RISK MANAGEMENT MAGAZINE

Vol. 17, Issue 3
September – December 2022

AIFIRM

In collaboration with prometeia

Direttore responsabile (Editor in chief): Maurizio Vallino – Condirettore (Managing Editor): Corrado Meglio
INDEPENDENT MAGAZINE WITH NO PUBLIC FUNDING (Law 250/1990)
Poste Italiane - Spedizione in abbonamento postale – 70% aut. DCB / Genova nr. 569 anno 2005

IN THIS ISSUE

Scientific journal recognized by ANVUR and AIDEA

Risk Management Magazine
Volume 17, Issue 3, September - December 2022

Direttore Responsabile (Editor in Chief)
Maurizio Vallino (AIFIRM General Manager; Banca Carige, Italy)

Condirettore (Managing Editor)
Corrado Meglio (AIFIRM Vice President; Banca Credito Popolare, Italy)

Ownership, Newsroom and Secretariat:
Associazione Italiana Financial Industry Risk Managers (AIFIRM), Via Sile 18, 20139 Milano

Registration number at Court of Milan n. 629 dated 10th September 2004

ISSN Print 2612-3665 – **ISSN Online** 2724-2153
DOI 10.47473/2016rrm

E-mail: risk.management.magazine@aifirm.it; Tel. +39 389 6946315

Printing
Algraphy S.n.c. - Passo Ponte Carrega 62-62r 16141 Genova

The authors bear sole responsibility for the opinions expressed in the articles.

MAILED TO AIFIRM SUBSCRIBERS WHO ARE RESIDENT IN ITALY AND DULY REGISTERED

Journal printed on 20th November 2022

Peer review process on papers presented for publication

The papers that are presented to our magazine for publication are submitted anonymously to a double level of peer review.

The first level is a review of eligibility, implemented on the paper by the members of the Editorial Board, who assess the adequacy of the paper to the typical topics of the magazine.

The second level is a review of suitability for publication, implemented on the paper by two referees, selected within the Editorial Board, the Scientific Committee or externally among academics, scholars, experts on the subject who assess the content and the form.

Editorial regulation

"Risk Management Magazine" is the AIFIRM (Italian Association of Financial Industry Risk Managers) magazine, fully dedicated to risk management topics.

The organization includes the managing editor, a joint manager and an Editorial Board and a Scientific Committee composed by academics.

The magazine promotes the diffusion of all content related to risk management topics, from regulatory aspects, to organizational and technical issues and all articles will be examined with interest through the Scientific Council.

The papers shall be presented in Microsoft Word format, font Times New Roman 10 and shall have between 5.000 and 12.000 words; tables and graphs are welcome.

The bibliography shall be written in APA format and shall accuratel specify the sources.

An Abstract in English is required (less than 200 words) highlighting the Key words.

The authors bear sole responsibility for the opinions expressed in the articles.

The Statement on ethics and on unfair procedures in scientific publications can be found on our website www.aifirm.it.

Processo di referaggio degli articoli proposti per la pubblicazione

Gli articoli che sono proposti alla rivista per la pubblicazione sono sottoposti in forma anonima a due successivi livelli di referaggio.

Il primo livello di referaggio (di ammissibilità) viene effettuato sull'articolo dai membri dell'Editorial Board che ne valutano la congruità ai temi trattati dalla rivista.

Il secondo livello di referaggio (di pubblicabilità) viene effettuato sull'articolo da due referee scelti all'interno dell'Editorial Board, del Consiglio Scientifico o all'esterno tra accademici, ricercatori, esperti della materia, che ne valutano il contenuto e la forma.

Regolamento redazionale

"Risk Management Magazine" è il periodico di AIFIRM (Associazione Italiana Financial Industry Risk Managers) ed interamente dedicato ai temi del risk management.

La sua struttura organizzativa prevede, oltre al direttore responsabile, un condirettore, un Editorial Board e un Consiglio Scientifico formato da accademici.

La rivista favorisce la diffusione di tutti i contenuti afferenti i temi del risk management, dagli aspetti normativi a quelli organizzativi e alle technicalities e vaglierà con interesse, per mezzo del Comitato Scientifico, i contributi che le perverranno.

Gli articoli proposti dovranno pervenire in formato Microsoft Word carattere Times New Roman 10 ed avere un numero di parole compreso tra 5.000 e 12.000; è gradita la presenza di tabelle e grafici.

La bibliografia deve essere redatta in formato APA, specificando accuratamente le fonti.

Si richiede la predisposizione di un Abstract in lingua inglese (meno di 200 parole) con indicazione delle Key words.

Le opinioni espresse negli articoli impegnano unicamente la responsabilità dei rispettivi autori.

Sul sito www.aifirm.it è pubblicata la Dichiarazione sull'etica e sulle pratiche scorrette nella pubblicazione dei lavori scientifici.

Referees Year 2022

Implications of IFRS 17 in European financial stability: accounting methodology and evaluation modelling

Stefano de Nichilo (University of Cagliari)

Article submitted on 7th July 2022 and accepted on 18th September 2022, not submitted to double-blind peer review

Abstract

The purpose of this document is to provide an overview of IFRS 17 and its possible practices according to a context analysis conducted by EIOPA and ESMA. Furthermore, the implications of the standard will be evaluated with respect to a traditional life and pension products. This requires a deeper insight into the standard, the construction of fictive insurance product and the determination of measurements techniques. With the recent release of the standard, IFRS 17 is unexplored to many within the insurance industry. Thus, the effects of the standard on the financial statements of insurance companies and strategies for reaching objectives such as profit smoothing are yet unknown. Furthermore, the standard is principle based and does hence not specify a practice. Therefore, to determine a practice that complies with the standard and enables achievements of desired objectives is vital. In conclusion, IFRS 17 is expected to bring substantial benefits to financial stability in the EU, mainly through the transparency channel; the requirements in IFRS 17 may push insurance corporations to improve internal processes, including enhancing their internal risk management frameworks.

Keywords*:* Accounting Methodology and Evaluation Modelling, European Financial Stability, IFRS 17.

1. Introduction of IFRS 17

In the second quarter of 2017, the *International Accounting Standards Board* (IASB) published a new *International Financial Reporting Standard*: *IFRS 17 Insurance Contracts,* here referred to as IFRS 17 or the standard. The standard presents principles for recognition, measure, presentation and disclosure of insurance contracts (Tutino 2016). The goal is to provide accurate information to stakeholders and investors according to a context analysis conducted by EIOPA and ESMA. Furthermore, the standard will harmonize how risks are recognized and measured (PricewaterhouseCoopers 2020). Thus, the accounting principle is believed to increase the comparability of insurance companies. The standard will be mandatory from the year of 2023 and will apply to all insurance contracts including reinsurance contracts held and issued by entities.

Important impacts of the standard are how to value insurance contracts and when to recognize profits and losses. By the principles of the standard, the financial statements of insurance companies will be largely affected (Nissim 2010).

The standard proposes a hybrid of market consistent valuation and book value accounting (Tutino and Pompili 2017). Specifically, insurance contracts are valued by their fulfillment cash flows which are derived from:

a. the best estimate of future cash flows;
b. an adjustment to time value of money and financial risks;
c. an adjustment to non-financial risk.

The best estimate of future cash flows refers to an unbiased estimation of them using the probability weighted average The adjustment to time value of money and financial risks is carried out by means of a discount rate that should be consistent with the liquidity characteristics of the future cash flows. Thus, the return from bearing financial risks is captured by the discounted estimated future cash flows. Similarly, the return from bearing non-financial risks is captured by the non-financial risk adjustment.

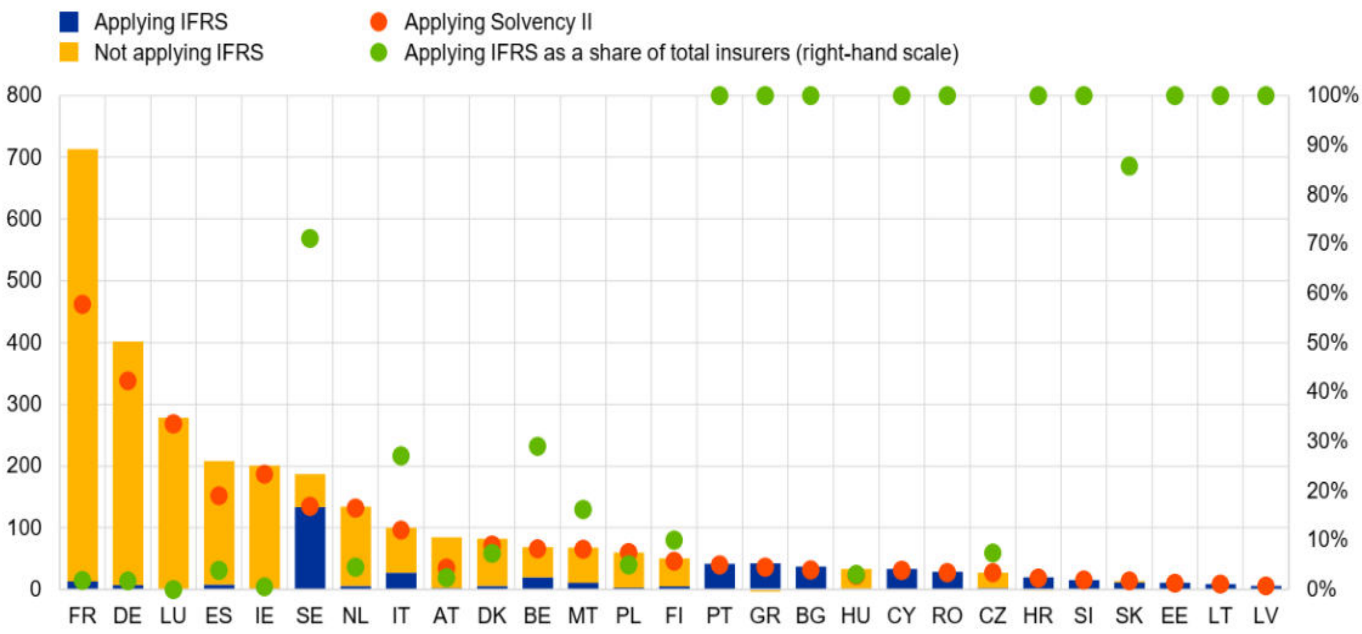
Under IFRS 17, profits are not recognized in a first initial step. Instead, any surplus in is captured in an item of the financial statement named *contractual service margin* (CSM). The CSM is subsequently adjusted for changes in fulfillment cash The CSM is also gradually released as the service is being fulfilled. Similarly, the risk adjustment is released as the contracts are released from non-financial risks. Profits are in turn recognized as the release of the CSM and the risk adjustment. The fact that surplus is captured by the CSM gives to the issuer a possibility of smoothing profits over time. Contrary, when there is a deficit in fulfillment cash flows a loss should be recognized instantly.

2. Methodology: forecasting of early adoption of IFRS 17

The implications of the standard will be analyzed and evaluated with respect to a traditional life and pension products. This requires a deeper insight into the standard, a construction of fictive insurance product and a determination of measurements techniques that compiles with the standard. A discussion of the data and methodology used by EIOPA and ESMA on the financial stability of IFRS 17 in its early adoption follows.

IFRS 17 will be applied to subpopulation of mainly large EU insurance corporations, as the requirement to apply IFRS 17 in the EU will cover consolidated financial statements of listed insurance corporations (European Financial Reporting Advisory Group 2020). In general, IFRS are required only for the consolidated financial statements of listed reporting entities across the EU. However, Member States can require or allow the use of IFRS for the consolidated financial statements of unlisted reporting entities and/or for their individual (separate) financial statements. According to data collected by the European Financial Reporting Advisory Group, only 500 insurers had been using IFRS at the end of 2018, out of a population of almost 3,000 insurance corporations in the EU (of which 2,402 had been applying Solvency II), with substantial cross-country heterogeneity (Figure 1). In general, it can be expected that smallest insurance corporations will not apply IFRS 17, with some exception in certain EU Member States that also allow or require IFRS for individual financial statements (Dobler, 2020). Consequently, in terms of size of their financial statement, those insurance corporations applying IFRS 17 can be expected to be the largest, representing a large share of the total financial statement of insurance corporations in the EU. The total financial statement of the 340 groups reporting regularly to the European Insurance and Occupational Pensions Authority (EIOPA) was approximately €10.477 trillion at the end of 2019, while the 2,716 insurance corporations reporting to EIOPA on a solo basis had a total balance sheet of €12.706 trillion. This shows that stand-alone insurers that are not part of a consolidated group accounted for no more than about €2.2 trillion (or about 20% of the size of those insurers that are part of a group).

Figure 1. Application of IFRS by insurers across EU countries at the end of 2018



Source: European Financial Reporting Advisory Group (2021)

3. Evidence of investigation: transparency, stability and risk management reporting

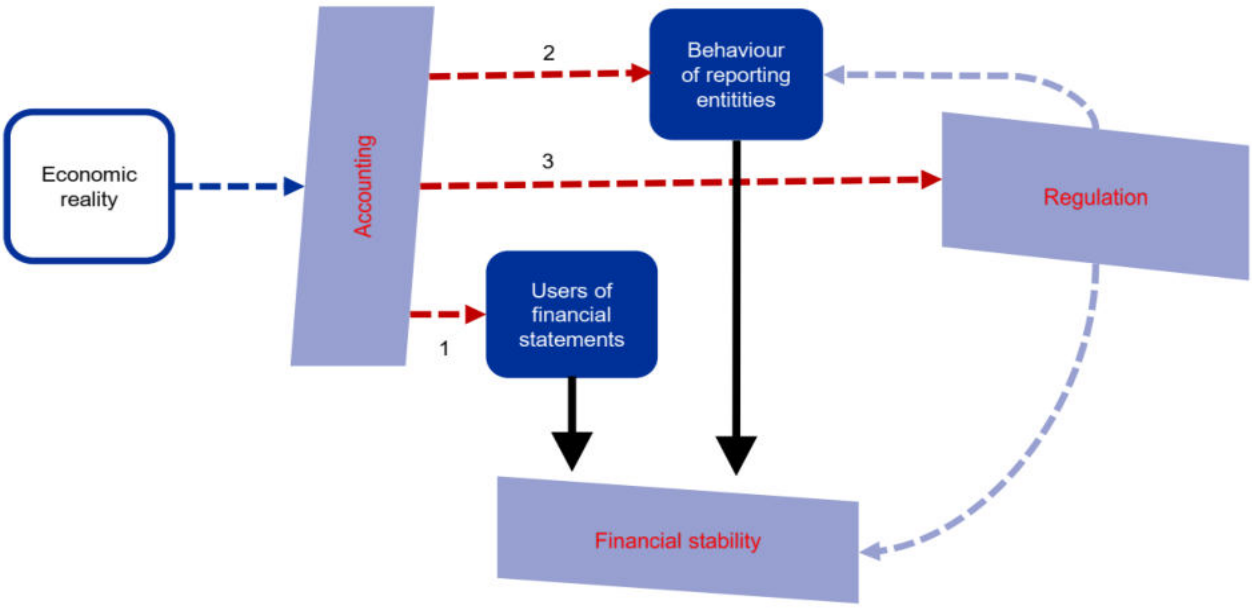
In 2018, EIOPA and the European Securities and Markets Authority (ESMA) have analyzed the impact of IFRS 17, touching upon financial stability considerations. EIOPA issued a report analyzing IFRS 17 from a financial stability perspective (European Financial Reporting Advisory Group 2021a). The paper concluded that IFRS 17 is expected to increase transparency and comparability in the insurance sector (Bertolotti and Van Wyk de Vries 2019), as it:

(i) provides stakeholders with better insights into insurers' business models, exposures and performance;
(ii) better reflects economic reality;
(iii) supports efficient risk management.

