dt) set equal to a working day, making the pricing platform particularly flexible and suitable for accurate analysis of the price formation and of the risk measures associated with the product.

4) Implementing the financial characteristics of the most popular and traded certificates

The particular characteristics of today's financial markets, characterized by low yields and deep negative risk-free interest rates, have led private investors to request structured products in order to have a higher return compared to standard financial products, although they need to be aware of a greater risk.

Certificates represents a valid solution to this demand. Drawing up a taxonomy of all possible commercial structured products on the market is a difficult, if not impossible, undertaking, even more so considering the possibility of creating ad-hoc structures to reflect the particular needs of wealth management and the most advanced investors.

In this regard, we recall that a classification of the structured products was carried out by the EUSIPA - European Structured Investment Product Association - professional association which proposed the well-known EUSIPA Derivatives Maps, an exhaustive classification of the most popular structured products (EUSIPA, 2020).

The main market players have added names to this "standardized" classification and they are certainly more bizarre and attractive from a commercial point of view than a technical (and sometimes numerical) identification (for instance Phoenix or Snowball certificate rather than autocallable product with conditional coupons having memory effect)

From the point of view of the pricing process, it is important for an engineer to focus on the definition of the pay-off and on the mechanics of the product for retrieving the key concepts with the aim of facing the pricing problem in the most general manner.

Having made this premise and having programmed an engine capable of producing a set of simulated values for a desired future time, as described in paragraph 3, the present section focuses on the analysis of the most common features that can be associated with a structured product.

However, in order to make the reasoning more concrete, we will specify in the following part, which among the products on the market have the described technical characteristics.

- from a commercial perspective, one of the most interesting features is the attractiveness of the potentially very high coupon amount (obviously in return for a typically greater risk of loss of part of the invested capital at the maturity date). Coupons can be guaranteed, or if the certificate does not foresee to be called up before the contractual expiry, they are always paid on the dates indicated in the prospectus or they are conditional. In the latter case, the coupon is paid if the asset characterized by the worst performance is above a threshold level (Coupon Trigger), typically expressed as a percentage of the underlying initial fixing value. Thus in this case there are as many digital cash-or-nothing options as there are coupons that can potentially be paid to the investor. A feature that is often associated to this kind of coupons is the presence of a so-called “memory effect” to be applied on the coupon strip. In such case, when the condition that all the assets be above the Coupon Barrier is not reached, nothing has to be paid to the holder of the structured product, as in the previous case, but if the event occurs again in the future such that all assets in the basket are above the threshold, then an amount will be paid equal to the sum of all the coupons that should have been paid previously, starting from the last coupon cash flow received. From the point of view of the Monte Carlo model, the simulation of each path will be considered in correspondence with the event dates of potential coupon payment. If the prospective value of the worst asset has a higher performance than the Coupon Trigger Percentage then the coupon amount due on the date will be added to the Net Present Value of the instrument after having discounted it for the appropriate discount value, otherwise zero. If the “memory effect” is present, it is necessary to store the date of the last coupon paid in an array in order to match the total amount of past unpaid coupons if the positive event occurs again. It is reasonable to model guaranteed coupons as a degenerate case of conditional coupons by setting the coupon trigger barrier equal to zero. Recent market trends have led to the structuring of certificates with coupon strips belonging to the most different categories, including Reverse Convertible (the pay-off at the redemption date is characterized by a short position in a vanilla put option), Barrier Reverse Convertible (the pay-off at the redemption date is characterized by a short position in a knock-in barrier put option), partially guaranteed capital products (the pay-off at redemption date is characterized by a call spread option strategy) and autocallable (certificates having a potential automatic early-redemption determined by the exceeding of the basket performance threshold).

- the pricing platform has to take into account that many products on the market have the feature to be potentially redeemed before the maturity date. Early redemption typically depends on the issuer (Issuer Callability) or on the achievement of a market condition (Autocallability clause). The former category, less common among retail investors, provides that the issuer of the structured product has the right to recall the certificate on certain pre-fixed dates, while the latter and much more widespread category, provides that the product is automatically redeemed upon the occurrence of a market condition (regardless of the will of the issuer or the holder). Typically, this happens when the performance of the worst assets in the basket reaches a value higher than a threshold value (autocallable barrier) on certain predetermined dates (autocallable dates). This trigger is usually expressed as a percentage (decreasing, constant or increasing over time) of the initial fixing prices of the assets in the basket of the certificate.

From a mathematical perspective, the autocallability can be straightforwardly modeled: it is necessary to simulate the underlying on the event dates, choose the asset in the basket that has had the worst performance and compare it with the threshold level: if it is above the threshold level, the certificate is called and, usually, the last coupon is paid at the same time, otherwise it continues until the next date of the potential automatic early redemption where the autocallability test will be repeated or, if no other future dates are foreseen, then the products will be redeemed at maturity.

The modeling of the convenience of early exercise by the issuer for such complex products is undoubtedly more challenging. In this case, similarly to standard financial products, the Bellman principle of optimality should be applied (Giribone, 2021). In this specific context it is necessary to quantify the convenience for the issuer to redeem the certificate earlier by comparing the price simulations at maturity with those at the earliest issuer call date.

The expectation of the instrument is as follows:

$$prob_{issuer\ call} \cdot Price_{issuer\ call\ date} + (1 - prob_{issuer\ call}) \cdot Price_{redemption\ date} \quad (5)$$

It is worth observing that, given the discretion that the issuer has to recall the certificate, factors that cannot always be directly measured or quantified could potentially affect the overall instrument valuation.

Structured products that typically include the autocallability features, technically called Autocallables, are often referred to with the commercial name of “Express Certificates” and when these include conditional coupons they are referred to as “Phoenix” or if they pay coupons with a memory effect, they are often called “Snowball”.

The Issuer Callability feature is generally associated with the family of certificates belonging to barrier reverse convertibles.

- The pay-off at maturity determines which macro-category the certificate belongs to and therefore plays a fundamental role in determining the fair value. Given the multiple characteristics, its definition in the pricing platform is extremely customizable. Here is a non-exhaustive list of the most common pay-offs to be applied at maturity date:

A call spread can be used to structure a product that allows a bounded participation below (capital protection floor) and / or above (cap) an equity underlying at maturity.

A short position in a put is used to represent the pay-off of a reverse convertible or discount certificate. Similarly in a barrier reverse convertible or in a barrier discount certificate, a short position in a knock-in put option can be used. In this last case, the barrier level plays the role of conditionally protecting the capital up to that value.

As a result, the knock-in barrier level in the put option plays a fundamental role in the conditional protected certificate because if this is not reached (at maturity in the case of "European" monitoring or throughout the life of the product in the case of continuous monitoring, called "American") by the worst of the assets in the basket, it protects the investor who receives the entire amount invested (100).

Otherwise, if only one of the underlyings of the certificate exceeds the barrier level, then in this case the short position of the put is activated at maturity and therefore the holder could proportionally lose part of the invested amount.

The same can also be said for Autocallables, which in their standard versions (strike percentage equal to 100% of the initial fixing) share the same characteristics at maturity as the previous ones.

5) The output measures for a complete analysis of the certificate

The simulation platform allows to monitor and track all the relevant variables for a complete analysis of the product day after day through textboxes and iterative graphs.

The main Graphical User Interface allows to display the most important quantitative measures, among which:

- the simulated paths for each underlying.

- the most significant market input used for the SDE integrations.

- The hitting probabilities for the main triggers associated with the basket product, such as barrier levels and coupon levels.

- The hitting probabilities for the main triggers associated with the product, such as the automatic early redemption and the issuer callability.

- The distance to barriers.

- The theoretical product price, splitted into its components.

- the platform allows to modify in real-time and dynamically the model parameters for a complete what-if analysis.

- the simulator allows to implement stress tests through the implementation of constant or time-varying bumps applied to each of the variables.

- The pricing platform is also able to estimate the main Greeks thanks to the possibility to handle the parameters of the random number generator. Allowing to manage the seeds which generate the simulations, the sensitivities of the products can be computed using the traditional finite difference method with the typical 1% bump to be applied to the input parameters. These sensitivity measures are estimated at product level and for each asset of the certificate basket.

- The software can generate empirical probability distributions for the product price and for the risk measures, associating a custom statistical distribution to each input. These types of simulations can be very useful for studying the behavior of the valuation of the certificate under extreme scenarios (5% or 1% percentiles).

In addition to these analyses, all the intermediate results are available in dedicated modules of the simulator.

The user is therefore able to perform a deep analysis and inspect the values of variables up to a single day for each variable indexed to time, among which: $r(t)$, $q(t)$, $S(t)$ and $\sigma(t)$.

6) Pricing and Risk Analysis: Market Cases

The first pricing example is related to the quantitative analysis of a Barrier Reverse Convertible with Issuer Callability. The financial characteristics of the Certificate are summarized in Table A.1.