Overall, IFRS 17 was found to potentially strengthen financial stability in the EU. More recently, ESMA and EIOPA responded to EFRAG's draft endorsement advice. ESMA referred to financial stability aspects, highlighting that the provision of more transparent information has a beneficial effect on ensuring that financial market participants receive comparable and timely information. In addition, ESMA started that the effectiveness of IFRS 17 in depicting economic mismatches that may arise from the interplay between insurance liabilities and financial and non-financial assets backing those liabilities is particularly beneficial for financial stability (European Securities and Markets Authority, 2021). Similarly, EIOPA highlighted the benefits of IFRS 17 in strengthening financial stability in the EU (European Insurance and Occupational Pensions Authority, 2021). Regarding annual cohorts, EIOPA noted that, even though several ideas to replace this requirement were tentatively explored by EFRAG, none of them were found to be viable. Furthermore, EIOPA warned that an endorsement advice to exclude "intergenerationally-mutualised and cash-flow matched contracts" from the annual cohort requirement in IFRS 17 could lead to undesired (negative) consequences. ESMA expressed similar concerns and warned against any attempt to simply remove the annual cohort requirement without an appropriate replacement, noting that the requirements on the level of aggregation, including disaggregation into annual cohorts, are integral to the functioning of the entire standard.

Although the accounting process is not primarily meant as a tool to foster financial stability (Crenca 2017), there are at least three channels through which relevant and reliable financial information from the accounting process can affect financial stability through transparency (1), the behavioral response of the reporting entities (2), and regulation (3) (Figure 2).

Figure 2. Accounting and financial stability



Source: Author's elaboration, based on European Central Bank (2006) and International Accounting Standards Board (2017)

Transparency in accounting (EIOPA 2018), understood as the access to relevant, reliable and timely financial information about an economic agent, enables users of financial statements to make informed decisions of economic nature. It discourages economic agents (the reporting entities) from engaging in risky behaviors, transactions and supports internal and external decision-making processes. These are all beneficial to financial stability.

In this respect, financial statements prepared in accordance with a given set of accounting standards must provide information to financial market participants on the risks being taken on by the reporting entity and their impact in terms of financial position, performance and cash flows. Financial statements are helpful when they fairly depict the underlying economic reality of the reporting entity, thus reflecting the best objective evidence that is available at a certain reporting date, while not omitting any relevant information. Accounting standards can be seen as a "reporting language", translating economic reality into standardized metrics, such as assets and liabilities, and profits and losses. Changes in the "reporting language" can also change the translation of the economic reality.

There may be a trade-off between (individual) short-term effects and (broader) long-term effects of transparency on financial stability. Accounting standards can promote transparency. However, if negative issues relating to economic agents are disclosed to the public, this can trigger episodes of short-term instability. For example, the disclosure of large losses obtained by a financial institution informs investors about its deteriorated economic situation. This information can lead to a decline in its share price and to an increase in the interest rates on its debt securities, triggering second-round and spillover effects for similar institutions. However, these episodes of short-term instability are typically followed by a long-term strengthening of financial markets, as weaker entities are identified in a timely manner and forced to improve their performance. In the long run, transparency prevents the build-up of risks and vulnerabilities, thereby enhancing overall financial stability.

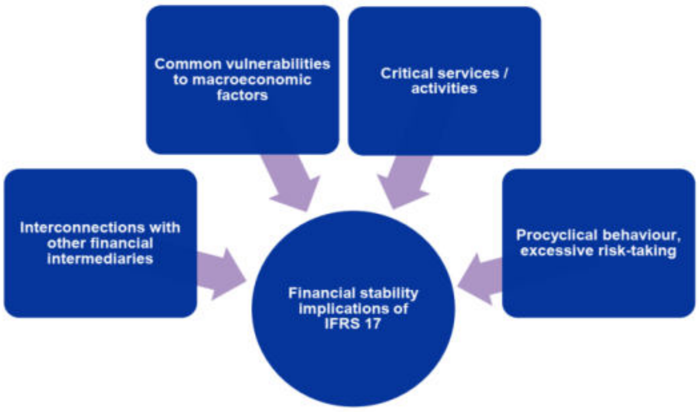
Some behavioral responses can be particularly detrimental to financial stability and should not be incentivized by accounting standards (Bloomer 2004). Behavioral responses that could harm financial stability include (Figure 3), for example:

(i) Procyclical behaviors (in asset allocation, pricing, recognition of gains and losses, etc.),
(ii) Excessive concentration of exposures and/or interconnections,
(iii) Inappropriate or excessive involvement in certain activities and/or products,
(iv) Discouragement of desirable activities and/or products,
(v) Excessive risk-taking (particularly when not appropriately addressed by the relevant prudential framework), and
(vi) Pertaining to investors as users of financial statements, collective behavior that could exacerbate market price movements.

In our assessment, IFRS 17 is expected to make a substantial contribution to financial stability by promoting internationally comparable accounting practices and by increasing transparency in the insurance sector. The current accounting standard for insurance liabilities, that is IFRS 4, is found to be inappropriate by many stakeholders, hampering transparency and comparability with other sectors or within the insurance sector (European Securities and Markets Authority, 2021).

IFRS 17 defines a clear treatment of insurance liabilities and should contribute to providing a fair view of the financial position and performance of insurance corporations. In addition, detailed and consistent disclosures are expected to increase transparency in the sector. There have been concerns as to whether IFRS 17 could disincentivize the use of reinsurance, an activity considered to be desirable from both a safety and soundness and a financial stability perspective.

Figure 3 Sources of insurance-related systemic risk and their impact on financial stability implications of IFRS 17



Source: Author's elaboration, based on European Systemic Risk Board (2015 and 2018)

4. Conclusion

In general, IFRS 17 is expected to bring substantial benefits to financial stability in the EU, mainly through the transparency channel (European Financial Reporting Advisory Group 2021b). By fostering comparable accounting practices and by increasing transparency in the insurance sector, IFRS 17 addresses the shortcomings of the current accounting standard for insurance contracts (IFRS 4). That is particularly important in a macroeconomic environment that is highly challenging for insurance corporations (Chong-Tai Bell, Windsor and Yong 2020). Through the transparency channel, the adoption of IFRS 17 is expected to provide more accurate and timely information to users of financial statements, which they can use to make informed economic decisions. IFRS 17 can also encourage insurance corporations to avoid overly risky behaviors and transactions, such as situations in which losses are not immediately recognized or remain "hidden". In addition, the requirements in IFRS 17 may push insurance corporations to improve internal processes, including enhancing their internal risk management frameworks. At the same time, IFRS 17 is not found to exacerbate systemic risk in the insurance sector through any of the channels previously identified by EIOPA and the ESRB.

However, this research has identified some features of IFRS 17 that deserve particular attention in the implementation of the standard in order to ensure its financial stability benefits.

References

- Bertolotti, A., Van Wyk de Vries, K. (2019). IFRS 17: The impact on internal audit, pwc.blogs.com, 30 May 2019.
- Bloomer, J. (2004). Impact of Insurance Accounting on Business Reality and Financial Stability, *The Geneva Papers on Risk and Insurance*: Issues and Practice, 29, 56-62.
- Chong-Tai Bell, M., Windsor, P. and Yong, J. (2020), Accounting standards and insurer solvency assessment, FSI Insights on policy implementation, *Financial Stability Institute*, No 26, July.
- Crenca G., (2017). L'attuario e l'Enterprise Risk Management, ANRA.it, 6 Settembre 2017.
- Dobler, M. (2020), The European Union's endorsement of Amendments to International Financial. *The journal of Corporate Accounting & Finance*, Volume 31, Issue 3, July 2020 pag 190-196
- EIOPA report 18-717, EIOPA's analysis of IFRS 17 Insurance Contracts. Issue 18, October 2018.
- European Financial Reporting Advisory Group (2020), IFRS 17 and IFRS 9 – Issues Paper, April.
- European Financial Reporting Advisory Group (2021), IFRS 17 Insurance Contracts as amended in June 2020 – Draft Final Endorsement Advice – Annex 1 – Application of annual cohorts to intergenerationally-mutualized and cash flow matched contracts, March.
- European Insurance and Occupational Pensions Authority (2021), EIOPA's comments on EFRAG's draft endorsement advice on IFRS 17 Insurance Contracts, letter, January.
- European Securities and Markets Authority (2021), Ref: EFRAG's Draft Letter to the European Commission Regarding Endorsement of IFRS 17 Insurance Contracts as amended in June 2020, letter, January.
- Nissim, D. (2010). *Analysis and valuation of insurance companies-industry study number two*. Centre for Excellence in Accounting & Security Analysis, Columbia Business School. Retrieved from http://www8.gsb.columbia.edu/ceasa/research.
- PricewaterhouseCoopers (2020), IFRS 9 for insurers, April.
- Tutino, M. (2016). *Fair Value Accounting nei Bilanci delle Società*, Kluwer, Padova.
- Tutino M., Pompili M. (2017). Fair Value Accounting and Management Opportunism on Earnings Management in Banking Sector: First Evidences. *Corporate Ownership and Control* 15(2), 59-69.

Risk-Adjusted Loan Pricing

Franco Fiordelisi (University of Essex, UK and University of Rome III, Italy), Carlo Palego (Banca di Asti, Italy), Annalisa Richetto (Intesa SanPaolo, Italy), Giulia Scardozzi (Università degli Studi Internazionali di Roma, Italy)

Article submitted to double-blind peer review, received on 28th April 2022 and accepted on 20th October 2022

Abstract

We analyze what are the main pricing components for performing loans. By exploiting a survey conducted by the authors in AIFIRM (2021), we provide empirical evidence about whether and to what extent various pricing criteria are related to interest income within the internal model framework. Our main findings are that banks' interest income is positively related to the adoption of advanced internal risk-based models, the calculation of the break-even rate, and the implementation of the risk-adjusted profitability measures in the pricing, while it is negatively linked to higher market competition, a decentralized pricing function (allowing more customer-oriented loans prices). The results make urgent to monitor and develop improve current risk models to support both central offices and the sales network in the process of formulating loan prices and monitoring the value consequently created.

Keywords: Pricing, Interest income

Acknowledgment: We would like to thank AIFIMR, and especially Corrado Meglio and Maurizio Vallino for kindly inviting us to develop this paper and allowing us to report the results of the survey carried out in the *AIFIRM's Position Paper on "*Pricing and risk-adjusted return measures". A special thanks to Andrea Favretti (Prometeia) for his outstanding support and active role in the AIFIRM (2021).

1. Introduction

The accurate pricing of lending activities has become crucial for banks over the last decades given the unprecedented low levels of market interest rates in the Eurozone. The European Authorities have been inviting banks to adopt a risk-adjusted pricing framework adequately integrated with banks' business model, risk profile, and overall risk governance. The methodological and organizational process of determining risk-adjusted pricing is made even more complex by the ongoing Covid19 pandemic. Through the highly asymmetrical impacts on customer segments and industrial sectors, loan pricing assesses the risk component of the sectors themselves even more relevant from a prospective and macroeconomic perspective (such as Covid19 pandemic).

How do banks price their loans? We address this question in the first part of the paper by identifying the main pricing components adopted by the banks that joined the survey. Loan pricing is the sum of three components: the "break-even rate" (or "hurdle rate"), the "market", and the "client" components. The "break-even rate" or "hurdle rate" component is the rate that generates interest flows to cover operating costs, expected losses, and the remuneration of production factors (in particular, capital and liquidity) employed (also called price to value). This rate is considered a threshold or minimum rate. The "market" component is interpreted as the spread (usually a mark-up) on the hurdle rate aimed at achieving expected revenues on the loans granted taking care of the market considerations.

This component may therefore be determined according to the market segment to which the borrower belongs, the size, and the technical form of the loan. The sum of these two components (hurdle rate and "market") represents the "internal benchmark rate" of a credit transaction (so-called price-to-market). The "client" component is interpreted as the spread (either markup or markdown) on the "reference rate" of the loan transaction. This component is determined based on the "specific" or "idiosyncratic" characteristics of the borrower and cannot be determined a priori: it is based on the specific relationship with the borrower, the possibility of cross-selling and similar considerations. The sum of these three components provides the "actual rate" charged to the borrower on the loan transaction.

What is the relationship between pricing components and banks' profits? We address this question in the second part of the paper. To this aim, we run an empirical exercise where variables capturing pricing components are obtained by transforming answers obtained in the survey in the AIFIRM (2021) into a dummy or categorical variable. Bank profits are measured by a few variables by the ratio of interest income on total loans. To this aim, we developed a standard panel data regression model and collect a sample of Italian banks between 2017 and 2020.

The remainder of the paper is structured as follows: first, we summarize past scientific papers dealing with loan pricing (section 2), then we describe what are the three main pricing components and their organizational issues (section 3). The discussion of these three pricing components relies on AIFIRM (2021), which is a working paper realized by the authors in collaboration with the Italian Association of Financial Risk Management Industry summarizing the main pricing methodological and organizational issues and running a survey conducted among Italian banks. In section 4, we report the main results of the survey carried out by the authors in AIFIRM (2021)[1].

[1] We would like to thank AIFIRM, and especially Corrado Meglio for the great support provided us during the survey. We also would like to thank all participants of the AIFIRM (2021) working group, and banks participating in the survey. A special thanks to Andrea Favretti (Prometeia) for his

2. Literature review

Loan pricing is widely discussed in the literature. Specifically, risk-adjusted pricing was first discussed in the 1990s and subsequently implemented by banks (Greenspan, 1995) when the benefits were highlighted in the literature. Risk-adjusted pricing is a suitable practice to improve the performance of banks, which experience an increase in profits because of adopting these practices (Jung and Strohhecker, 2009). Competition in the banking market has led to lower interest rates in the past, which has eroded banks' operating profits (Motley, 2006) thus necessitating risk-adjusted pricing policies by banks. The literature has grown considerably in the aftermath of the introduction of the new regulatory frameworks by the Basel Committee. Hasan and Zazzara (2006) derived mathematical formulas to allow bankers to calculate credit risk reserves regarding Basel II regulations; specifically, the authors proposed a methodology to estimate risk-adjusted interest rates for bank loans in the corporate sector according to Basel II capital requirements. Similarly, Curcio and Gianfrancesco (2009) derived a similar formula using as input data the factors to be used for the application of internal models to estimate credit risk. Ruthenberg and Landskroner (2008) also conducted the same study using data from Israelian banks, showing that larger banks can attract the best borrowers because they can apply internal models through which risk-adjusted pricing results in lower interest rates.

This paper aims at formalizing how risk-adjusted pricing formulas can be amended under IFRS9. The main change introduced by IFRS9 compared to IAS39 is the calculation of reserves from a forward-looking perspective: the interaction of IFRS9 with the Basel regulations is thus able to promote financial stability (Novotny-Farkas, 2016) through a more correct measurement of credit risk from a lifetime rather than an annual perspective. The IFRS9 regulations provide little detail regarding the possibilities of measuring lifetime expected loss. Chawla et al. (2016a), Skoglund (2017), Xu (2016), and Bellini (2019) provide some suggestions for calculating credit risk under the IFRS9 framework. Engelmann (2021) derives in more detail some formulas for calculating lifetime expected credit loss according to IFRS9, both for fixed and floating rates, concluding that the most adopted formulas are inconsistent with measurements based on discounted cash flow methods, the latter being more onerous for banks.

The new method of calculating reserves to cover credit risk, therefore, has an impact on banks' regulatory capital: Kruger et al. (2018), and Seitz et al. (2018) showed that reserves are calculated according to IFRS9 would allow banks to be promptly recapitalized in economically adverse times. However, the impact is not homogeneous across countries: Loew et al. (2019) analyzed European countries and found that financially distressed countries, such as Greece and Italy, experience a greater impact of the new IFRS9-compliant reserve calculation methods than financially stronger countries.

3. Pricing components

Loan pricing is defined as the determination of the lending rate on bank loans charged by the bank to ordinary customers. This is the sum of three components: the "break-even rate" component (section 3.1), the market component (section 3.2), and the "client" component (section 3.3).

3.1 Price to value

The "price to value" or "hurdle rate" is designed to cover the underlying costs of the transaction. It is therefore a break-even price that can vary significantly depending on the cost components considered (cost of interest rate risk, funding, capital, credit risk costs, direct and indirect transaction costs) and how they are calculated. In the main practice adopted by the industry, the hurdle rate represents the minimum remuneration level (break-even) of the credit transaction.

The basic costs, i.e., those typically included by the banking industry in the algorithm for calculating the hurdle rate, are[2]: Cost of interest rate risk, funding cost, cost of credit risk, cost of the remuneration of employed capital expected by shareholders, and operating costs.

The cost of interest rate risk (base rate) is the cost of hedging the interest rate risk generated by the lending operation. It is calculated based on the market risk-free curve possible movements or volatility and according to the financial characteristics of the operation, such as duration, amortization, and type of rate, as well as to the banking book asset and liability durations. The funding cost is the cost of using the liquidity provided by the bank for its lending operation by paying an additional spread over the market risk-free rate. This cost is a function of the duration of the loan and the corresponding funding source and of the other specific liquidity characteristics of the source itself. The cost of funding thus represents the remuneration of the liquidity factor used for granting the loan and it is defined considering the characteristics of the bank's funding sources and the liquidity conditions of the interbank market. Credit risk cost covers the expected loss generated by the credit transaction. The bank calculates an expected credit loss and an unexpected credit loss on each credit exposure. The former represents the cost a bank can expect on average from the possible default of the financed counterpart, in respect to which the bank must, in each year in which that exposure remains in its assets, set aside a corresponding amount (provision), in line with the accounting principles and in compliance with the indications of the Supervision Authority too. The cost of the remuneration of employed capital expected by shareholders is related to minimum capital requirements

outstanding support and active role in the AIFIRM (2021). Last but not least, we would like to thank Maurizio Vallino, and Corrado Meglio for their kind inviting us to develop this paper and allowing us to report the results of the survey carried out in AIFIRM (2021).

[2] The cost components described are not exhaustive of all possible cost components that may come into play in any loan transaction. The exact identification/listing (and subsequent quantification) of cost components is the responsibility of the individual lending bank.

to cover the unexpected credit loss over the remaining life of the exposure. The capital must be remunerated at market prices or, in the absence of reliable market references, based on alternative criteria for adequate shareholder remuneration. The cost of equity, the function of the unexpected credit loss (UCL), is thus the remuneration of the "capital" input (or production factor) absorbed by the credit transaction. In final, operating costs are those directly attributable to the operation (direct costs) and the costs indirectly attributable as a share of overall costs ("management costs" or "industrial product costs").

The sum of the individual cost components, determined using specific calculation models, is used to determine the hurdle rate (*hr*), as shown in model (1).

$$hr = r + \frac{S_{fund} + S_{ECL} + S_{UCL} + S_{man}}{n} \quad (1)$$

where r is the base rate calculated based on the risk-free market curve for the date of the loan; S_{fund} is the spread over the base rate against the cost of funding calculated at the date of the loan; S_{ECL} is the spread (in percentage) for the cost of credit risk; S_{UCL} is the spread (in percentage) for the cost of unexpected loss (or absorbed capital factor); S_{man} is the spread (in percentage) for operating costs, and n represents the contractual term of the loan (expressed in year and fraction of year).

3.2 Credit risk cost Lifetime compliant

The spread covering the cost of credit risk represents the component covering the credit loss that the bank is expected to incur during the residual life of the granted loan (LifeTime Expected Credit Loss - LTECL). LTECL differs from the expected credit loss (ECL), which is calculated by the bank's internal credit loss measurement systems and typically has a time horizon of one year[3]. The lifetime expected loss is estimated as follows:

$$LTECL = \sum_{i=1}^{n} PD_{f,i} * LGD_{f,i} \quad (2)$$

where $PD_{f,i}$ and $LGD_{f,i}$ are the Probability of Default (PD) and Loss Given Default (LGD) *forward* parameters respectively estimated concerning each time bucket i ($i=1, \dots, n$) into which the remaining lifetime of the loan for which the lender's rate is being determined is conventionally divided.

They are generally obtained from transition matrices calculated based on AIRB ratings, possibly adjusted, under a management perspective, to neutralize the strictly prudential and regulatory components)[4]; and n is the last bucket into which the residual lifetime of the loan for which the rate is being determined is conventionally divided.

Compared to the annual ECL, the Life-Time Expected Credit Loss, therefore, introduces the components of the forward credit loss, i.e., expected from the financed counterpart's default event at the various future points in time into which the remaining life of the loan is divided:

$$\boldsymbol{S_{ECL}} = \sum_{i=1}^{n} \frac{\boldsymbol{PD_{forward,i} * LGD_{forward,i}}}{\boldsymbol{(1 - PD_{forward,i} * LGD_{forward,i})}} * \frac{\boldsymbol{1}}{\boldsymbol{(1+r_i)^i}} \quad (3)$$

where S_{ECL} is the previously mentioned spread, r_i is the risk-free curve rate for maturity i. Several elements can be deduced from formula (3).

First, *ECL* is calculated from the credit risk parameters (usually produced by the AIRB system in its management application - AIRBGest[5]), as PD and LGD. The adoption of these parameters implies that the credit risk costs to be covered by the lending rate are represented by the unrecoverable costs and foregone recoveries at the closure of a currently performing loan position, conditional on the event of default of the latter before the final maturity of the exposure. The credit risk parameters of PD and LGD, applied to the estimated future exposure at the time of default (EAD), replace the size of the accounting provisions in the calculation of the "cost of credit risk" spread.

Second, the expected credit loss (in the hurdle rate framework) is calculated in lifetime logic based on forwarding or multi-period PDs. In principle, forward values should also be used for the LGD parameter in the lifetime formulation of the ECL. The adoption of lifetime logic, i.e., about the entire residual life of the transaction, is also very important in determining the cost of credit risk because it adjusts the risk with the duration and therefore the greater uncertainty of the transaction.

[3]Assuming for simplicity of representation the exposure (EAD) to be constant and unitary the ECL is given by the product between the Probability of Default (PD) and the Loss Given Default (LGD).
[4]Forward parameters are estimated to represent the measured risk (probability of default, loss at default) over a longer time horizon than the typical AIRB models time horizon (1-year) and therefore can be extended to the entire residual life of the position. So the application of such parameters is necessary in calculation processes that must determine metrics with reference to the entire life of an exposure, such as credit pricing. Finally, AIRB models are typically used as the basis for estimating forward parameters from a management perspective, i.e. they are stripped of the strictly prudential components required to calculate RWA for regulatory purposes, such as LGD adjustments for downturns, MoC and others.
[5]With regard to credit risk, supervisory regulations provide for two methods of calculating capital requirements: the Standardized Approach and the Internal Rating Based (IRB) method, in which risk weights are a function of banks' internal assessments of debtors. The internal ratings-based approach is divided into a Foundation Internal Rating Based (FIRB) and an Advanced Internal Rating Based (AIRB) IRB, which are differentiated by the risk parameters that banks must estimate; in the foundation approach, banks use their own estimates of PDs and regulatory values for other risk parameters, while in the advanced approach all the relevant risk parameters are internally estimated.

The lifetime logic adopted by the banking sector for the calculation of the ECL in the calculation of the hurdle rate for Stage 1 positions is like that adopted under IFRS9 and related supervisory rules, to calculate the cost of credit in the balance sheet for performing positions classified as Stage 2. In addition to being consistent with the recurring nature of the costs that the annual lending rate is required to cover, this approach allows for a better estimate of risk from a Lifetime perspective and greater consistency with the accounting evidence.

The logic applied for pricing purposes, however, has some peculiarities within the IFRS9 framework. The lifetime ECL IFRS 9-compliant approach requires the use of macroeconomic scenarios expected over a given time horizon, usually three years, for the valuation of Stage 2 positions (the predominant approach used in the industry is the so-called "multi-scenario"); by contrast, for pricing purposes, the industry's practices in adopting macro scenarios vary, with no scenarios applied for lifetime expected credit loss computation purposes (that is the most widespread banking practice), with some banks excluding Stage 1 positions from some scenarios, others weighting the scenarios differently and others (very few) adopting the IFRS9 multi-scenario in full. The IFRS9-compliant frameworks adopted by the banks provide for the use of forwarding LGD curves to measure the expected loss on Stage 2 positions: that is true, particularly for larger banks; however, as noted above, algorithms for calculating the credit cost component of the hurdle rate typically consider the AIRB LGD parameter to be constant over time and equal to the parameter produced for the managerial one year expected credit loss computation purposes.

3.3 Price To Market

Price to market refers to the strategic direction and the expected risk-adjusted profitability of the credit institution. Specifically, the price is calibrated considering the range and type of products and services offered to customers, the target market, and therefore the type of business model specific to each financial institution. A fundamental regulatory element, which can influence an institution's risk policies, concerns the regulation of usury, which, especially in conditions of high risk-return, is an essential constraint on pricing policies and which, in certain contexts, could influence lending policies, with direct consequences for the local and national economy and the related social implications.

Monitoring and reporting activities are worth mentioning. To guarantee an adequate market placement and sustainable profitability over time, it is necessary to carefully monitor the discipline of the prices applied and the market shares by segment and product, and also to activate the necessary corrective actions to the pricing strategies by the strategic supervisory bodies.

Two types of pricing can be envisaged in the pricing-to-market: "public prices", which are communicated to customers in prospectuses for all product types, and “benchmark prices”, which are defined for specific product types and considered the specific risk level of both the product and customer group.

The formulation of "Public prices" takes place considering also external factors (e.g., reference regulations), specifically for all types of products, and in respect of the defined target market, considering and considering the strategic and business initiatives promoted by the institution; in this context, prices are defined in line with the reference regulations which act as the maximum standard value applied to customers. Public prices are therefore the prices communicated in the information provided to customers (transparency).

In summary, price-to-market is aimed at identifying and structuring possible mark-ups for price-to-value (hurdle rate), through "external" benchmarks (assessment of market prices on clusters of comparable customers, products, areas, etc.) or "internal" benchmarks (assessment of prices applied on the bank's customer portfolio, also in this case employing an expert or statistical clustering analysis). It should be noted that the basic logic underlying the formulation of the price-to-market must also consider the competition of the market, which does not always allow for the setting of mark-ups on hurdle rates fully consistent with the desired profitability or risk/return objectives. In addition, analytical calculation models for the determination of the price-to-market component are not yet widespread in the Italian banking industry at least, showing a significant difference from the quantitative-analytical approach that has long been used by Italian banks to determine the price-to-value component of the final interest rate applied to customers (borrowers).

3.4 Price To Client

The price-to-client leads to the final price applied to the customer for the given credit transaction, possibly even in derogation of the list price foreseen for that transaction. While the Price to Value and the Price to Market components are two structured processes, this component introduces subjectivity in the pricing process since the final rate may be subject to further evaluation at the overall level of the customer or of a specific loan portfolio.

Pricing actions can therefore be defined with a structural logic or dedicated to individual positions with the counterparty's membership of specific clusters, consistency with strategic/commercial objectives, the counterparty's overall profitability characteristics, and the overall assessment of the relationship with the counterparty. The final price, therefore, reflects market logic that leads the price-to-client to a value that may differ from the list price due to discounts, agreements, competitive pressures, incentive systems, and so on. In the case of the Price to Client, therefore, the adjustment is linked both to methodological aspects (e.g., definition of metrics for measuring the risk-adjusted profitability of the transaction and the client as a whole, analysis, and assessments relating to the performance of the relationship with the counterparty, etc.), and to process aspects (e.g., influence on the price by portfolio policies, commercial campaigns, cross-selling, etc.).

4 The Survey

To support the analysis of the pricing process, we discuss some of the results from the AIFIRM (2021) survey[6], which involved 20 Italian banks (eight "Significant" and twelve "Less Significant" banks).

First, we focus on the model used by banks to measure credit risk. As shown in Table 1, the standard approach is the most widely adopted (65%). The internal model (specifically AIRB) is adopted by the remaining 35% of respondent banks for, at least, a part of their loan portfolio. Not surprisingly, Less Significant banks mostly use a standard approach to calculate Risk-Weighted Assets (RWA) for credit risk measurement.

Table 1 Survey respondents' composition

The table shows the number of banks using standard or internal models (e.g., AIRB) for Risk-Weighted Assets computation. Source: Authors' own production using data of AIFIRM (2021).

Model	Less Significant banks	Significant banks	Total
Standard	10	3	13
AIRB	2	5	7
Total	12	8	20

Surprisingly, not all Significant banks adopted an internal model. In general, banks using internal models mostly do so for corporate banks (see Figure 1).

Figure 1 Segments for which the Price to Value has been applied

The figure reports the answers to the question: "To which macro-segments the Price To Value has been applied?" Source: Authors' own production using data of AIFIRM (2021).

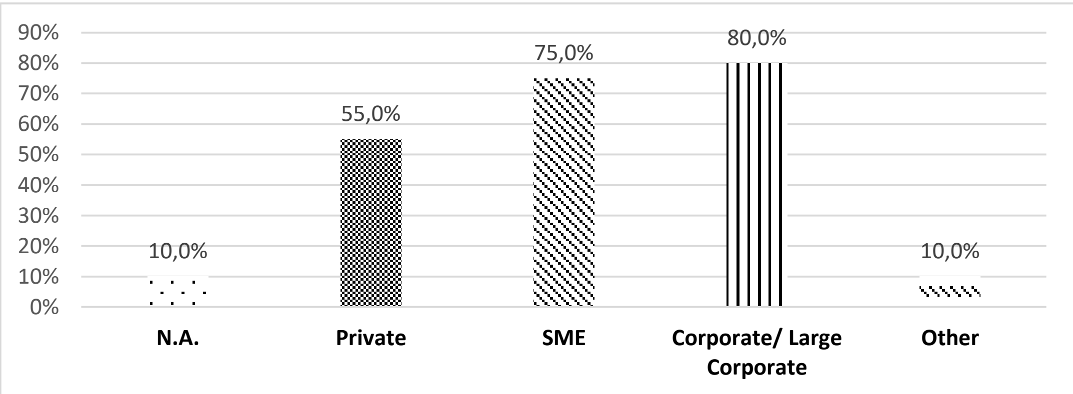
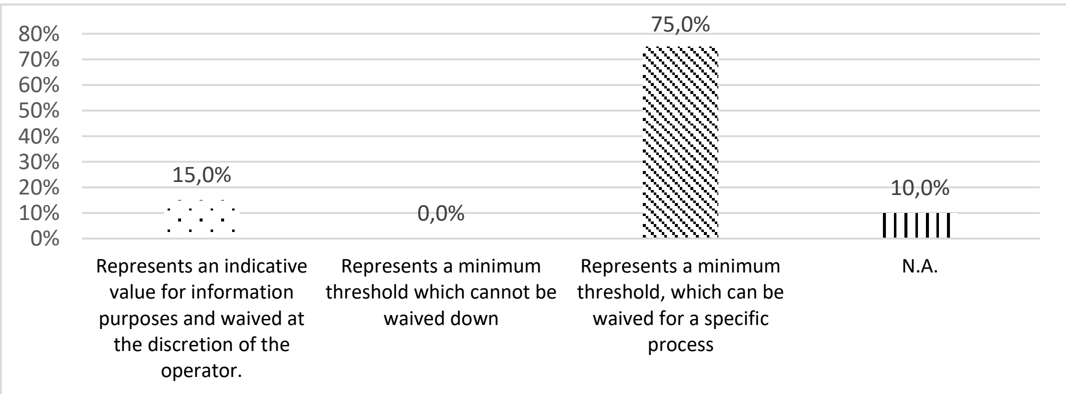


Figure 2 Weight of the Price to Value in the formulation of the final price to the customer

The figure reports the answers to the question: "What weight does the Price to Value take in the formulation of the final price to the customer?" Source: Authors' own production using data of AIFIRM (2021).