The valuation date for the structured product is June 8^{th}, 2021 and the market data used for pricing the assets are summarized below (Source: Bloomberg®):

- Table A.2 contains the interest rates term-structure used for the discounting of the expected future cash flows, $r_{DISC}(t)$.
- Table A.3 contains the interest rates term-structure used for the forwarding of the three underlyings in US dollars, $r_{FWD}(t)$.
- Table A.4 contains the implied dividend yields, $q(t)$ and the spot prices $S(t=0)$ for the three assets. Together with such market data, the key parameters associated with the underlyings for the representation of the certificate mechanics are reported: Strike level (100% of the Initial Fixing) and the knock-in barrier level (54.6% of the Initial Fixing).
- Figures A.1, A.2 and A.3 show the implied volatility surfaces associated with the three assets. For the BS engine, $\sigma(t)$ has been calibrated in accordance with the moneyness of the Strike Price, while for the "Local Volatility" model, these surfaces have been used for the generation of the volatility "à la Dupire", $\sigma(t, S(t))$.
- Given that there is a quanto effect, due to the fact that the currency of the discount curve (EUR) is different from the currency of the three assets (USD), Table A.5 shows the input of the model related to the forex.

Starting from the historical prices of the underlying basket and the exchange rate, the intermediate variables leading to the final result have been estimated. In particular, the matrix of the correlations between assets for the random number generation in accordance with the Cholesky decomposition, the correlation between asset and the exchange rate ($\rho_{S,FX}$) and the forex volatility, σ_{FX}.

The fair value of the certificate without taking into consideration the Issuer Callability has been estimated by the pricing platform equal to 106.06, the issuer call probability is high and equal to 92.65%. As a result, the probability to hit the continuous-time barrier is low and equal to 7.36%

Considering that the product can be called by the issuer at 100 on June 23^{rd}, 2021, the fair value converges to 97.22 and such price is aligned with the Bloomberg® (ALLQ) quotes that displays a bid price of 97.13.

Regarding the main first-order risk measures:

- The overall delta for the product is 0.2245. The sensitivity respect to the spot prices can be divided out among the assets of the basket as follows: $\Delta_{\text{MCD}} = 0.0694$, $\Delta_{\text{PM}} = 0.0783$ and $\Delta_{\text{KHC}} = 0.0768$.
- the overall vega for the product is -0.5382. The sensitivity respect to the volatility can be divided out among the assets of the basket as follows: $\vartheta_{\text{MCD}} = -0.1286$, $\Delta_{\text{PM}} = -0.1688$ and $\Delta_{\text{KHC}} = -0.2408$.
- The rho of the product, that is the sensitivity respect to the interest rates, is 0.1142.

Figure 1 shows the GUI of the pricing platform highlighting all the main calculations useful for a deep understanding of the analyzed products.

The second market case regards the pricing of an Express Certificate with conditional memory coupon. Its main financial characteristics are synthetized in Table B.1

Similarly to what was done in the previous analysis, the market data used for pricing are summarized below (Source: Bloomberg®, Valuation Date: June 8^{th}, 2021).

- Table B.2 contains the interest rates term structure used for both the discount factors calculations and the forward rates used during the projection of the assets, $r(t)$. In this case, the product is not characterized by a quanto effect, given that the only currency is the Swiss Franc (CHF).
- Table B.3 contains the key values for the underlyings: strike prices (100% of the Initial Fixing), barrier levels (60% of the Initial Fixing), coupon barriers (60% of the Initial Fixing), autocallability levels (100% of the Initial Fixing), spot prices and implied dividend yields.
- Figures B.1, B.2 and B.3 show the implied volatility surfaces of the three assets: for the BS model, $\sigma(t)$ has been tuned along the moneyness of the Strike Price; for the "Local Volatility" model these surfaces have been used as the starting point for the creation of the volatilities "à la Dupire", that is $\sigma(t, S(t))$.

The fair value of the analyzed certificate is equal to 96.82 and it is aligned with the Bloomberg theoretical price (the Local Vol model implemented in the DLIB module gives a value of 96.66) and the counterparty quote (ask price is 96.85).

Regarding the main first-order Greeks:

- the delta of the product is 0.607728 and the sensitivity respect to the spot prices can be divided out among the assets as follows: $\Delta_{\text{BAER}} = 0.046698$, $\Delta_{\text{CSGN}} = 0.50166$ and $\Delta_{\text{ZURN}} = 0.05937$.
- the vega of the product is -0.84556 and the sensitivity respect to the volatility can be divided out among the assets as follows: $\vartheta_{\text{BAER}} = -0.0644$, $\vartheta_{\text{CSGN}} = -0.6810$ and $\vartheta_{\text{ZURN}} = -0.10016$.
- the product has a rho (sensitivity respect to interest rates) equal to -1.2429.

Figure 2 shows the GUI of the simulator with all the computed analytics.

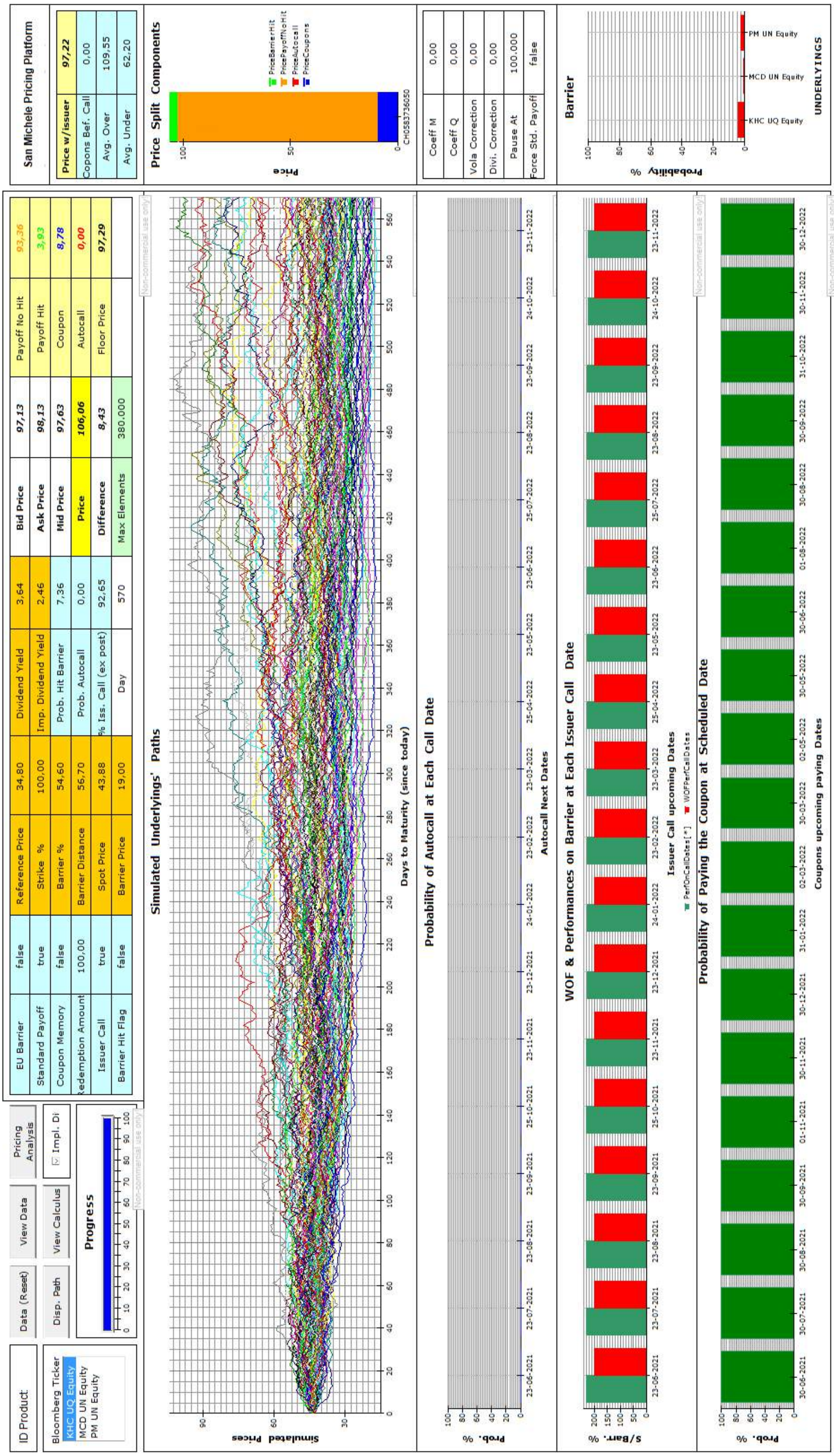


Figure 1 Quantitative Analysis for the Barrier Reverse Convertible

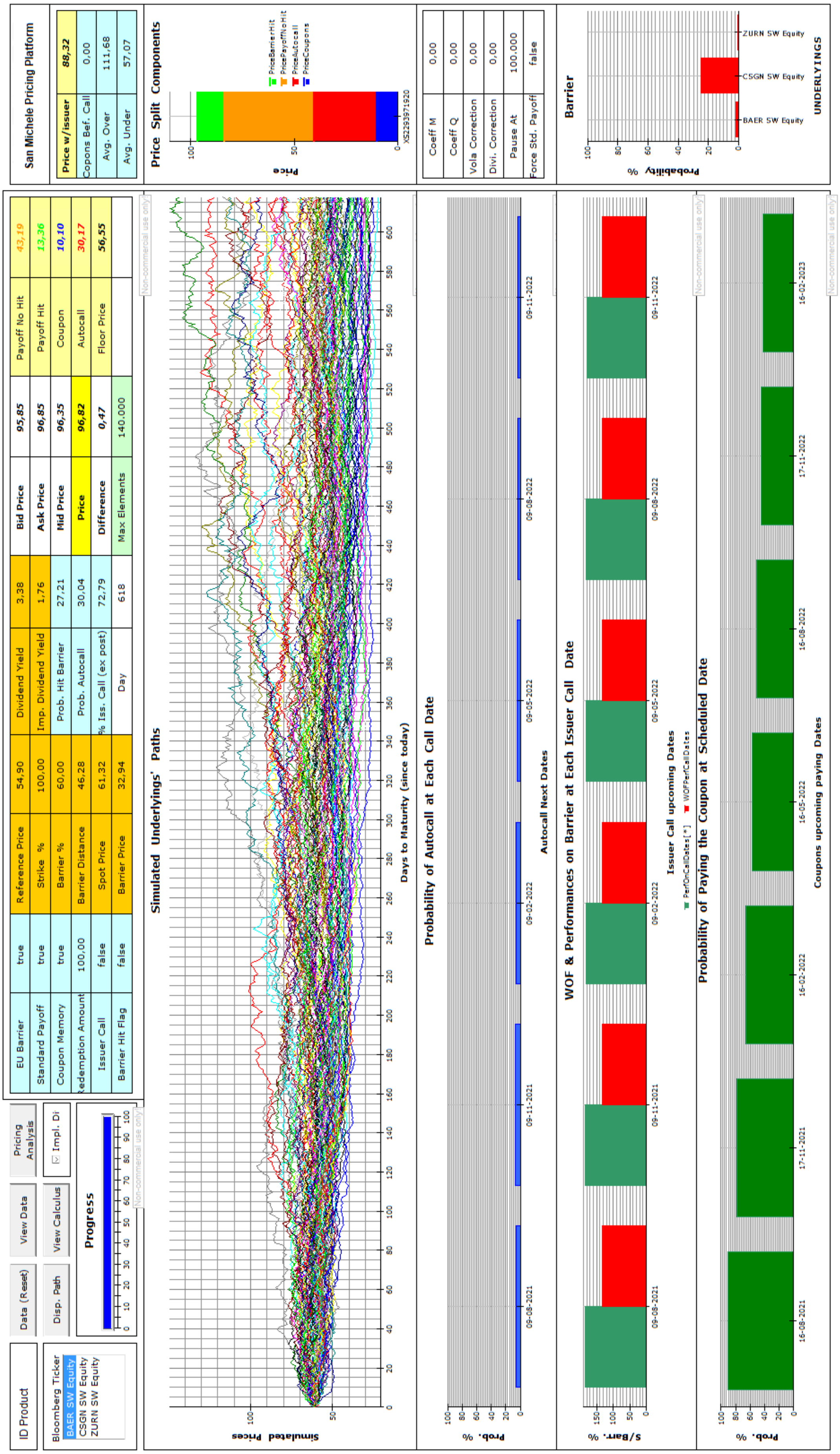


Figure 2 Quantitative Analysis for the Express Certificate

7) Conclusions

This paper has shown how Discrete Event Simulations (DES) and System Dynamics (SD) techniques can also be successfully applied to a context that is quite different from traditional engineering: in particular the authors proposed a very innovative application to quantitative finance and risk management.

One of the main results of this study is the creation of a simulator, able to perform a complete quantitative analysis associated to highly structured certificates, financial products mainly reserved to private banking and Finance professionals.

One of the main advantages of using a visual block-oriented simulator rather than more traditional programming techniques is that it allows the user (not necessarily a high-skilled quant or a financial engineer) to understand the most important factors in the formation of the fair value of the structured product and to consequently develop a high sensitivity to the product.

The innovative platform was therefore created with the intention of proposing an innovative tool for product design, for checking the price provided by the counterparties and for conducting scenario or what-if analyses in a very easy, precise and intuitive manner. Furthermore, an analyst is able to model the dynamics, the market data, the financial characteristics of the products and the pay-off without having specific skills in programming.

This can undoubtedly be considered of great help in daily operations: a trader can calculate the simulated day-by-day portfolio profit in a very easy way without the need to explicitly code scenarios or what-if analyses or sensitivities exposure: it is enough to drag-and-drop or linking objects or editing parameters directly from the platform.

An interesting further development of this study consists in implementing different stochastic differential equations suitable for representing dynamics that differ from Equities or Indexes, such as commodities and interest rates.

Another possible extension of the platform is to apply the same pricing framework to credit derivatives for which the Probability of Default can be modelled using the KMV model, which is a particular extension of the traditional Merton approach.

Given that the underlying SDE is the Geometric Brownian Motion in this case as well, the mathematical model already implemented in the platform can be easily extended for this purpose.

Bibliography

Asay, M. R. (1982). “A Note on the Design of Commodity Option Contracts”. In: Journal of Futures Markets 52, pp. 1–7.

Bachelier, L. (1900). “Theory of Speculation in "The Random Character of Stock Market Prices"”. In: MIT Press.

Bagnato, M. and P. G. Giribone (2021). “Implementation of a Commitment Machine for an adaptive and robust Expected Shortfall estimation”. Network Models For Financial Contagion and Systemic Risk – NMFCSR 2021 Conference Proceedings

Bates, D. S. (1991). “The crash of ’87: Was it expected? The evidence from options markets”. In: Journal of Finance 46.3, pp. 1009–1044.

Black, F. and M. Scholes (1973). “The Pricing of Options and Corporate Liabilities”. In: Journal of Political Economy 81, pp. 637–654.

Black, F. (1976). “The Pricing of Commodity Contracts”. In: Journal of Financial Economics 3, pp. 167–179.

Bloomberg Quantitative Analytics (2016). “Overview of Bloomberg models for Equity Derivatives in DLIB”.

Boyle, P.P. (1986). “Option valuation using a three-jump process”. In: International Options Journal 3, pp. 5–12.

Boness, A. (1964). “Elements of a Theory of Stock-option value”. In: Journal of Political Ecomomy 72, pp. 163–175.

Brandimarte, P. (2006). Numerical methods in finance and economics.Wiley Interscience.

Burro, G., P. G. Giribone, S. Ligato, M. Mulas and F. Querci (2017). “Negative interest rates effects on option pricing: back to basics?” In: International Journal of Financial Engineering 4.2.

Cafferata, A., P. G. Giribone, and M. Resta (2017). “The effects of negative nominal rates on the pricing of American calls: some theoretical and numerical insights”. In: Modern Economy 8.7.

Cassettari L., P. G. Giribone and R. Mosca (2012). “Reliable Control of Convergence in Monte Carlo Pricing Methods for Options based on MSPE Technique”. WSEAS International Conference on Mathematics and Computers in Business and Economics - MCBE'12 Conference Proceedings.

Chacur, A. A., M. M. Ali, and J. G. Salazar (2011). “Real options pricing by the finite element method”. In: Journal of Computers and Mathematics with Applications 61, pp. 2863–2873.

Company, R., Jódar, L., & Pintos, J. R. (2012). A consistent stable numerical scheme for a nonlinear option pricing model in illiquid markets. Mathematics and Computers in Simulation, 82(10), 1972-1985.

Conze, A. and Viswanathan (1991). “Path Dependent Options: the case of lookback options”. In: Journal of Finance 46, pp. 1893–1907.

Cooke, D. L. (2004, July). Using system dynamics models to enhance the visualization of stochastic price processes. In Proceedings of the International Conference of the System Dynamics Society 2004 (pp. 25-29).

Corrado, C. J. and T. Su (1996). “Skewness and kurtosis in S&P 500 Index Returns implied by option prices”. In: Journal of Financial Research XIX, pp. 175–192.

Cox, J. C. and S. A. Ross (1976). “The valuation of options for Alternative Stochastic Processes”. In: Journal of Financial Economics 3, pp. 145–166.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). “Option pricing: a simplified approach”. In: Journal of Financial Economics 7, pp. 229–263.