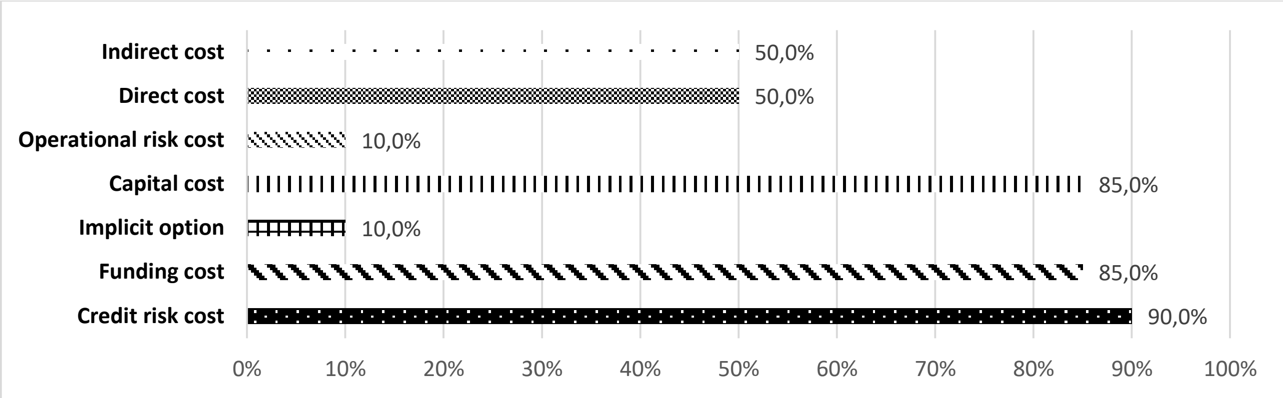


[6] The questions asked in the survey are listed in the Appendix (Table A5).

The survey also shows that the cost of credit risk is the main component in the determination of the hurdle rate (Figure 3).

Figure 3 Cost components in Price to Value determination

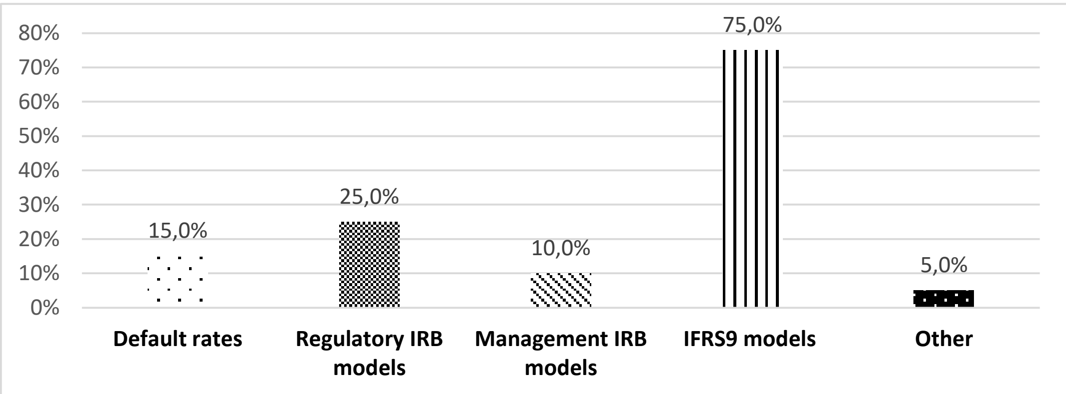
The figure reports the answers to the question: "Indicate the presence of the following components in the calculation of the Price to Value" Source: Authors' own production using data of AIFIRM (2021).



Regarding the IFRS9 logic, we observe a progressive adoption of such metrics, although not yet complete (e.g.: not widespread adoption of macro-scenarios). From an economic perspective, some elements of this framework may penalize pricing concerning the higher cost of expected risk. From a risk management perspective, they offer the possibility of more accurate measurements, more informed pricing decisions, and greater adherence to supervisory guidelines. Figure 4 shows that banks primarily (66.7%) calculate credit risk using IFRS9 logic.

Figure 4 Credit risk calculation

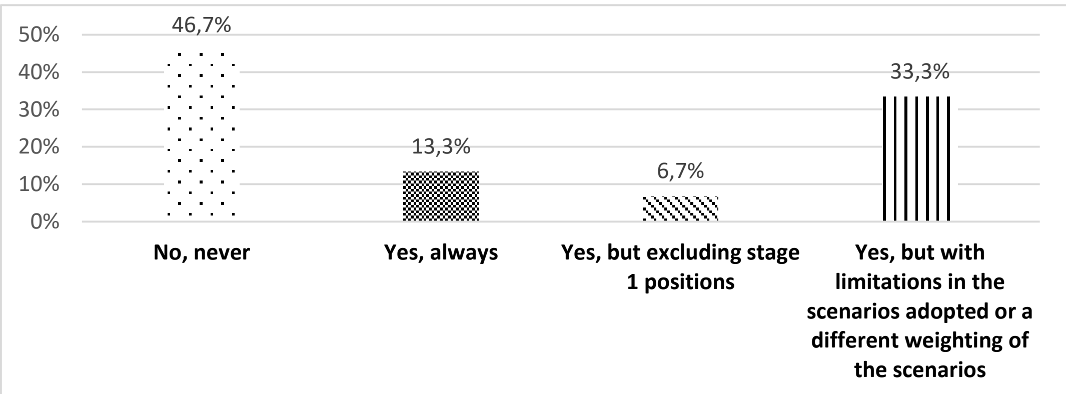
The figure reports the answers to the question: "How is the cost of credit risk calculated?". Source: Authors' own production using data of AIFIRM (2021).



However, our survey shows a substantial lack of adoption of alternative macro-scenarios when adopting IFRS9 models (Figure 5).

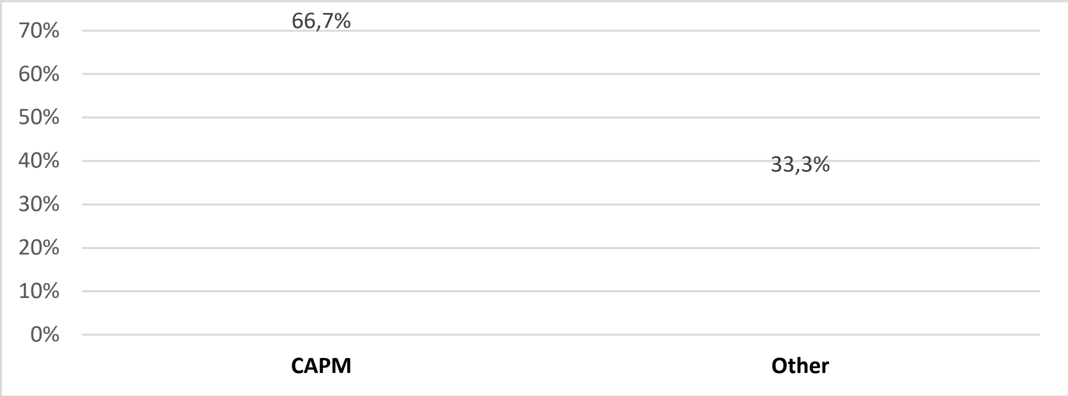
Figure 5 Alternative scenario implementation

The figure reports the answers to the question: "If IFRS9 models are adopted, are alternative scenario analyses applied?" Source: Authors' own production using data of AIFIRM (2021).



Concerning the setting of the rate of return on absorbed capital (k_e), i.e., the percentage remuneration ideally expected by the bank shareholder, the survey shows that most of the banks involved (about 66.7% of the sample) use the CAPM approach, also for reasons of procedural and managerial simplification.

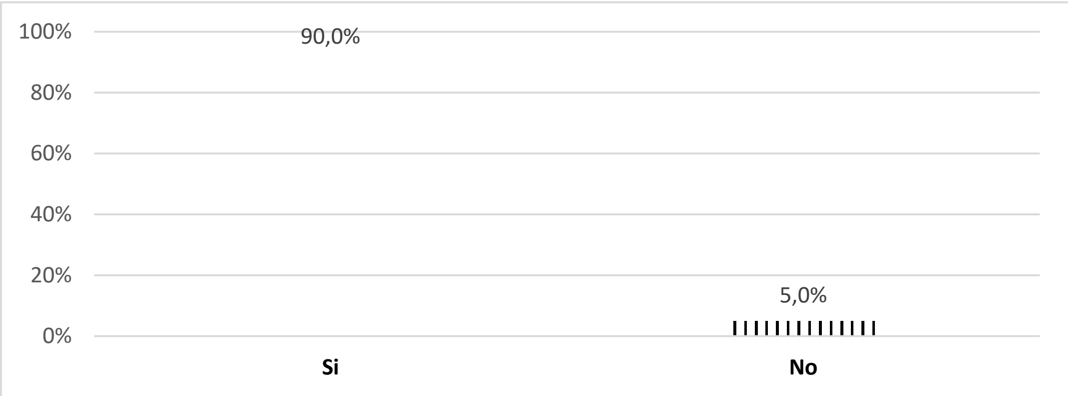
Figure 6 Methodologies applied to determine the rate of return on capital
The figure reports the answers to the question: "What methodologies are applied to determine the rate of return on capital?" Source: Authors' own production using data of AIFIRM (2021).



Banks' ability to modulate their k_e, according to the absorption of the capital factor, is still limited on average. However, it should be remembered that the latter is dependent, with the same notional exposure and technical form of credit, on the regulator's SREP Decision on each company and on the credit risk measurement methodology adopted by the company itself.

Concerning the Price to Market implementation, the survey highlights that most of the respondent banks define the Price to Market in their pricing process.

Figure 7: Price to Market calculation
The figure reports the answers to the question: "In the broader process of pricing, are there price-to-market logics?" Source: Authors' own production using data of AIFIRM (2021).



5 Empirical Analysis

What is the relationship between pricing components and banks' profits? To address this question, we run an empirical exercise where variables capturing pricing components are obtained by transforming answers obtained in the survey in the AIFIRM (2021) into a dummy or categorical variables. Variables related to banks' balance sheets are *Orbis Bankfocus*.

To this aim, we developed a standard panel data regression model and collect a sample of Italian banks between 2017 and 2020. Table 2 reports the descriptive statistics about the sample: not surprisingly, banks are heterogeneous in terms of size, amount of loans granted, credit quality, capitalization, and profitability (Table 2)[7].

Table 2 Summary statistics.

The table shows the summary statistics of the independent variables of the sample. The data have been collected for the period 2017-2020 from Orbis BankFocus and the survey AIFIRM (2021). The number of observations is given by the product between the number

[7] The description of the variables is reported in the Appendix (Table A1).

of banks (20) by the years collected (4) minus 2 because one bank does not have financial data for 2017 and 2018. Source: Authors' own production using data of AIFIRM (2021).

	N	Mean	Std. Dev	Min	Median	Max
Total Assets (billions of €)	78	110.602	202.923	0.885	24.462	1002.614
Loans	78	66.037	108.811	0.584	15.718	489.272
Performing Loans (billions of €)	78	61.000	102.976	0.450	12.254	468.29
Cost of funding	78	0.005	0.003	0.000	0.004	0.012
NPL ratio	78	0.093	0.081	0.001	0.079	0.525
Equity/Total Assets	78	0.072	0.019	0.038	0.069	0.135
Loans/Total Assets	78	0.665	0.143	0.216	0.688	0.914
Interest income on Total Loans	78	0.004	0.018	-0.069	0.004	0.070
Interest income on Performing Loans	78	0.004	0.021	-0.090	0.004	0.072
AIRB	78	0.359	0.483	0.000	0.000	1.000
Price to Value	78	0.359	0.483	0.000	0.000	1.000
Price to Market	78	0.769	0.424	0.000	0.000	1.000
Multi-scenario	78	0.359	0.483	0.000	0.000	1.000
Other Capital Remuneration	78	0.308	0.465	0.000	0.000	1.000
Indirect cost	78	0.513	0.503	0.000	0.000	1.000
Monitoring	78	0.744	0.439	0.000	0.000	1.000
Risk Adjusted Profitability	78	0.769	0.424	0.000	0.000	1.000
Simulations	78	0.641	0.483	0.000	0.000	1.000

Our empirical analysis aims to show the correlation between bank interest income and pricing components. To this aim, we analyze the relationship between each pricing component (using the answers obtained in the survey) and bank interest income ratio, after controlling for various factors (such as funding cost, credit quality, capitalization, and bank size). Accurate pricing is expected to allow banks to increase their interest income ratio, obtained as the ratio between the interest income over the volume of the loans granted by the bank. A greater and positive ratio is expected when the pricing is implemented by the bank via a more accurate estimate of risk, especially when the valuation is developed from a lifetime perspective.

Specifically, we use the following regression model in which the pricing components are included among our independent variables:

$$Y_{i,t} = X_{i,t} + Z_{i,t} + \varepsilon_{i,t} \tag{4}$$

where the dependent variable $Y_{i,t}$ is the ratio of interest income on total loans granted (as a robustness test, we also use the ratio of interest income on the fraction of performing loans) for the *i-th* bank at the time *t*. $X_{i,t}$ is a vector of variables capturing the pricing components, as: 1) the adoption of an AIRB model using a dummy equals to 1 if the bank applies at least for one fraction of the portfolio the internal model for credit risk computation[8], and zero otherwise; 2) the two main pricing components (i.e., *Price to Value, and Price to Market*)[9]; 3) the application of alternative scenarios (*Multi-scenario*) using a dummy equal to 1 for the banks that compute the pricing using multi scenario approach[10], and zero otherwise; 4) the capital remuneration factor (*Other Capital Remuneration*) using a dummy equals to 1 for the banks that use a different measure of capital remuneration rather than CAPM[11], and zero otherwise; 5) the taking into account of the indirect cost (*Indirect cost*) using a dummy equal to 1 for the banks that include the indirect costs for the Price To Value determination[12], and zero otherwise; 6) the monitoring process application (*Monitoring*) using a dummy equals to 1 if there is a structured monitoring and reporting system on the waivers made on Price To Market[13], and zero otherwise; 7) the application of risk-adjusted techniques for profitability (*Risk Adjusted Profitability*) using a dummy equal to 1 if in formulating the final price assessments related to risk-adjusted profitability are made[14], and zero otherwise; and 8) the application simulations (*Simulations*) using a dummy equal to 1 if simulations are performed in the price formulation with respect to target values of these metrics[15], and zero otherwise. Moreover, $Z_{i,t}$ is a vector of control variables that may influence the link between interest income and pricing determinants, such as the cost of funding (calculated as the ratio between interest expenses and total liabilities),

[8] question 1.2.1 of the survey
[9] question 1.2.4 of the survey
[10] question 2.2.8 of the survey
[11] question 2.3.4 of the survey
[12] question 2.4.1 of the survey
[13] question 3.2.2 of the survey
[14] question 4.1.1 of the survey
[15] question 4.1.3 of the survey

the capitalization (calculated as the ratio between the equity and the total assets - Equity/TA), and the bank risk represented by the NPL ratio (given by the ratio between impaired loans and total loans). In final, we estimate the model (4) by saturating the model with Time fixed-effects years to control for time-variant unobservable factors and using robust standard error to control for heteroscedasticity problem.

5.1 Results

The results of model 1 are reported in table 3, where the dependent variable is the interest income ratio on total loans (column 1) and the interest income ratio on performing loans (column 2).

Table 3 Regression analysis.

*The table shows the results of model (1). The dependent variables are the ratio between the interest income and total loans granted (column 1), and the ratio between the interest income and performing loans (column 2). The sample period is 2017- 2020. The model includes year fixed effects. Standard errors in parentheses. ***, **, * means that p<0.01, p<0.05, and p<0.1, respectively. Source of data: Orbis BankFocus and Aifirm (2021).*

	(1) *Interest Income on Total Loans*	(2) *Interest Income on Performing Loans*
Cost of funding	0.618	0.569
	(0.397)	(0.468)
Equity/TA	0.062	0.050
	(0.051)	(0.056)
Log of Total Assets	-0.001**	-0.001
	(0.001)	(0.001)
NPL ratio	0.014	0.078***
	(0.012)	(0.010)
AIRB	0.009***	0.010***
	(0.002)	(0.003)
Price to Value	0.004**	0.004**
	(0.002)	(0.002)
Price to Market	-0.007***	-0.008***
	(0.002)	(0.003)
Multi-scenario	0.003	0.004
	(0.002)	(0.003)
Other Capital Remuneration	-0.005*	-0.007**
	(0.002)	(0.003)
Indirect cost	0.001	0.001
	(0.001)	(0.001)
Monitoring	-0.012***	-0.015***
	(0.002)	(0.003)
Risk Adjusted Profitability	0.003*	0.005***
	(0.002)	(0.002)
Simulations	0.002	0.003**
	(0.002)	(0.002)
Significant	0.000	-0.003*
	(0.002)	(0.002)
Constant	0.043***	0.035***
	(0.010)	(0.011)
Observations	78	78
R-squared	0.681	0.812
R-squared adjusted	0.743	0.760

Our results suggest various considerations. The AIRB dummy variable is positive and statistically significant at the 1% confidence level: this suggests that banks using advanced internal models show a higher interest income ratio. As the probability of default distribution is exponential, that implies that implementing an advanced internal model allows pricing better (and higher) for the riskier customers: the availability of an advanced rating based internal model whose performance, in terms of discriminating capacity, is sufficiently high, constitutes a decisive factor for a bank's capacity of optimizing the risk/return ratio of its loans. As a matter of fact, a typical commercial bank usually pursues the maximum profitability of its lending activity, by means of opportunely managing the natural trade-off relationship existing between the riskiness of a loan and the return of the latter. It's therefore clear that being able to exactly measure the expected loss and the risk (unexpected loss) associated with any loan facility (a capacity in principle assured by the availability of a proper A-IRB model), enables a bank to exploit the loan crucial risk/return trade-off till the very limit which allows reaching the goal of its earning maximization with sustainable levels of credit risk.

Looking at pricing components, coefficient estimates for the price-to-value are positive and significant at a 10% confidence level: the price-to-value concerns a scientific valuation of the costs of the loan for the bank, hence the banks that implement the calculation of the Price-To-Value component fully cover all the relevant costs of the loan transaction, this enhances greater interest income. Conversely, Price-to-Market coefficient estimates have the opposite sign (negative) and are statistically significant at the 1% confidence level. This is not surprising since the survey has been conducted in a low-interest rate and quite competitive market. Hence, we argue that more competitive banks must somehow implicitly consider caps on their loan prices: high competition erodes interest revenues, and this impacts more on banks intensively adopting the Price-to-Market approach. Furthermore, the adoption of a model different than CAPM to measure shareholders' expectation return shows a negative link with the interest income ratio: thus, the CAPM adoption seems to overestimate expected returns.

Coefficient estimates for Monitoring are statistically significant at the 1% confidence level and negative. Although this might be somewhat surprising, could be interpreted as a signal of a decentralized pricing function: the negative sign of the association monitoring practices decreasing loan prices captures the observed-in-the-industry-practice that banks characterized by more decentralized loan pricing powers are usually more inclined to match the customers' requests for lower loan interest rates than the banks with more centralized pricing decisions or pricing processes (physiologically less sensitive to the ordinary customers' instances). Even if the result of the coefficient Monitoring is significant, this has not a direct effect on the earnings of the commercial lending activity, but an indirect one on loan prices. As concerns the coefficient of the Risk-adjusted profitability variable, we note a positive sign that is statistically significant at a 10% confidence level or less. The coefficient sign is as expected: this variable concerns a better calibration of the trade-off risk-return, the correlation with the risk is higher, hence the loan prices tend to be more favourable for banks that more intensively adopt risk-adjusted profitability indicators.

The other independent variables included in the model are not statistically significant at the 10% level or less: hence, the adoption of alternative scenarios (*Multi-scenario*), the consideration of the indirect costs in the Price To Value determination, and, the implementation of the simulations concerning the target values (Simulation variable) does not show a statistically significant link with banks' interest income ratio.

6. Conclusion

Our paper analyses the main loan pricing components in the Italian banking system, and empirically addresses their link with the bank's economic performance[16]. By using data collected in the survey conducted by the authors in AIFIRM (2021), we provide readers with new insights into the link between pricing components and bank interest income ratio. We implement an empirical model[17] showing that the application of advanced internal risk-based models, the calculation of the break-even rate, and the implementation of the risk-adjusted profitability measures in the pricing improve banks' performance. Conversely, market competition, a decentralized pricing function allowing more customer-oriented loans prices, and the use of non-CAPM models to estimate shareholders' expected capital remuneration erode the interest income ratio.

We argue that the decision to apply the Lifetime Expected Loss criteria following IFRS 9 in calculating the hurdle rate for lending to Stage 1 positions should be left to the discretion of individual banks. The individual banks should make this choice autonomously in terms of opportunities, methods, and the general incorporation of multi-scenario and sector analysis tools into their business processes, based on the degree of refinement of the IFRS 9 framework and considering new regulations in existence or soon to be applied, which will then obviously be subject to market and competitive scrutiny.

In final, the monitoring of the macroeconomic context (induced by the Covid-19 pandemic outbreak) deserves specific attention. The impacts on pricing and methodologies are difficult to interpret, at least for the following reasons: 1) the presence of government interventions that introduce "distorting" factors to the normal process/methodological framework for assessing, granting, and pricing credit on the one hand and for limiting the risk taken on the other (due to the combined effect of high average PDs and low LGDs for guarantee schemes), but with a possible acceleration of credit quality deterioration in 2021; 2) the still uncertain course of the pandemic; and c) the highly asymmetrical impact on industrial sectors. Thus, it is urgent to monitor and develop improve current risk models, incorporate multi-scenario prospective evaluations at the sector level, enhance organizational processes for waivers and frameworks for monitoring the impact of waivers on value creation, and in final develop adequate tools, as the IT tools, to support both central offices and the sales network in the process of formulating loan prices and monitoring the value consequently created.

Our paper provides readers with a first step in showing what are the main pricing components for performing loans and their effect on banks' interest income. This topic deserves future research by focusing on each of the pricing components. The lack of publicly available data is a major issue to run further research on this area and we are grateful to AIFIRM to let us use data on the 2021 survey to run this paper.

[16] The analysis conducted deserves future researches in different papers studying the pricing components effects on other items of the banking business.
[17] We are aware about the limit of our sample of 20 banks. We deserve future research in case a new survey will be conducted with many banks.

Appendix

Table A1 Variables description

The table reports the description and the acronymous of the variables used in the analysis. Source: Authors' own production.

Variable	Acronymous	Description
AIRB	*AIRB*	A dummy variable taking the value of 1 for banks that apply (at least for a fraction of the loan portfolio) internal risk-based models, and zero otherwise
Capitalisation	*Equity/TA*	The ratio of the bank's equity and total assets
Cost of funding	*Cost of funding*	The ratio between interest expenses and total liabilities
Indirect cost	*Indirect cost*	A dummy variable taking the value of 1 for banks that include the indirect costs for the Price to Value determination
Interest income ratio	*Interest Income Ratio 1*	It is a ratio among the interest income and the total volume of loans
Interest Income ratio performing	*Interest Income Ratio 2*	It is a ratio among the interest income and the fraction of performing loans
Loans	*Loans*	Total loans (assets) of the bank in billions of euros
Monitoring	*Monitoring*	A dummy variable taking the value of 1 for banks that apply monitoring processes when waivers are made related to Price to Market computation, and zero otherwise
Multi-scenario	*Multi-scenario*	A dummy variable taking the value of 1 for the banks that compute the pricing using a multi-scenario approach
Non-Performing Loans ratio	*NPL ratio*	The ratio of impaired loans over total loans
Other Capital Remuneration	*Other Capital Remuneration*	A dummy variable taking the value of 1 for the banks that use a different measure of capital remuneration rather than CAPM, and zero otherwise
Price To Market	*Price to Market*	A dummy variable taking the value of 1 if the bank calculated Price to Market, and zero otherwise
Price To Value	*Price to Value*	A dummy variable taking the value of 1 if the bank calculated Price to Value, and zero otherwise
Risk-adjusted profitability	*Risk Adjusted Profitability*	A dummy variable taking the value of 1 if the bank in formulating the final price implements risk-adjusted profitability assessments, and zero otherwise
Simulations	*Simulations*	A dummy variable taking the value of 1 if simulations are performed in the price formulation with respect to target values of these metrics, and zero otherwise
Total Assets	*Total Assets*	Total assets of the bank in billions of euros

Table A2 The post estimation results

Panel 1: *The table reports the Variance Inflation Factor Indicator post estimated by model (1).*

VIF	1/VIF
4.610	0.217
4.580	0.218
4.090	0.245
4.020	0.249
3.870	0.258
3.700	0.270
3.100	0.323
2.550	0.392
2.200	0.455
2.170	0.460
1.990	0.502
1.900	0.527
1.860	0.538
1.750	0.572
1.710	0.586
2.940	

Panel 2: *the table reports the p-value associated at the Breusch-Pagan test to check for heteroscedasticity.*

Breusch-Pagan test for heteroskedasticity	
Assumption: Normal error terms	
Variable: Fitted values of Int_Inc_Loans	
H0: Constant variance	
chi2(1)	1.17
Prob > chi2	0.2788

Table A3 The matrix of correlation

The table reports the matrix of correlation of the variables included in model (1).

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1) Interest Income on Total Loans	1.000										
(2) AIRB	0.241	1.000									
(3) Price To Value	0.008	-0.098	1.000								
(4) Price To Market	-0.269	-0.098	0.133	1.000							
(5) Multi-scenario	0.205	0.109	0.156	-0.098	1.000						
(6) Other Capital Remuneration	-0.042	-0.036	0.101	0.101	-0.499	1.000					
(7) Indirect cost	-0.220	-0.126	0.075	-0.169	-0.126	-0.017	1.000				
(8) Monitoring	-0.099	0.439	0.236	-0.322	0.439	-0.372	0.133	1.000			
(9) Risk adjusted Profitability	0.056	0.156	0.133	0.133	0.156	0.101	-0.169	0.236	1.000		
(10) Simulations	-0.059	-0.109	0.098	0.351	-0.109	0.036	-0.088	0.050	0.351	1.000	
(11) Significant	0.127	0.463	-0.038	0.209	0.245	-0.330	-0.021	0.251	0.209	0.407	1.000

Table A4 The Hausman (1978) specification test

The table reports the Hausman specification test for model (1). The p-value suggests that the preferred model is the one using fixed effects.

	Coef.
Chi-square test value	45.833
P-value	0.000

Table A5 The survey

The table reports the question and the set of possible answers submitted to the respondent banks. Source: AIFIRM(2021).

1.1.1	**Name of Respondent**
1.1.2	**Company/ Institute of Respondent**
1.1.3	**Organizational Unit of Respondent**
1.2.1	**Which is the authorized regulatory approach for calculating the Credit RWA?**
a	Standard
b	FIRB (at least for part of the portfolio)
c	AIRB (at least for part of the portfolio)
1.2.2	**If authorized to use the FIRB/ AIRB approach, (at least for part of the portfolio) indicate which models are used and for which segments**
a	PD Retail
b	CCF Retail
c	LGD Retail
d	PD SME Retail
e	CCF SME Retail
f	LGD SME Retail
g	PD Corporate
h	CCF Corporate
i	LGD Corporate
l	PD Banks
m	CCF Banks
n	LGD Banks
o	PD Public authority/ institution
p	CCF Public authority / institution
q	LGD Public authority / institution
1.2.3	**If NOT authorized to use FIRB/ AIRB approach, are there still credit risk internal models for management use? For which customer segments?**
a	None
b	PD Retail
c	CCF Retail
d	LGD Retail
e	PD SME Retail
f	CCF SME Retail
g	LGD SME Retail
h	PD Corporate
i	CCF Corporate
h	LGD Corporate
1.2.4	**Which components are included in the internal pricing process for performing credit operations? (Mark with x the included components)**
a	Price to value
b	Price to market
c	Price to client
1	Comments
2.1.1	**Does the bank consider the calculation of a "Price to value" or "hurdle rate" in the pricing determination process?**
a	Yes
b	No
2.1.2	**If it does, indicate for which macro segments**
a	N.A.
b	Household customers
c	SME
d	Corporate/ Large Corporate
e	Other
2.1.3	**Which is the weight of the Price to Value in formulating the final price to the customer?**
a	It is an approximate value provided to the manager for information, and which can be waived at the manager's discretion
b	It is a firm minimum threshold
c	It is a minimum threshold that can be waived through a specific process
d	N.A. (price to value not determined)
2.1.4	**Mark with x the presence of the following components in the calculation of the Price to Value**
a	N.A.