Damiani, L., Giribone, P., Guizzi, G., Revetria, R., & Romano, E. (2016). Different approaches for studying interruptible industrial processes: Application of two different simulation techniques. In Handbook of Research on Computational Simulation and Modeling in Engineering (pp. 69-104). IGI Global.

Derman, E. and M. Kamal (1999). “When you cannot hedge continuously: the corrections of Black-Scholes”. In: Risk Magazine 12, pp. 82–85.

Duffie, D. J. (2006). Finite Difference methods in financial engineering. Wiley Finance.

Dupire, B. (1994). “Pricing with a smile”. In: Risk Magazine, 18-20

EUSIPA - European Structured Investment Product Association (2020). The EUSIPA Derivative Map©. Website: https://eusipa.org/wp-content/uploads/European_map_20200213_web.pdf

Fabbri, M. and P. G. Giribone (2019). “Design, implementation and validation of advanced lattice techniques for pricing EAKO - European American Knock-out option”. In: International Journal of Financial Engineering 6.4.
Figlewski, S. and B. Gao (1999). “The adaptive mesh model: a new approach to efficient option pricing”. In: Journal of Financial Ecomomics 53, pp. 313–351.
Franco, M. Di, F. Polimeni, and M. Proietti (2002). Opzioni e titoli strutturati. Il Sole 24 ore.
FINMAB, Mathworks (2019). “Finance MathWorks Advisory Board, Frankfurt”.
French, D. W. (1984). “The Weekend effect on the distribution of stock prices”. In: Journal of Financial Ecomomics 13, pp. 547–559.
Garman, M. and S.W. Kohlhagen (1983). “Foreign Currency Option Values”. In: Journal of International Money and Finance 2, pp. 231–237.
Giribone, P., R. Revetria, and A. Testa (2013). “Simulation Models for Supporting Hedging in Illiquid Markets”. 7th International Conference on Applied Mathematics Simulation, Modelling (ASM)
Giribone, P. G. (2021). “Mathematical modeling in Quantitative Finance and Computational Economics”, Part II. Deterministic and Stochastic Optimal Control. Ph.D. Thesis in Economics. Supervisors: Marco Guerrazzi (UNIGE – DIEC) and Ottavio Caligaris (UNIGE – DIME).
Giribone, P. G., R. Mosca, L. Cassettari and I. Bendato (2015). “Monte Carlo method for pricing complex financial derivatives: an innovative approach to the control of convergence”. In: Applied Mathematical Sciences 9.124.
Giribone, P. G., S. Ligato, and M. Mulas (2017). “The effects of negative interest rates on the estimation of option sensitivities: the impact of switching from a log-normal to a normal model”. In: International Journal of Financial Engineering 4.1.
Glasserman, P. (2003). Monte Carlo methods in Financial Engineering. Springer.
Goldman, B. M., H. B. Sosin, and M. A. Gatto (1979). “Path dependent option: "Buy at the Low Sell at the High"”. In: Journal of Finance 34.5, pp. 1111–1127.
Grabbe, O. J. (1983). “The Pricing of Put and Call Options on Foreign Exchange”. In: Journal of International Money and Finance 2, pp. 239–253.
Hagan, P. S., D. Kumar, A. Lesniewski and D. E. Woodward (2002). “Managing Smile Risk”. In: Wilmott Magazine September.
Haug, E. G. (2007). The Complete guide to option pricing formulas. McGraw-Hill.
Hoggard, T., A. E. Whalley, and P. Wilmott (1994). “Option Pricing and Replication with Transactions Costs”. In: Advances in Futures and Options Research 7, pp. 21– 35.
Hull, J. C. (2018). Options, Futures, and Other Derivatives. Prentice Hall.
Hull, J. and A. White (1988). “An analysis of the bias in option pricing caused by stochastic volatility”. In: Advances in Futures and Options Research 2.
Huynh, H. T., V. S. Lai, and I. Soumare (2008). Stochastic simulation and applications in finance with Matlab programs.Wiley.
Jarrow, R. and A. Rudd (1982). “Approximate Option Valuation for Arbitrary Stochastic Processes”. In: Journal of Financial Economics 10, pp. 347–369.
Kochkodin, B. (2019). “Negative Rates are rewriting the rules of Modern Finance”. In: Bloomberg Markets September 2019.
Leland, H. (1985). “Option Pricing and Replication with Transactions Costs”. In: Journal of Finance XL.5, pp. 1283–1301.
Lieu, D. (1990). “Option Pricing with Futures-Style Margining”. In: Journal of Futures Markets 10, pp. 327–328.
Lo, A. and J. Wang (1995). “Implementing Option Pricing when Asset Returns are Predictable”. In: Journal of Finance 50, pp. 87–129.
Merton, R. C. (1973). “Theory of Rational Option Pricing”. In: Bell Journal of Economics and Management Science 4, pp. 141–183.
Pena, A. (2004). “Option Pricing with Radial Basis Functions: a tutorial”. In: Wilmott Magazine.
Reiner, E. and M. Rubinstein (1991a). “Breaking down the barriers”. In: Risk Magazine 4.8, pp. 28–35.
Reiner, E. and M. Rubinstein (1991b). “Unscrambling the Binary Code”. In: Risk Magazine 4.9.
Rubinstein, M. (1991). “Options for Undecided”. In: Risk Magazine 4.1.
Smith, C. W. (1976). “Option pricing: a review”. In: Journal of Financial Economics 3, pp. 3–51.
Sprenkle, C. (1964). “Warrant prices as indicators of expectations and preferences in the random character of stock markets prices”. In: MIT Press.
Stafford, P. (2019). “Bloomberg urges users to get set for negative US rates”. In: Financial Times June 2020.
Stewart, I. (2013). In pursuit of the unknown: 17 equations that changed theWorld. Profile Books.
Turnbull, S. M. and L. M.Wakeman (1991). “A quick algorithm for pricing European Average Options”. In: Journal of Financial and Quantitative Analysis 26, pp. 377–389.
Wilmott, P. (2000). Paul Wilmott on Quantitative Finance. John Wiley and Sons.

Appendix: Financial Characteristics and Market Data

A. Barrier Reverse Convertible with Issuer Callability Pricing Data

Underlyings [Currency]	KHC UQ [USD] - MCD UN [USD] - PM UN [USD]
Product Type Description	Barrier Reverse Convertible (Worst of Assets)
Initial Fixing Date	23-Dec-2020
Issue Date	30-Dec-2020
Final Fixing Date	23-Dec-2022
Redemption Date	30-Dec-2022
Issue Price	100
Quoting Type Description	Dirty
Call Frequency Description	Monthly
Barrier Trigger Percentage	54.6%
Next Issuer Call Date	23-Jun-2021
Barrier Type Description	American
Call Type Description	Issuer Call
Currency Description	EUR
Quanto Product	Yes
Coupon Type Description	Guaranteed Coupon
Coupon Frequency Description	Monthly
Coupon Per Period	0.458
Next Coupon Observation Date	23-Jun-2021
Coupon Paid	2.29
Coupon Rate Per Annum	5.5
Coupon Paid Difference	0.458
Strike Percentage	100%
Redemption Amount	100

Table A.1 Financial Characteristics of the Barrier Reverse Convertible

Term	Implied Rates	Implied DF
1 WK	-0.522452	1.000102
2 WK	-0.536585	1.000209
3 WK	-0.594567	1.000347
1 MO	-0.587893	1.000523
2 MO	-0.581348	1.000986
3 MO	-0.577738	1.001478
6 MO	-0.578874	1.002947
9 MO	-0.607666	1.004619
1 YR	-0.593523	1.006036
18 MO	-0.578299	1.008874
2 YR	-0.562421	1.011502
3 YR	-0.500185	1.015344
5 YR	-0.362468	1.018555

Table A.2 Discount Curve, Reference Date: 8th June 2021, Currency: EUR. Source: Bloomberg®

Term	Par Rate	Zero Rate	Discount
EDM1	0.122479	0.124161945	0.99967009
EDU1	0.119495	0.122697264	0.999368225
EDZ1	0.16871	0.138457361	0.998942215
EDH2	0.152632	0.14245755	0.998556952
EDM2	0.186173	0.152143165	0.998051134
EDU2	0.244482	0.167715414	0.997434723
2 YR	0.22217	0.222090614	0.995555923
3 YR	0.412	0.412111124	0.987701626
4 YR	0.6356	0.637426342	0.974808227
5 YR	0.8411	0.845907807	0.958564366

Table A.3 Forward Curve, Reference Date: 8th June 2021, Currency: USD. Source: Bloomberg®

Underlying	Bloomberg Ticker	Initial Fixing Level	Barrier Level (54.60%)	Strike Level (100%)	Spot Level	Dividend Yield
Kraft Heinz	KHC UQ	34.80 USD	19 USD	34.8 USD	43.88 USD	2.461%
McDonald's Corp	MCD UN	212.02 USD	115.76 USD	212.02 USD	231.69 USD	1.706%
Philip Morris Int	PM UN	82.20 USD	44.88 USD	82.2 USD	98.56 USD	4.435%

Table A.4 The Underlying characteristics, Reference Date: 8th June 2021. Source: Bloomberg®

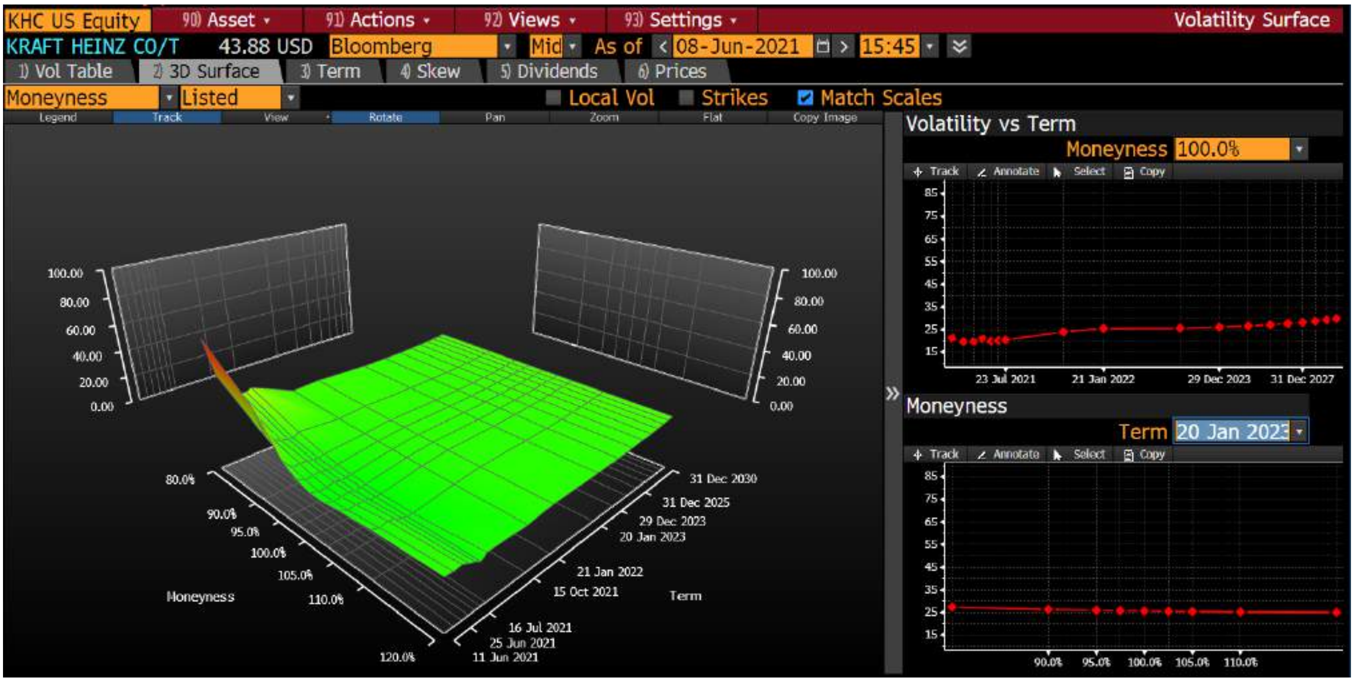


Figure A.1 Implied Volatility Surface of KHC US Equity. Reference Date: 8th June 2021. Source: Bloomberg®

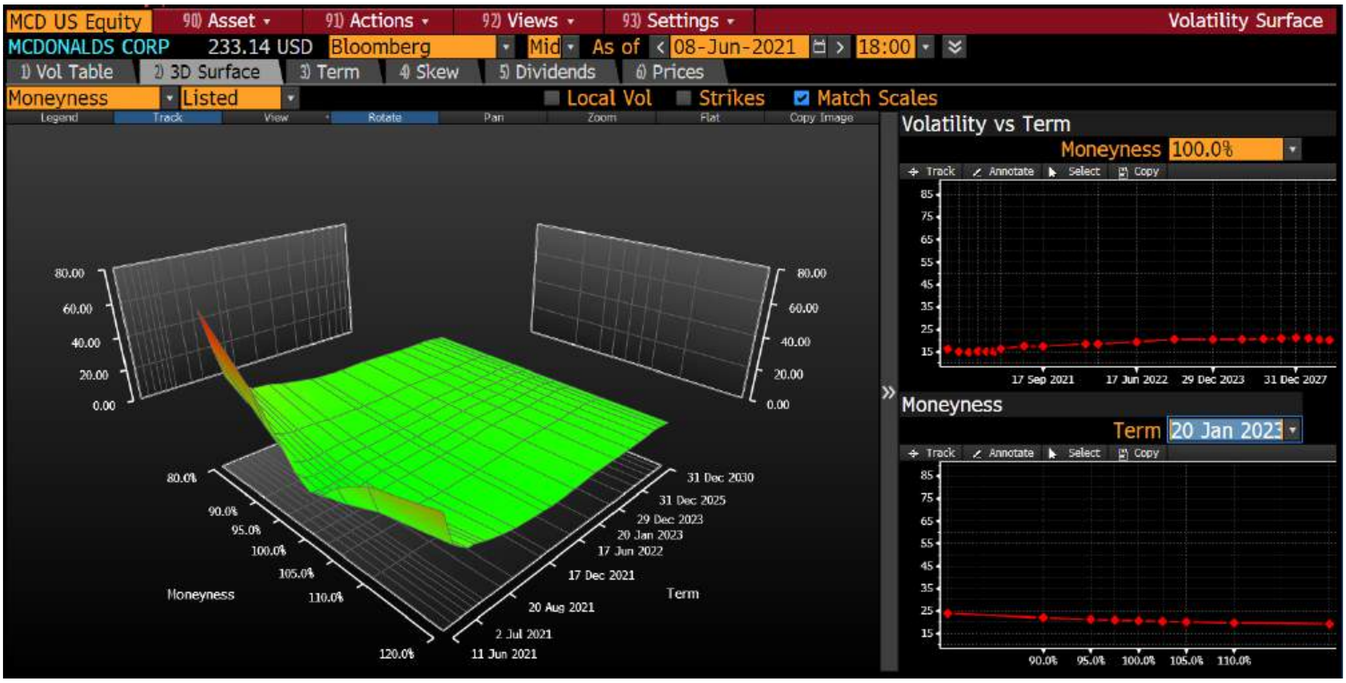


Figure A.2 Implied Volatility Surface of MCD US Equity. Reference Date: 8th June 2021. Source: Bloomberg®

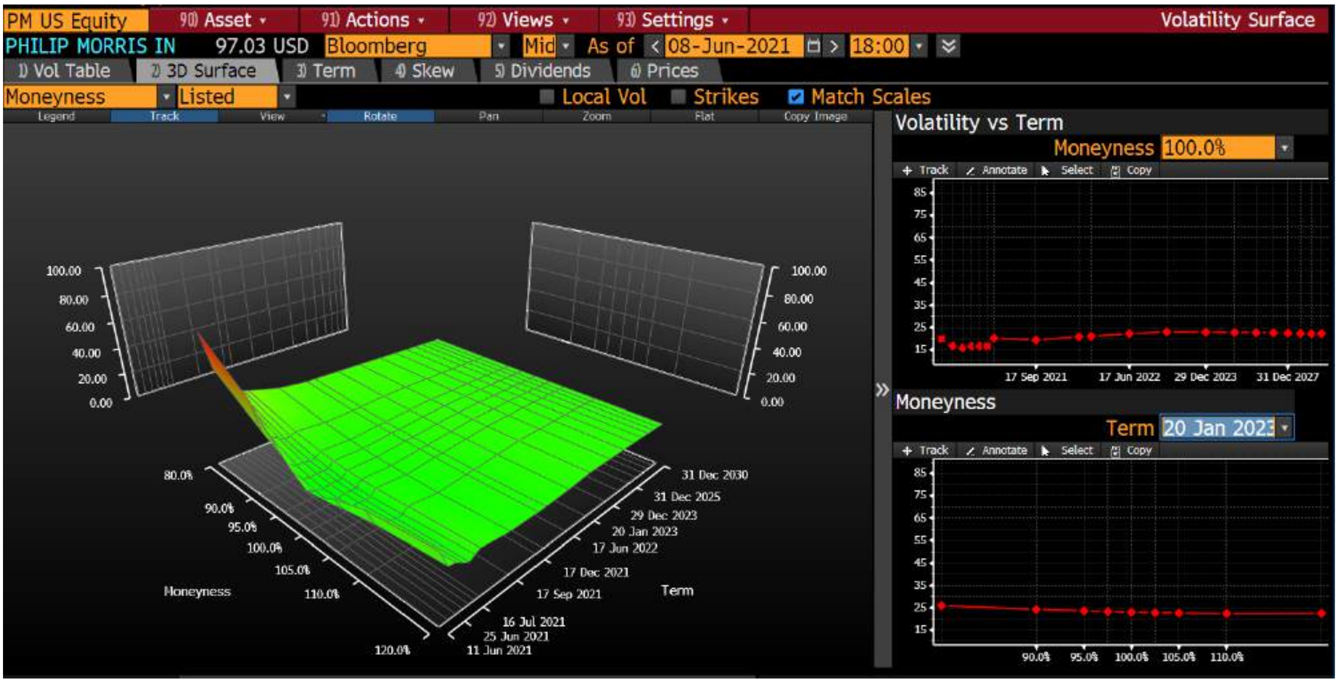


Figure A.3 Implied Volatility Surface of PM US Equity. Reference Date: 8th June 2021. Source: Bloomberg®