b	Cost of credit risk
c	Cost of funding
e	Embedded options
f	Cost of Capital
g	Operating Risk cost
h	Costs directly connected to the operation
i	Indirect costs (for personnel, …)
j	Other (free text):
2.2.1	**Which is the logic behind the calculation of the credit risk cost of the operation?**
a	The logic of expected loss
b	The logic of accounting credit cost
2.2.2	**In which way is the operation credit risk cost calculated for the majority of segments that considers calculating the price to value?**
a	Observed default rates
b	Regulatory IRB models
c	Management IRB models
d	IFRS9 models
e	Other
2.2.3	**In case of use of "management" IRB models, which differences are envisaged compared to regulatory IRB models?**
a	Adjustment to short term default rates (Point in Time logic) for a greater correspondence to current risk levels
b	Adjustment to long term default rates ("long run average" logic) for higher stability of risk compared to the operation life cycle
c	Different adjustments based on the type of product / duration
d	Exclusion of certain prudential effects linked to the economic cycle trend
e	Introduction of specific remedies linked to the type of product / operation
f	Other (free text):
2.2.4	**Which is the main reason behind the use of IFRS9 models for calculating Price to Value – Credit Risk Spread?**
a	They are the only available internal models to estimate credit risk
b	They allow a better consistency with the accounting credit cost
c	They allow a better risk evaluation in a lifetime perspective
d	Other (free text):
2.2.5	**In case of use of IFRS9 models, are they implemented with any management change / remedy?**
a	**No**
b	**Yes**
2.2.6	**In case of use of IFRS9 models, how are the migrations between stages handled?**
a	Consistently with IFRS9, Stage 1 positions are associated with 1Y parameters and Stage 2 positions with Lifetime parameters
b	Differently from IFRS9, Lifetime parameters are associated to all positions, independent of their stage
c	Differently from IFRS9, 1Y parameters are associated to all positions, independent of their stage
d	Other
2.2.7	**In case of handling of stages consistent with the IFRS9 framework, are there any mechanisms to include in the pricing the migration risk from stage 1 to stage 2 or 3 and the resulting increase of risk (and of accounting cost)?**
a	No
b	Yes, through covenants which allow to change the spread according to the stage
c	Yes, through an estimate of the migration probability
d	Yes, other
2.2.8	**In case of use of IFRS9 models, are alternative scenario analyses implemented?**
a	No, never
b	Yes, always
c	Yes, but excluding positions in stage 1
d	Yes, but with limitations in the adopted scenarios or with a different weighting
2.3.1	**For defining the cost of capital which is the concept of capital used?**
a	Standard supervisory capital requirement
b	IRB supervisory capital requirement
c	Internal capital calculated as IRB minimum requirement with adjustments
d	Economic capital calculated through portfolio models
e	Other figures representing economic capital
2.3.2	**Are other metrics used to incorporate portfolio diversification effects (different from the portfolio models used to estimate ECAP)?**
a	No

b	Yes, management recommendations such as sector indications from Credit Strategies, …
c	Yes, quantitative indications of a different kind
2.3.3	**Which is the rationale behind the choice of the capital figure used?**
a	Regulatory supervisory cost of capital
b	“Management” regulatory approach, consistent with the risk capital and the competitive context
c	Internal metrics (e.g. ECAP) to optimise return compared to the actual allocation of capital
d	Other
2.3.4	**Which methods are implemented to determine the rate of return of capital?**
a	CAPM
b	Other (expected/ target ROE, ad hoc management assessments, …)
2.3.5	**Which are the advantages and weaknesses related to the used methods as of the previous question?**
a	Advantages (free text)
b	Weaknesses (free text)
2.4.1	**Are components of indirect cost considered in the formulation of the Price to Value?**
a	Yes (state which ones)
b	No
2.4.2	**Which are the main difficulties when estimating the indirect costs and which proxies are used? (explain in free text in box below)**
a	Free text
2.4.3	**Which methods are implemented to transfer the indirect costs on the price? (Explain in free text in box below)**
a	Free text
3.1.1	**Are there any “Price to market” logics in the broader pricing formulation process?**
a	Yes
b	No
3.1.2	**If present, in which way is the “Price to market” expressed?**
a	Minimum and maximum spreads applicable and not suspendable
b	Minimum and maximum spreads suspendable through a structured process
c	Minimum and maximum spreads with information value and suspendable at manager’s discretion
d	Other (free text):
3.2.1	**In case of suspendable price to market, under which criteria can an exception be made/ requested?**
a	Objective criteria that do not need any authorisation
b	Objective criteria that need authorisation
c	Subjective criteria
d	Other (free text)
3.2.2	**Is there a structured system for monitoring and reporting the exceptions?**
a	No
b	Yes, only on the number of exceptions
c	Yes, on number and extent of exceptions
d	With differences according to the line of analysis (segments, products, …)
e	Yes, other ways
3.2.3	**Which drivers are used to differentiate the price to market? (mark with x the implemented drivers)**
a	Segment
b	Product
c	Rating class
d	Turnover class
e	Sector
f	Geographic area
g	Other
3.2.4	**How important are market prices in determining the price to market values?**
a	Not important
b	Half important
c	Very important
3.2.5	**Are internal benchmarks also used in determining the price to market values?**
a	No, only use of risk drivers or external benchmarks
b	Yes, through benchmarks on comparable divisions
c	Comments
4.1.1	**Are estimates linked to the risk adjusted return implemented in formulating the final price?**
a	Yes
b	No
4.1.2	**If they are, which metrics are used?**
a	Rorac

b	Raroc
c	Rarorac
d	EVA operation
e	EVA customer
f	Other
4.1.3	**Are estimates related to the target values of such metrics implemented in formulating the price? (Any further detail to the answer can be made in the Comments)**
a	Yes, vs target minimum values
b	Yes, vs portfolio targets
c	Yes, other
d	No
e	Comments
5.1.1	**Has the new current and prospect macroeconomic context caused by Covid-19 determined changes in the models or logic for pricing? (Further comments in relevant box)**
a	Yes, on the models implemented to measure the components of the credit risk cost
b	Yes, on the other cost components
c	Yes, on the ways to define the price to market
d	Yes, on the ways to define the price to client
e	Yes, other
f	No
5.1.2	**Are the measures introduced by the Government or the authorities impacting on prices for the customers in a positive way, a negative way or not at all? (Further comments in relevant box)**
a	The measures do not impact the prices or are compensated
b	The measures impact the prices for customers in a positive way, allowing for lower prices
c	The measures impact the prices for customers in a negative way, causing higher prices
5.1.3	**Are the measures introduced by the Government or the authorities impacting the bank's margins? (Further comments in relevant box)**
a	The measures do not impact the margins
b	The measures impact the margins positively
c	The measures impact the margins negatively
d	Comments

References

AIFIRM (2021). Pricing and risk-adjusted return measures, *AIFIRM Position Paper n.27*.

Greenspan, A. (1995). The case for risk-based pricing, *ABA Banking Journal*, pp. 58–60.

Bellini, T. (2019). IFRS 9 and CECL Credit Risk Modelling and Validation: A Practical Guide with Examples Worked in R and SAS, *Academic Press*, London.

Chawla, G., Forest, L.R., Aguais, S.D. (2016). Point-in-Time loss given default rates and exposures at default models for IFRS 9/CECL and stress testing, *Journal of Risk Management in Financial Institutions*, 9(3), pp. 249-263.

Curcio, D., Gianfrancesco, I. (2009). Bank loans pricing and Basel II: a multi-period risk-adjusted methodology under the new regulatory constraints, *Banks and Bank Systems*, 4(4), pp. 56-66.

Engelmann, B. (2021). Calculating lifetime expected loss for IFRS 9: which formula is measuring what? *The Journal of Risk Finance.*

Hasan, I., Zazzara, C. (2006). Pricing risky bank loans in the new Basel 2 environment, *Journal of Banking Regulation*, 7(3), pp. 243–267.

Jung, T., Strohhecker, J. (2009). Risk-Adjusted Pricing Strategies for the corporate loans business: Do they really create value?, System Dynamics Review, 25(4), pp.251-279.

Kruger, S., Rosch, D., Scheule, H. (2018). The impact of loan loss provisioning on bank capital requirements, *Journal of Financial Stability*, 36(C), pp. 114-129.

Loew, E., Schmidt, L., Thiel, L. (2019). Accounting for financial instruments under IFRS 9–first time application effects on European banks' balance sheets, *Regulation of Financial Institutions eJournal*, available at SSRN.

Motley, LB. (2006). What "aggressive pricing" really means, *ABA Banking Marketing*.

Novotny-Farkas, Z. (2016). The interaction of the IFRS 9 expected loss approach with supervisory rules and implications for financial stability, *Accounting in Europe*, 13, pp. 197-227.

Ruthenberg, D., Landskroner, Y. (2008). Loan pricing under Basel II in an imperfectly competitive banking market, *Journal of Banking and Finance*, 32(12), pp. 2725-2733.

Seitz, B., Dinh, T., Rathgeber, A. (2018). Understanding loan loss reserves under IFRS 9: a simulation-based approach, *Advances in Quantitative Analysis of Finance and Accounting*, 16, pp. 311-357.

Skoglund, J. (2017). Credit risk term-structures for lifetime impairment forecasting: a practical guide, *Journal of Risk Management in Financial Institutions*, 10(2), pp. 177-195.

Xu, X. (2016). Estimating lifetime expected credit losses under IFRS 9, available at SSRN.

The impact of negative interest rates on the pricing of options written on equity: a technical study for a suitable estimate of early termination

Anna Bottasso (University of Genoa – Department of Economics - DIEC), Lorenzo Bruno (EGO Energia – Risk Management), Pier Giuseppe Giribone (Banca CARIGE, Gruppo BPER – Financial Engineering and Data Mining and University of Genoa – Department of Economics - DIEC)

Article submitted to double-blind peer review, received on 25th October 2022 and accepted on 11th November 2022

Abstract

This work aims to investigate the main problems that impact the pricing models and the sensitivity measures of American options written on shares without a pay-out, in the presence of negative interest rates with a specific focus on the Monte Carlo method. The first paragraph carries out a review of the anomalies caused by such an odd condition and focuses thereafter on the core topic of the research by treating a wide range of numerical models suitable for unbiased evaluation of the early exercise, thus expanding the existing literature. The two following paragraphs are dedicated to describing the models used for the correct estimation of fair value: binomial lattice models (Cox-Ross-Rubinstein - CRR Tree, Leisen Reimer - LR Tree, Jarrow-Rudd - JR Tree and Tian Tree), trinomial stochastic trees, Finite Difference Method (FDM) scheme and the Longstaff-Schwartz Monte Carlo. Particular attention is paid to this last approach which allows to combine the flexibility of traditional numerical integration schemes for stochastic processes on equity with the estimation of the convenience of exercising the American option ahead of time. After conducting quantitative tests both on pricing and on the estimation of sensitivity measures, the LR Tree was selected as the most performing deterministic algorithm to be compared with the Monte Carlo stochastic technique. The final part of the work focuses on quantifying the valuation gap introduced by negative interest rates in the valuation of American options written on an unprofitable underlying comparing the traditional valuation approach and the deterministic Leisen Reimer model and the Longstaff-Schwartz stochastic model.

Key Words:

Negative Interest rates, American Option pricing, early-exercise valuation, extreme market conditions, sensitivity measures, lattice models, Cox-Ross-Rubinstein (CRR) Tree, Leisen Reimer (LR) Tree, Jarrow-Rudd (JR) Tree, Tian Tree, CRR Trinomial Tree, Finite Difference Method (FDM), Stochastic Differential Equation (SDE), Longstaff-Schwartz Monte Carlo

1) The main problems caused by negative interest rates

The first paragraph introduces the main problems related to the presence of negative nominal interest rates. After a brief introduction, in which the main historical facts that led central banks to reduce interest rates to negative values will be discussed, more specific issues will be addressed, linked to the technical issues caused by the zero lower bound of rates. The last of these issues, namely the effect that negative interest rates have on options written on equity that do not pay dividends, will be the core topic of this work.

1.1) Negative interest rates and historical-economic context

It all began in the United States of America in 2006, the year in which the so-called subprime mortgage crisis broke out: the term subprime refers to those loans with high financial risk, mainly mortgage loans, that many American banks disbursed in favor of customers with a high risk of default. Between 2000 and 2003, the Fed drastically reduced interest rates, from 6.5% to 1%. This decreasing trend in rates, together with other factors, triggered an increase in the demand for mortgage loans, fueled in turn by a growing real estate market, characterized by speculative practices and also by a parallel financial market based on the securitization of the same mortgage loans, in which large banks, retail banks and institutional investors held and traded very complex financial instruments such as MBS - Mortgage-Backed Securities or CDO - Collateralized Debt Obligation (Holt, 2009). The crisis started to appear in the second half of 2006 when the US housing bubble began to deflate in the face of a rate hike by the FED, from 1% to 5.25%, which occurred in a rather short period of time, from 2004 to 2006. The graph in Figure 1 traces the trend of the Federal Fund Rate. The sudden rise in rates caused a high percentage of default on subprime loans. Consequently, the prices of financial instruments built on these loans, which constituted a market whose capitalization vastly exceeded the total value of the underlying loans, collapsed along with the real estate market (Bernanke, 2010). The consequences were rather disastrous, involving the main US Investment Banks, which suffered very heavy losses caused by the sharp depreciation of MBS and CDOs, and collaterally by CDS (Credit Default Swaps). The market values of the banks also dropped significantly. The most striking and emblematic case occurred in September 2008, when Lehman Brothers, one of the most important American financial institutions, founded in 1850, filed for bankruptcy (Lehman Brothers Holding Inc, 2008). The American Government decided not to save the bank to give a strong signal to the financial system, and to prevent future moral hazard behavior by other banks or financial institutions typically considered "Too Big To Fail". This complex series of circumstances led to a strong mistrust in the financial system and to a contagion effect that caused the so-called credit crunch: with this term, which literally means a "tightening of credit", we mean a significant and sudden contraction in the supply of credit, at the end of a prolonged period of expansion. The banks initially froze the interbank credit market out of mutual distrust, wary of the solvency of their respective counterparties. This led to a general contraction of credit even in the retail market and corporate market, triggering a process of economic recession that brought a strong impact on the real economy. The credit crisis globally spread like wildfire, also affecting the Eurozone, where many institutions were saved with public funds or with bail-in procedures. The strong distrust in the financial system generated panic in certain cases, as in the case of Northern Rock, a British institution specializing in real estate loans, which in 2007 suffered an out-and-out bank run (Dunkley, 2017). To face those circumstances, the ECB adopted a series of measures aimed at implementing an expansive monetary policy, with the aim of stimulating the economy and ensuring price stability in the Eurozone. The ECB therefore cut interest rates until they reached negative levels in 2014 (ECB, 2014). The goal was to disincentivize banks from depositing liquidity with the central bank, whose negative interest rate would have led to a loss on deposits, in favor of the use of such liquidity for the provision of loans to businesses and consumers. The graph in Figure 2 shows the evolution of the Deposit Facility Rate, i.e., the interest rate that banks receive on deposits with the European Central Bank. The graph clearly shows the systematic cut of the DFR, aggressively from 2008 until the first half of 2009, then more gradually, reaching the zero level in 2012, and finally assuming negative values from 2014.

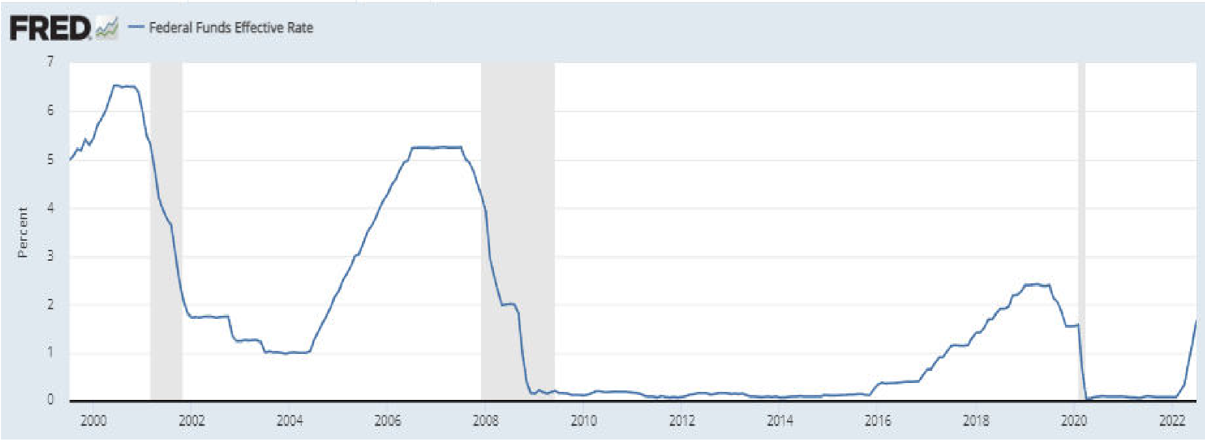


Figure 1 - Federal Fund Rate between 2000 and 2022 (Source: FRED Economic Data)

Figure 2 - ECB Deposit Facility Rate years 2000-2022 (Source: FRED Economic Data).