Spot	1 WK	2 WK	1 MO	3 MO	6 MO	9 MO	1 YR	2 YR	3 YR	5 YR
0.8215	0.8214	0.8213	0.8210	0.82	0.8186	0.8169	0.8154	0.8085	0.799	0.7726

Table A.5 USD/EUR Forward Curve. Reference Date: 8th June 2021. Source: Bloomberg®

B. Express Certificate with Conditional Memory Coupons pricing data

Underlyings [Currency]	BAER SW - CSGN SW - ZURN SW
Product Type Description	Express Certificate (Worst of Assets)
Initial Fixing Date	09-Feb-2021
Issue Date	17-Feb-2021
Final Fixing Date	09-Feb-2023
Redemption Date	16-Feb-2023
Issue Price	100
Quoting Type Description	Dirty
Call Frequency Description	Quarterly
Barrier Trigger Percentage	60%
Call Trigger Percentage	100%
Coupon Trigger Percentage	60%
Next Autocallable Date	09-Aug-2021
Barrier Type Description	European
Call Type Description	Auto Call Fixed
Currency Description	CHF
Quanto	No
Coupon Type Description	Conditional Memory Fixed Coupon
Coupon Frequency Description	Quarterly
Coupon Per Period	2.175
Next Coupon Observation Date	09-Aug-2021
Coupon Paid	2.175
Coupon Rate Per Annum	8.7
Coupon Paid Difference	2.175
Strike Percentage	100%
Redemption Amount	100

Table B.1 Financial Characteristics of the Express Certificate

Term	Par Rate	Zero Rate	Discount
6 MO	-0.7062	-0.7173	1.0036
SFFR0AG	-0.707	-0.7139	1.0042
SFFR0BH	-0.7004	-0.7104	1.0048
SFFR0CI	-0.6934	-0.7057	1.0053
SFFR0DJ	-0.6841	-0.7036	1.0059
SFFR0EK	-0.6749	-0.7004	1.0064
SFFR0F1	-0.6655	-0.6966	1.0070
SFFR0I1C	-0.6463	-0.6857	1.0087
2 YR	-0.6461	-0.6484	1.0131
3 YR	-0.5745	-0.5761	1.0175
4 YR	-0.4875	-0.4893	1.0198
5 YR	-0.395	-0.3969	1.0201

Table B.2 Forward and Discount Curve, Reference Date: 8th June 2021, Currency: CHF. Source: Bloomberg®

Underlying	Bloomberg Ticker	Initial Fixing	Strike Level (100%)	Knock-in Barrier Level (60%)	Coupon Barrier Level (60%)	Autocall Barrier Level (60%)
Credit Suisse AG	CSGN SE	12.30	12.30	7.38	7.38	12.30
Julius Baer Group	BAER SE	54.90	54.90	32.94	32.94	54.90
Zurich Insurance	ZURN SE	375.00	375.00	225.00	225.00	375.00

Underlying	Bloomberg Ticker	Spot Level	Dividend Yield
Credit Suisse AG	CSGN SE	9.806	0.515%
Julius Baer Group	BAER SE	61.08	1.174%
Zurich Insurance	ZURN SE	372.35	2.59%

Table B.3 The Underlying characteristics, Currency: CHF. Reference Date: 8th June 2021. Source: Bloomberg®

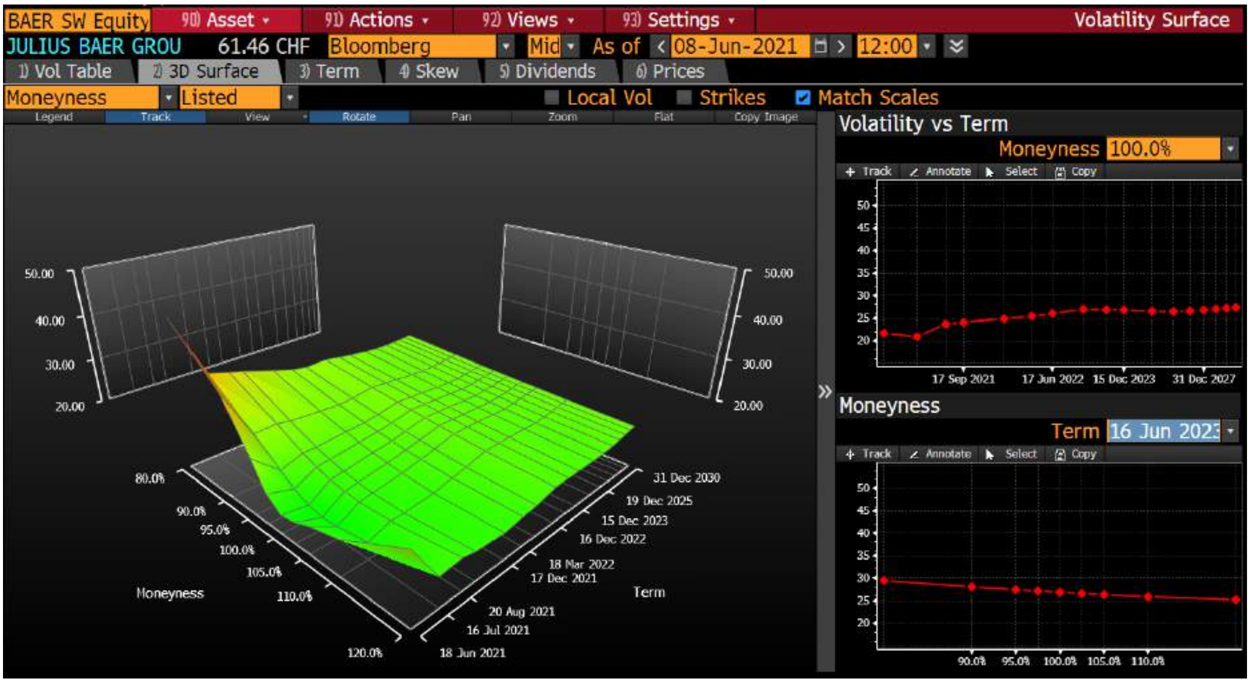


Figure B.1 Implied Volatility Surface of BAER SW Equity. Reference Date: 8th June 2021. Source: Bloomberg®

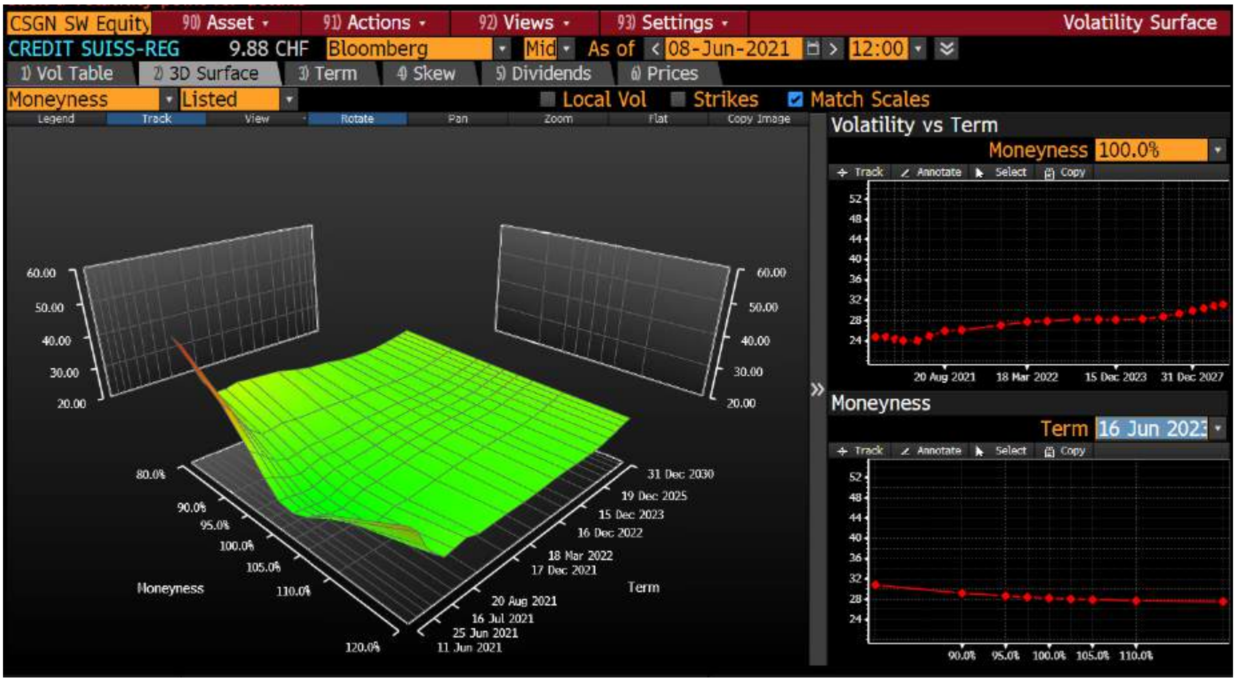


Figure B.2 Implied Volatility Surface of CSGN SW Equity. Reference Date: 8th June 2021. Source: Bloomberg®