After briefly discussing the economic and financial causes that led to negative nominal interest rates, the main effects caused by this anomalous condition are discussed hereafter.

1.2) Problems related to negative interest rates

For the first time in history, apart from a brief period in Japan in the 90s, a scenario of negative nominal interest rates occurs (Ansa, 2014). This peculiar circumstance brings a series of consequences within the financial system. The unexpected impacts that never occurred before are numerous and are accompanied by a dense literature which tries to analyze the potential repercussions on markets and financial instruments, whose issues and exchanges usually took place under the assumption of the presence of positive levels of interest rates.

1.2.1) Potential effects on the investment choices of investors

In the presence of negative interest rates, Government bonds with floating coupons linked to Euribor (for example CCTs) could theoretically yield negative coupons, which will be set to zero (floor) and this could lead investors to select riskier investments, but with a positive return. In this way, the investor might invest his money in financial instruments characterized by greater volatility, therefore potentially unsuitable to his risk appetite.

1.2.2) Application of negative rates on deposits

There is also the possibility that banks apply negative interest rates on the deposits of their account holders, inducing the latter to withdraw their money and deposit it with other institutions which, on the contrary, do not charge their customers on deposits. This issue can trigger a competition mechanism within the banking system, which results in the bank taking over this cost, with obvious repercussions on the financial statements.

1.2.3) Anomalies in the interest rates term structure

Moving on to more quantitative problems, one of them is certainly linked to the anomalies found in the interest rate curve (Cafferata et al., 2019). Let us consider the 6-month Euribor, one of the most relevant interest rates as it is typically used as a parameter for indexing mortgages, bonds and derivatives. The historical and prospective evolution of the 6-month Euribor has been analyzed by financial analysts and traders from all over the world. However, most of the models for representing the dynamics of this short-term rate are affected by the issue of negative interest rates, since their structure does not allow negative values within the formulas of the stochastic differential equations (Giribone, 2020).

1.2.4) Anomalies in the surface of the implied volatilities in caps and floors

Implied volatility provides an estimate of the expected volatility of the underlying made by the market maker, during the residual life of an option. It is one of the most important parameters in evaluating the price of an option. Before the advent of negative interest rates, the main framework used for determining the implied volatility was the Black framework, i.e., the log-normal model (Black, 1976). In a context with negative interest rates, the log-normal volatility surfaces observed in financial markets are incomplete. To address this problem, market makers have replaced Black's log-normal model with Bachelier's normal model (Haug, 2007). This model guarantees the integrity of the volatility surfaces when interest rates are negative. Normal volatility is expressed in basis points and is obtained by numerically inverting the Bachelier formula starting from the premiums of actively listed options on the market. The problem of moving from the log-normal model to the normal model exposes the counterparties to model risk, i.e., the risk associated with carrying out transactions with a different model, therefore at different prices (Giribone, Ligato & Mulas, 2017). Furthermore, the problem of changing the model has caused various issues to existing contracts, since, if the reference pricing model is contractually specified, such calculation method cannot be changed. To solve these data missing problems, rather complex techniques have been applied to rebuild the incomplete volatility surfaces of the Black model, also related to Machine Learning techniques (Caligaris, Giribone & Neffelli, 2017).

1.2.5) Valuation of options written on interest rates

One of the most debated issues in the sector's literature is the impact that negative interest rates have on options written on interest rates (Burro et al., 2017). Interest Rate options are financial instruments that have been widely used in recent years. They are generally embedded, i.e., incorporated within bonds or in bank assets such as mortgages. Among the most common we can mention Caps, Floors and Collars. Those derivative contracts are called yield-based options and are characterized by a cash settlement which amount is the difference between the value of the underlying and a strike price. Before the advent of negative interest rates, the main framework for pricing this type of contract has always been Black's log-normal model. With the advent of negative rates, this model can no longer work due to the negative input (i.e., the forward rate F) which should be inserted within the auxiliary variables in the well-known Black closed formula. This issue arises because the logarithm of a negative value does not exist in real numbers. This makes it impossible to use pricing techniques based on the log-normal model for valuating options written on interest rates. (Giribone & Ligato, 2016). A very similar phenomenon occurs for swaptions: negative interest rates do not allow for a correct estimate of the fair value and of the sensitivity measures even for this type of interest rate derivative.

1.3) Effect of negative interest rates on the pricing of options written on equity that pays no dividend

A decidedly less debated issue is that relating to the effects that negative interest rates have on options written on non-profitable shares, that is, those that pay no dividends. According to the theory of options, there are fourteen fundamental properties that options must satisfy, regardless of the pricing model used (Hull, 2015). In particular, one of them states:

"Under the assumption that the underlying share pays no dividend, it will never be worth exercising an American call option prematurely, so it will be priced like its European analogue".

In mathematical terms, this can be written as:

$f_A(S,K,T,r,0,\sigma) = f_E(S,K,T,r,0,\sigma)$ (1)

$f_A(S,K,T,r,0,\sigma) \geq f_E(S,K,T,r,0,\sigma) \geq S_t - K\, e^{-rT}$ (2)

Where: S is the spot level of the underlying, K is the strike price of the option, T is the time to maturity, r is the risk-free rate, $q = 0$ is the continuous dividend yield and σ is the volatility. Furthermore:

$f_A(S,K,T,r,0,\sigma) \geq \max[0\,; S_t - K]$ (3)

If we combine (2) with (3) we obtain:

$S_t - K < St - K\, e^{-rT}$ (4)

Well, this property is no longer valid in the presence of negative interest rates, and a bias can therefore be observed between the price of the European call and of the American call option, with resulting effects also on the sensitivity measures of the options, i.e., on the Greeks (Cafferata, Giribone, & Resta, 2017). A considerable problem therefore arises, namely that of identifying robust alternative routines that allow the valuation of this type of option in the presence of negative rates. The most widely used techniques for pricing American options, especially short-term ones, are the so-called quasi-closed formulas, such as the Bjerksund-Stensland formula (Bjerksund & Stensland, 2002). The issue with those techniques is that their algorithms are characterized by an if-condition which, in the absence of a dividend yield, returns the price of the European option. However, the extreme situation of a market characterized by negative interest rates is not considered. It is therefore necessary to implement techniques that consider the possibility of having an early exercise, even if the underlying has no pay-out. In (Cafferata, Giribone & Resta, 2017), the authors analyze the issues mentioned above and integrate their work with a numerical experiment in which certain pricing techniques applied to an American call option are implemented in different market scenarios: a first case in which interest rates are positive and the underlying pays a dividend, a second case in which the rates are again positive but the underlying share is not profitable, and finally a third case where market conditions are the most extreme since, in addition to a zero dividend yield, negative interest rates are observed. The models used for pricing the options were essentially the following:

- The Barone-Adesi-Whaley (BAW) model (Barone-Adesi & Whaley, 1987).

- The 1993 Bjerksund-Stensland model (Bjerksund & Stensland, 1993).

- The 2002 Bjerksund-Stensland model (Bjerksund & Stensland, 2002).

- The trinomial model (Boyle, 1986).

The first three models consist of the so-called quasi-closed formulas, while the fourth is a lattice model. The Authors infer from the results of the experiment that the bias generated by the quasi-closed formulas, used for calculating the price of an American call and of a European call, is due to approximation errors, and that such bias can be reduced through the implementation of a trinomial stochastic tree (Cafferata, Giribone, & Resta, 2017). The purpose of the present article is to integrate that work with further deterministic numerical methods and to compare them to those based on the Monte Carlo methodology. In fact, the latter generally allows greater flexibility in the definition of payoffs which can be useful for modeling structured products.

2) The implemented deterministic numerical pricing methodologies

The main lattice techniques and the finite difference method implemented for pricing and determining the most common options sensitivity measures will be described in the following subparagraphs.

2.1) Binomial stochastic trees

The binomial method of Cox-Ross-Rubinstein represents one of the most widely used deterministic algorithms to evaluate options characterized by non-standard payoffs. The first formulation of the binomial method dates back to 1979 by John C. Cox, Stephen A. Ross and Mark E. Rubinstein (Cox J. C., Ross S. A. & Rubinstein M., 1979). They demonstrated how to build a binomial tree which could discretize and approximate a geometric Brownian motion, in such a way that, if a large number of time intervals were considered, the use of the binomial method to evaluate European options would be equivalent to using the continuously defined Black-Scholes-Merton formula (Hull, 2015). The most interesting aspect, however, is that the binomial model allows to evaluate American options and many exotic options, for which there is often no exact pricing formula. The analytical formulas of Black-Scholes-Merton are, in fact, almost always unsuitable for providing a fair value for options with non-standard characteristics, such as the possibility of exercising them before maturity (Bermuda /American options) or with particularly complex payoffs (exotic options). In all of these cases, therefore, a numerical methodology has to be used for the valuation of the derivative.
The technique essentially involves dividing the time between the option valuation date and its expiry date into numerous time intervals, assuming that during the intervals two possible changes may occur in the value of the underlying of the derivative.
By way of example, it will be assumed that the underlying is a share but, this valuation methodology may be extended in a generalized form to numerous types of underlying (GBS - Generalized Black-Scholes pricing framework) (Haug, 2007).
The value of the share in a binomial tree, after a time interval Δt, can increase by a fixed amount u with a probability of p or it can decrease by a fixed amount d with a probability equal to $1-p$. N corresponds to the number of time intervals into which the time between the option valuation date and its expiry has been divided.
In order to distinctly identify each node of the binomial tree, the reference time interval is defined with the index j and the possible value assumed by the financial instrument, moving from one node to the next, is defined with the index i. The first node in the tree is identified with the values ($j=0$, $i=0$). If the price of the asset increases by the amount u (such that $S \cdot u > S$), the second node will be identified with the values ($j=1$, $i=1$). If, on the other hand, the price of the asset decreases by the amount d (such that $S \cdot d < S$), the position of the tree will be identified with the values ($j=1$, $i=0$). This is shown in Figure 3, which shows, by way of example, a binomial tree with five time-intervals.

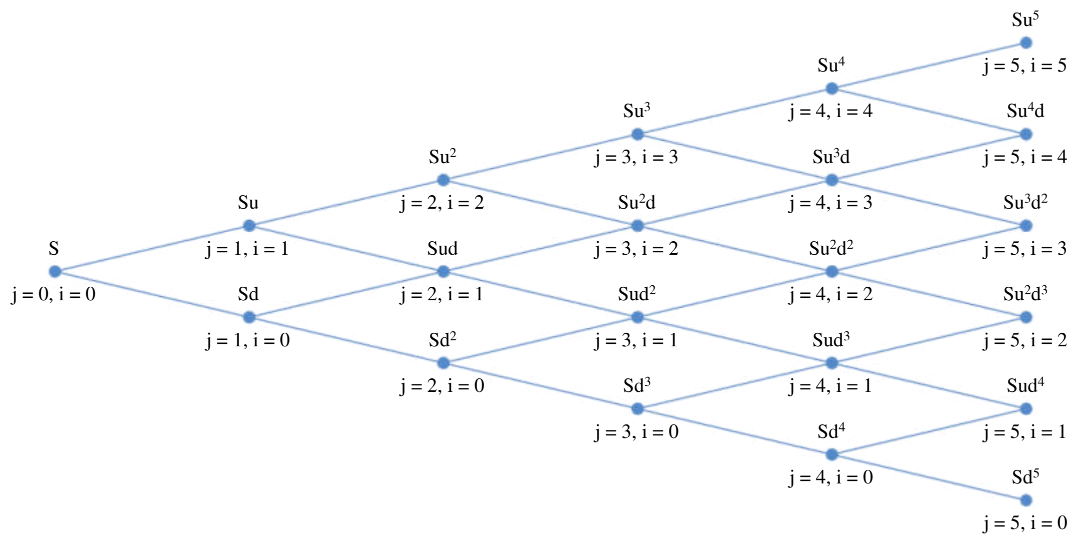


Figure 3. Example of a stochastic tree with arborescence $N = 5$

The number of paths leading to a general node (j, i) is equal to: $\frac{j!}{i!(j-i)!}$. Starting from this discretization scheme of the underlying, the pay-off $g(\cdot)$ is applied in the terminal nodes, and, proceeding backwards, the recombination of the tree is performed until the starting node ($j=0$, $i=0$) is reached, in which the price of the derivative is determined. In order to obtain a matching with the stochastic dynamics postulated by the Black-Scholes framework, Cox, Ross and Rubinstein suggested to select the parameters u and d so that, for each discrete time interval Δt, the assumed future values of the asset be consistent with the mean and the theoretical variance of the continuous model (Di Franco, Polimeni & Proietti, 2002). To this end, Cox, Ross and Rubinstein set the parameters u and d as follows: $u = \exp(\sigma\sqrt{\Delta t})$ and $d = 1/u = \exp(-\sigma\sqrt{\Delta t})$, where $\Delta t = T/N$ is the length of each time interval (i.e. the time interval

between price movements), T is the time to expiry of the option expressed in years, σ is the annualized volatility of the share price and N is the number of time intervals. Under those hypotheses, the probability that the share price increases between one interval and the next is defined as risk-neutral probability and its value is equal to: $p = (\exp(b\Delta t) - d)/(u - d)$, where b is the parameter known as cost-of-carry.
Depending on the value assumed by this parameter, we reach a pricing framework that can be used for a large number of option underlyings. In particular (Haug, 2007):

- if $b = r$ the definition is suitable for pricing options written on shares that pay no dividend.
- if $b = r - q$ the definition is suitable for pricing options written on shares or indices with a continuous dividend yield q.
- if $b = 0$ the definition is suitable for pricing options on futures.
- if $b = r - r_{FOR}$ the definition is suitable for pricing currency options.

The general formulation for pricing a European option is therefore:

$$Price = \exp(-rT) \sum_{i=0}^{N} \frac{N!}{i!(N-i)!} p^i (1-p)^{N-i} g(Su^i d^{N-i}, K) \quad (5)$$

The up and down jump factors (u, d) and the respective probabilities (p) of increasing/decreasing the price level of the underlying in the next step, $\Delta t = T/N$, depend on the model used.