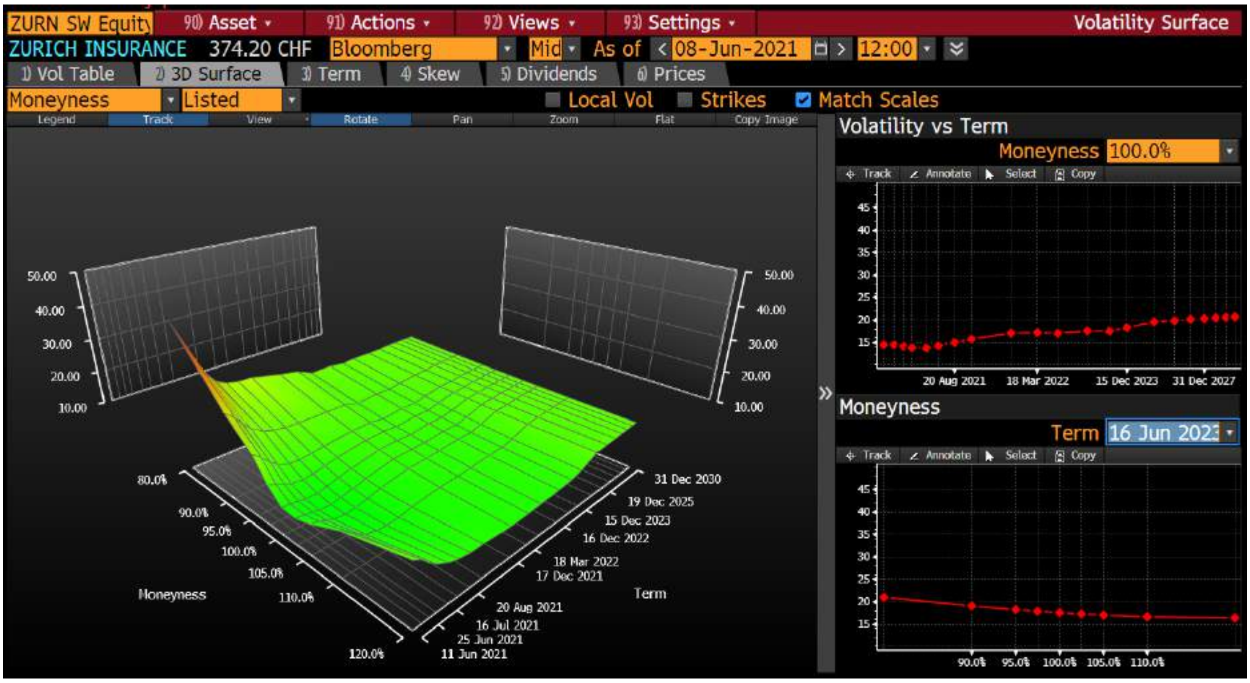


Figure B.3 Implied Volatility Surface of ZURN SW Equity. Reference Date: 8th June 2021. Source: Bloomberg®

C. The Pricing Platform

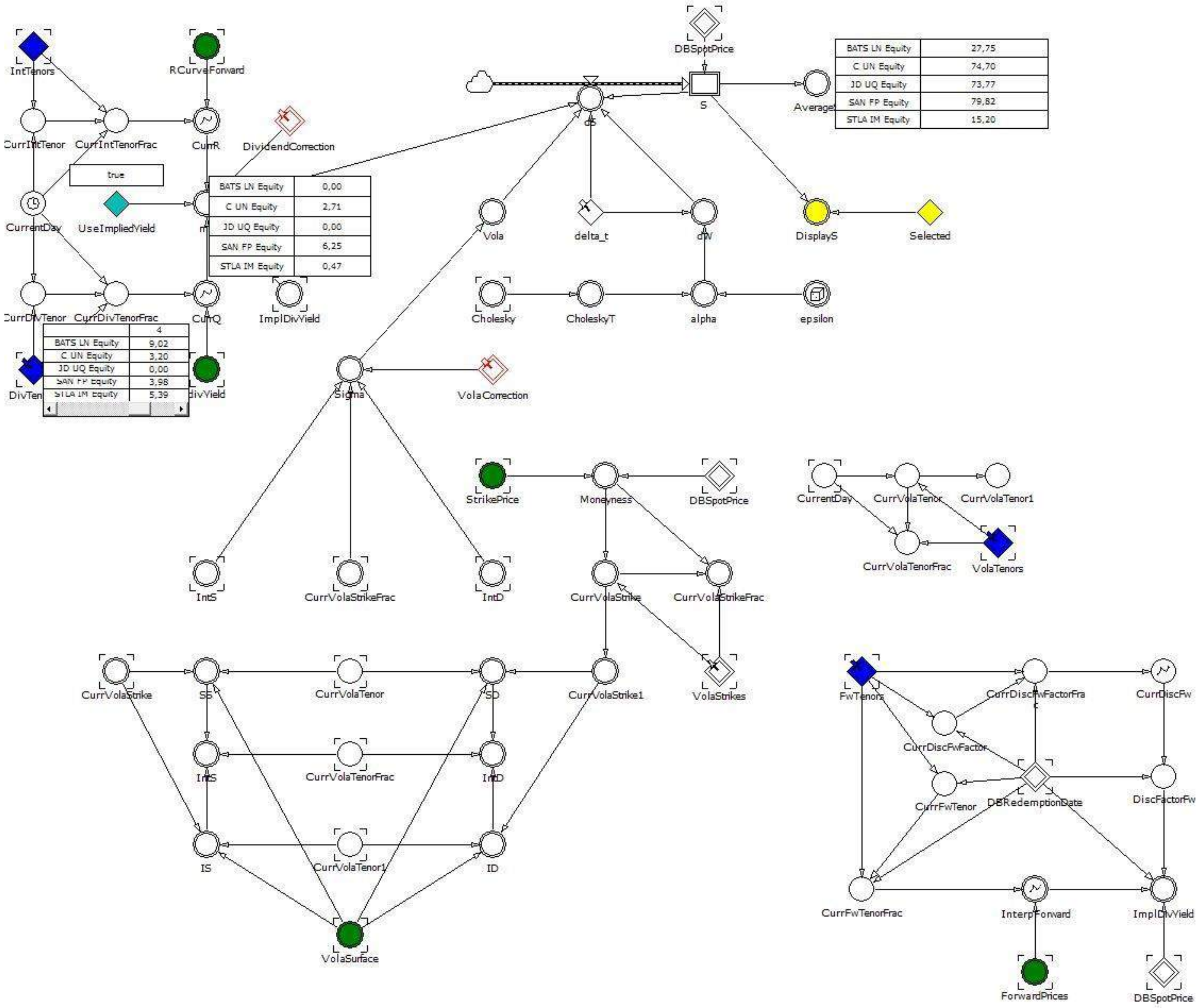


Figure C.1 Stochastic Simulations Core Engine

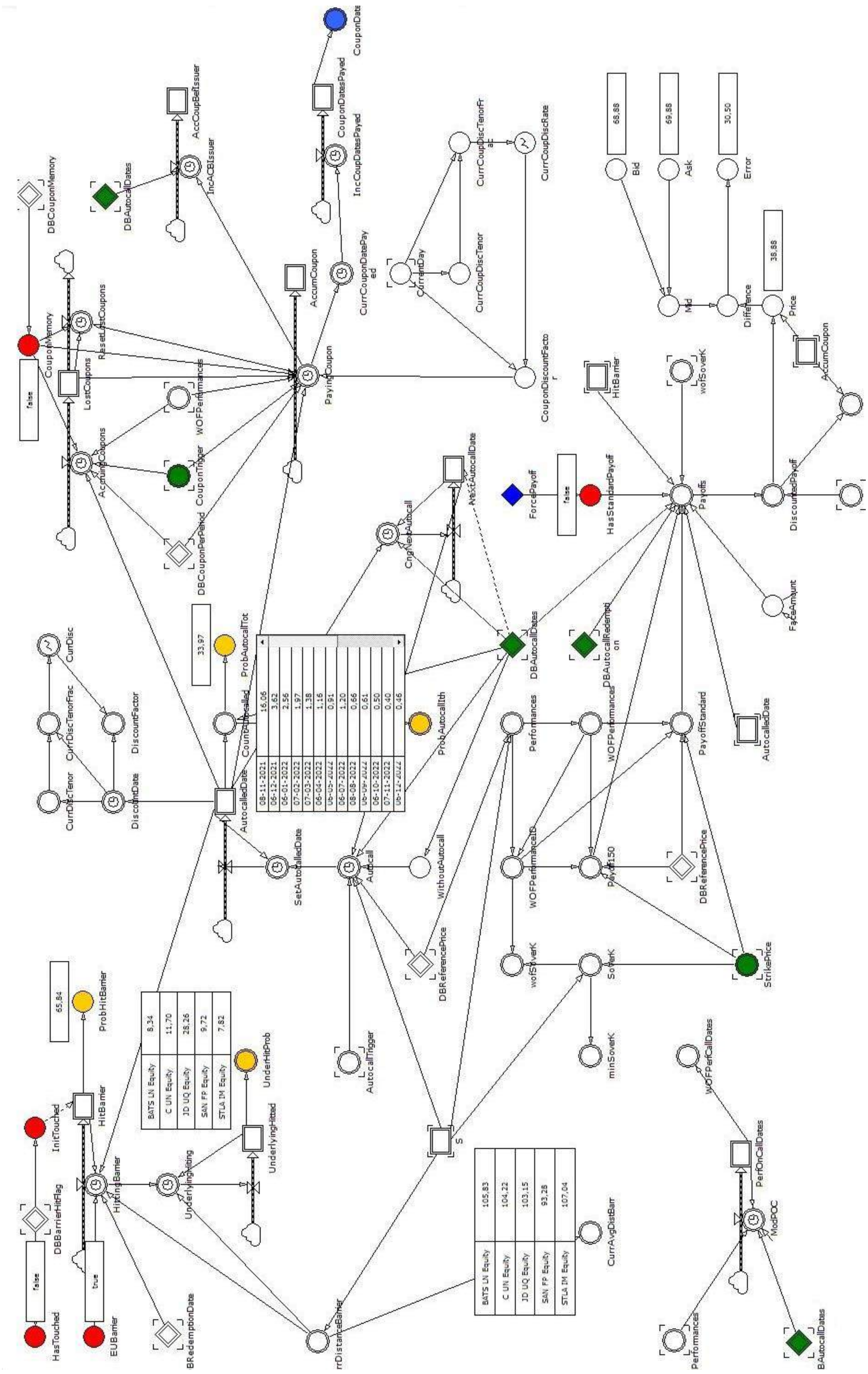
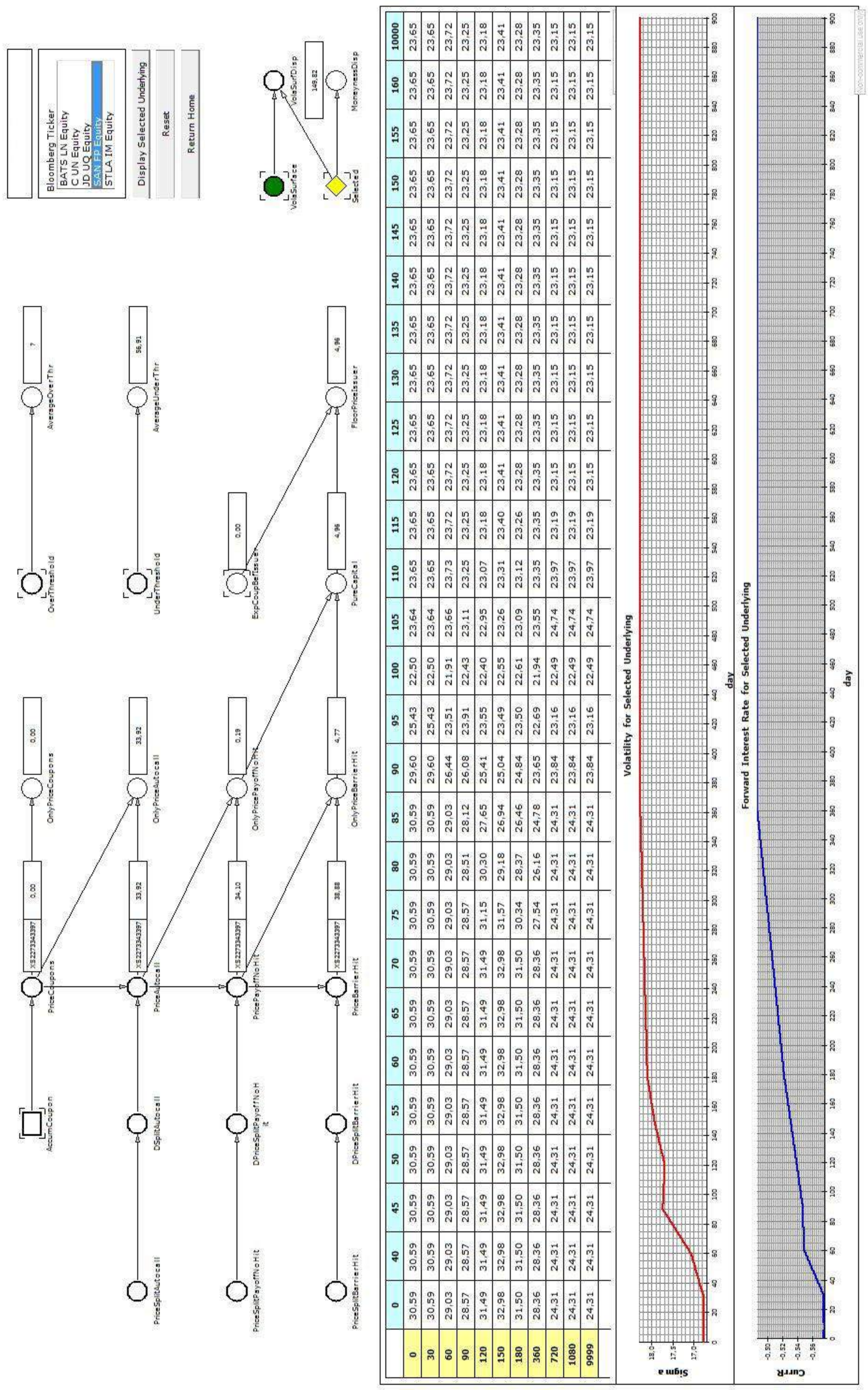


Figure C.2 Pay-Off Block Diagram



	0	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	10000
0	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	29,60	25,43	22,50	23,64	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65
30	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	30,59	29,60	25,43	22,50	23,64	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65	23,65
60	29,03	29,03	29,03	29,03	29,03	29,03	29,03	29,03	29,03	29,03	29,03	26,44	23,51	21,91	23,66	23,73	23,72	23,72	23,72	23,72	23,72	23,72	23,72	23,72	23,72	23,72	23,72
90	28,57	28,57	28,57	28,57	28,57	28,57	28,57	28,57	28,57	28,51	28,12	26,08	23,91	22,43	23,11	23,25	23,25	23,25	23,25	23,25	23,25	23,25	23,25	23,25	23,25	23,25	23,25
120	31,49	31,49	31,49	31,49	31,49	31,49	31,49	31,49	31,15	30,30	27,65	25,41	23,55	22,40	22,95	23,07	23,18	23,18	23,18	23,18	23,18	23,18	23,18	23,18	23,18	23,18	23,18
150	32,98	32,98	32,98	32,98	32,98	32,98	32,98	32,98	31,57	29,18	26,94	25,04	23,49	22,55	23,26	23,31	23,40	23,41	23,41	23,41	23,41	23,41	23,41	23,41	23,41	23,41	23,41
180	31,50	31,50	31,50	31,50	31,50	31,50	31,50	31,50	30,34	28,37	26,46	24,84	23,50	22,61	23,09	23,12	23,26	23,28	23,28	23,28	23,28	23,28	23,28	23,28	23,28	23,28	23,28
360	28,36	28,36	28,36	28,36	28,36	28,36	28,36	28,36	27,54	26,16	24,78	23,65	22,69	21,94	23,55	23,35	23,35	23,35	23,35	23,35	23,35	23,35	23,35	23,35	23,35	23,35	23,35
720	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	23,84	23,16	22,49	24,74	23,97	23,19	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15
1080	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	23,84	23,16	22,49	24,74	23,97	23,19	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15
9999	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	24,31	23,84	23,16	22,49	24,74	23,97	23,19	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15	23,15

Figure C.3 Price splitting and market data inspection