In the CRR (Cox-Ross-Rubinstein) Tree u, d, Π are chosen to match the first two moments of the price level distribution, as discussed before (Cox J. C., Ross S. A. & Rubinstein M., 1979):

$$u = \exp\left(\sigma\sqrt{\Delta t}\right) \quad (6)$$
$$d = \exp\left(-\sigma\sqrt{\Delta t}\right) \quad (7)$$
$$\Pi = \frac{\exp(b\Delta t) - d}{u - d} \quad (8)$$

There are other binomial methodologies in the literature, which allow the matching with the mean and the theoretical variance of the continuous Black-Scholes-Merton model. These are called alternative stochastic binomial trees. The most popular are:

In the JR (Jarrow-Rudd) Tree u and d are chosen in order to have a probability of $\frac{1}{2}$ (Jarrow & Rudd, 1993):

$$u = \exp\left[(b - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta T}\right] \quad (9)$$
$$d = \exp\left[(b - \sigma^2/2)\Delta t - \sigma\sqrt{\Delta T}\right] \quad (10)$$
$$\Pi = \frac{1}{2} \quad (11)$$

The model proposed by Tian equals the first three moments of the log-normal distribution followed by the underlying (Tian, 1993):

$$u = \frac{1}{2}\exp(b\Delta t) v \left(v + 1 + \sqrt{v^2 + 2v - 3}\right) \quad (12)$$
$$d = \frac{1}{2}\exp(b\Delta t) v \left(v + 1 - \sqrt{v^2 + 2v - 3}\right) \quad (13)$$
$$\Pi = \frac{\exp(b\Delta t) - d}{u - d} \quad (14)$$
$$v = \exp(\sigma^2 \Delta t) \quad (15)$$

The Leisen and Reimer tree sets the u and d factors so that the tree is centered around the strike price. This makes the convergence tend to the option value more smoothly and with a better performance (Leisen & Reimer, 1996). The parameters characterizing the chain are:

$$\Pi = h_{PP}(d_2) \quad (16)$$
$$u = \exp(b\Delta t) \frac{h_{PP}(d_1)}{h_{PP}(d_2)} \quad (17)$$
$$d = \frac{\exp(b\Delta t) - pu}{1 - p} \quad (18)$$
$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (19)$$
$$d_2 = d_1 - \sigma\sqrt{T} \quad (20)$$

Preizer-Pratt suggest two methods for the $h_{PP}(x)$ calculation (Giribone & Ligato, 2016).

The first inversion method (LR1 Tree) sets:

$$h_{PP1}(x) = \frac{1}{2} + \eta \left\{ \frac{1}{4} - \frac{1}{4}\exp\left[-\left(\frac{x}{N + \frac{1}{3}} \right)^2 \left(N + \frac{1}{6} \right) \right] \right\}^{\frac{1}{2}} \quad (21)$$

While the second inversion method (LR2 Tree) estimates:

$h_{PP2}(x) = \frac{1}{2} + \eta \left\{ \frac{1}{4} - \frac{1}{4} \exp\left[-\left(\frac{x}{N + \frac{1}{3} + \frac{0.1}{N+1}} \right)^2 \left(N + \frac{1}{6} \right) \right] \right\}^{\frac{1}{2}}$ (22)

Where: $\begin{cases} \eta = +1, x \geq 0 \\ \eta = -1, x < 0 \end{cases}$ (23)

Up to now we have shown the procedure to be used to value a European option, characterized by the fact that it can only and exclusively be exercised at maturity. It is therefore necessary to introduce the so-called early exercise feature into the model, which is the characteristic that distinguishes an American option: the possibility to exercise it at any time, from instant 0 to expiry T. Such peculiarity translates into the fact that, instead of valuating the pay-off only at maturity, and proceeding backwards through the backwardation algorithm, the pay-off also has to be determined in each discrete time interval, in order to verify whether it is convenient to exercise the option early or whether to bring it to maturity: the option price will be the higher of the values defined in the two respective scenarios (Hull, 2015). We can therefore infer that, in each node of the binomial tree, the value of the American call option is:

$$C_t = \max[C_{Dead}; C_{Alive}] = \max\left[S_t - K; \frac{C_u \Pi + C_d(1-\Pi)}{1+r}\right] \text{ (24)}$$

While for an American put option it is:

$$P_t = \max[P_{Dead}; P_{Alive}] = \max\left[K - S_t; \frac{P_u \Pi + P_d(1-\Pi)}{1+r}\right] \text{ (25)}$$

In order to generalize the procedure for a multi-step tree, two distinct indices have to be introduced: i which identifies the time-step and j which identifies the expected price: the equation which considers the right to early exercise in a general node of the binomial tree is the following (Haug, 2007):

- for the call option:

$$C_{i,j} = \max\left[S \cdot u_i \cdot d_{j-1} - K; \frac{C_{j+1,i+1}\Pi + C_{j+1,i}(1-\Pi)}{1+r}\right] \text{ (26)}$$

- for the put option:

$$P_{i,j} = \max\left[K - S \cdot u_i \cdot d_{j-1}; \frac{P_{j+1,i+1}\Pi + P_{j+1,i}(1-\Pi)}{1+r}\right] \text{ (27)}$$

The principle to be applied to valuate an option, taking the early exercise into account, is the same in all types of lattice models (Giribone & Raviola, 2019). In implementing the backwardation algorithm, the potential convenience of bringing the option to maturity has to be considered.

2.2) Trinomial stochastic trees

The construction of a trinomial tree is very similar to the procedure followed for developing a multi-step binomial tree (Boyle, 1986). Generally, the construction of the trinomial tree that represents the evolution of the price of the underlying occurs by using stochastic differential equations (SDE). The first step is to build the price chain of the underlying until maturity. The following step is to calculate the option price, starting from the pay-off function at maturity, and discounting the future expected values (backwards induction phase). Firstly, it is necessary to introduce the stochastic differential equation of a Brownian geometric motion, which describes the evolution of the price of the underlying (Hull, 2015):

$$dS = (r - q)Sdt + \sigma S dW_t \text{ (28)}$$

- $\mu = r - q$ is the annualized expected return earned by an investor over the time period dt.
- σ is the annualized volatility of the asset.
- dW_t is a Wiener process.

The variable $x = \ln(S)$ is defined by obtaining the modified stochastic differential equation:

$dx = \nu dt + \sigma dW \quad$ with $\quad \nu = r - q - \frac{1}{2}\sigma^2$ (29)

Now let us consider what happens to the variable x in a time interval Δt. The model supposes that it can assume three different values: it can increase (up) or decrease (down) by an amount equal to Δx or remain unchanged (no change). A probability is associated with each of the potential changes in the price of the underlying. To find the values of such probabilities, and obtain convergence with the Black-Scholes model, it is necessary to equal the mean and the variance in the interval Δx, and also impose the sum of the three probabilities equal to 1:

$E[\Delta x] = p_u(\Delta x) + p_m(0) + p_d(-\Delta x) = \nu \Delta t$ (30)
$E[\Delta x^2] = p_u(\Delta x^2) + p_m(0) + p_d(+\Delta x^2) = \sigma^2 \Delta t + \nu^2 \Delta t^2$ (31)
$p_u + p_m + p_d = 1$ (32)

Defining $\alpha = \frac{v\Delta t}{\Delta x}$ e $\beta = \frac{\sigma^2 \Delta t + v^2 \Delta t}{\Delta x^2}$, the values of the probabilities become as follows: $p_u = \frac{\alpha+\beta}{2}$, $p_d = \frac{\beta-\alpha}{2}$ e $p_m = 1 - \beta$.

We then proceed with the creation of the trinomial tree. Two indices are defined, n, which represents time, and j, which represents the price of the underlying. If S is the price of the underlying at $n = 0$, then the j-th price level is equal to $S_j^n = S\exp(j\Delta x)$.
We can therefore define the vector $\vec{S}$ as:

$\vec{S}[-N] = S\exp(-N\Delta x)$ (33)

$\vec{S}[j] = \vec{S}[j-1]\exp(\Delta x) \quad \text{with} \quad j = -N+1, \ldots, N$ (34)

Where N is the number of sub-periods in the interval $(0, T)$, T is the time to maturity, or $N\Delta t = T$.
Similarly to the development of the price of the underlying, the discretized values relating to the development of the price of the call option C are represented by the variable C_j^n.

The value of the call option at maturity is known, and its possible variants, respectively under the two different assumptions of continuous and discrete time, respectively, are given by:

$C(S, T) = \max(S - K, 0)$ (35)

$C_j^N = \max(S_j^N - K, 0)$ (36)

Finally, the price of the call option is determined at the n-th time interval as a discounted expectation under the hypothesis of risk neutrality, based on the value of the call option in the interval $n+1$:

$C_j^n = \exp(-r\Delta t)\left(p_u C_{j+1}^{n+1} + p_m C_j^{n+1} + p_d C_{j-1}^{n+1}\right)$ (37)

In short, the logical sequence of the steps to follow is:

1. The structure of the trinomial tree is created.
2. The value of the call option is initialized in the tree using the default probability values.
3. The pay-off vector C_j^N is calculated.
4. The values of the call options in the previous intervals are calculated C_j^n.

The first two steps represent the forward induction, while the second two steps implement the backward induction.
A very similar reasoning can be done to valuate a put option and to verify the convenience of early exercise during the backwardation phase.

2.3) The finite difference method

The finite difference method (FDM) is a numerical scheme that can be applied in quantitative finance for valuating options. The solution scheme of the explicit finite difference method for the fundamental Black-Scholes-Merton PDE (partial difference equation) is equivalent to the discounted expectations procedure of a trinomial tree (Duffy, 2006). Let us consider the PDE:

$$-\frac{\partial C}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 C}{\partial x^2} + v\frac{\partial C}{\partial x} - rC \quad (38)$$

We proceed from the expiry of the option backwards until the initial instant and the approximation of the explicit finite differences is constructed as follows:

$$-\frac{C_j^{n+1} - C_j^n}{\Delta t} = \frac{1}{2}\sigma^2 \frac{C_{j+1}^{n+1} - 2C_j^{n+1} + C_{j-1}^{n+1}}{\Delta x^2} + \frac{v\left(C_{j+1}^{n+1} - C_{j-1}^{n+1}\right)}{2\Delta x} - rC_j^{n+1} \quad (39)$$

Rearranging the terms of the equation we obtain:

$$C_j^n = p_u C_{j+1}^{n+1} + p_m C_j^{n+1} + p_d C_{j-1}^{n+1} \quad (40)$$

It is interesting to underline the fact that such equation, which represents the value of the call option in the interval n as the average, weighted by the probabilities of the three possible "states of the world" in the subsequent interval $n+1$, is the same which also describes the call option value in the pricing framework of a trinomial model, apart from the discount factor.

The values of the probabilities p_u, p_m e p_d are defined as follows:

$p_u = \frac{\Delta t\sigma^2}{2\Delta x^2} + \frac{\Delta t v}{2\Delta x}$ (41), $p_m = 1 - 2\frac{\Delta t\sigma^2}{2\Delta x^2} - r\Delta t$ (42) and $p_d = \frac{\Delta t\sigma^2}{2\Delta x^2} - \frac{\Delta t v}{2\Delta x}$ (43)

The probability values must be positive, and this entails certain restrictions on the amplitude of the step Δt. In general, the relationship between the time-steps and the spot price is as follows: $\Delta x = \sigma\sqrt{3\Delta t}$ (Clewelow & Strickland, 1998).

The ways in which early exercise can be taken into account for the FDM method are discussed below. An American option with a maturity T and with a function f for pay-offs can be exercised at any time until maturity. Let us define $P(S_t, t)$ as the option pay-off function, and $V(S_t, t)$ as the early exercise function, where $V: (0, \infty) \times [0, T] \to \mathbb{R}$.

$V(S_t, t)$ represents the price of the instrument at time t, which means that it values the future payoffs of the instrument, therefore $P(S_t, t)$ is the pay-off of the instrument if it were exercised at time t. This means that upon maturity:

$V(S, t) = P(S, t)$ with $S > 0$ (44)

Furthermore, based on the principle of non-arbitrage

$V(S, t) \geq P(S, t)$ with $(S, t) \in (0, \infty) \times [0, T]$ (45)

If the sign of the inequality would be strictly greater, it would not be convenient for the option holder to exercise it, since it would be more profitable to sell it at a price equal to $V(S, t)$, rather than exercise it for a value of $P(S, t)$. If, on the other hand, $V(S, t) = P(S, t)$, the optimal choice is to immediately exercise the option since by holding it till maturity, the holder risks losing money. Taking the above consideration into account, the general rule can be determined: the option holder must exercise it as soon as $V(S, t) = P(S, t)$. Up to the optimal exercise value, the relationship that dominates the dynamics of the option price is given by the fundamental Black-Scholes-Merton PDE (Hull, 2015):

$rS \frac{\partial V(S,t)}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} - rV(S, t) + \frac{\partial V(S,t)}{\partial t} = 0$ (46)

Where $V(S, t) > P(S, t)$.

At the optimal moment for exercise, the following equation applies:

$V(S, t) = P(S, t)$ (47)

These relations can be represented by the following free-boundary problem (Duffy, 2006):

$$\begin{cases} V(S,t) \geq P(S,t) \\ V(S,t) = P(S,t) \text{ or } rS \frac{\partial V(S,t)}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} - rV(S,t) + \frac{\partial V(S,t)}{\partial t} = 0 \\ V(S,T) = (K - S)^+ \quad \text{Terminal Condition} \\ \lim_{S \to 0} V(S,t) = K \quad \text{Left boundary condition} \\ \lim_{S \to \infty} V(S,t) = 0 \quad \text{Right boundary condition} \end{cases} \quad (48)$$

With $S > 0$ and $t \in [0, T]$.

The system describes the locus of points where $V(S, t) = P(S, t)$. In those points the system is not governed by the partial differential equation and the option holder of the option should exercise the option. In order to calculate the price of the American put option, the problem has to be transformed into a standard scheme for PDE. To do this, the traditional coordinate transformation has to be used (Giribone & Ligato, 2015). The established parabolic PDE is thus obtained from which the canonical resolution methods can be used.

3) The Monte Carlo methodology and the Longstaff-Schwartz algorithm

This stochastic pricing methodology, originally introduced by Boyle in 1977 (Boyle, 1977), can be used to value most options, but above all, thanks to its flexibility, it is mostly useful for pricing highly exotic derivatives or structured products. Since the value of a derivative is closely linked to the pattern of the price of the underlying financial asset $S(t)$ in the time period between the drafting of the contract and the maturity $t \in [0, T]$, it is necessary to mathematically describe a dynamic that represents the potential future trajectories of the asset on which the option is written. In this regard, the Monte Carlo method can be used to simulate a wide range of stochastic processes. The most common stochastic process and consistent with the Black-Scholes pricing framework is called Geometric Brownian motion and it is represented by the well-known Stochastic Differential Equation (SDE): $dS(t) = \mu S(t)dt + \sigma S(t)dW_t$. The SDE which identifies the Brownian geometric motion can be integrated through the Eulero-Maruyama numerical scheme and then implemented in a numerical processing software as follows (Kloeden & Platen, 1992):

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \to \Delta S = \mu S \Delta t + \sigma S \Delta W \to S_t = S_{t-1} + \mu S_{t-1} \Delta t + \sigma S_{t-1} \varepsilon \sqrt{\Delta t} \quad (49)$$

Stochastic calculus allows to formulate an analytical expression for the simulation of $S(t = T)$. Such result is considered extremely important for practical purposes, since it allows direct simulations of the asset to be performed at a general future time $t = T$, without needing to know the values assumed by the asset in the previous times $S(t < T)$. Starting from the hypothesis that the variable follows a stochastic process such as: $dS(t) = \mu S(t)dt + \sigma S(t)dW_t$, given Ito's lemma, we can state that there is a function $G(S(t))$ that follows the dynamics (Hull, 2015):

$$dG(S, t) = \left(\frac{\partial G(S,t)}{\partial S} \mu S + \frac{\partial G(S,t)}{\partial t} + \frac{1}{2} \frac{\partial^2 G(S,t)}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial G(S,t)}{\partial S} \sigma S dW_t \quad (50)$$

We define $G = \ln(S)$: $dG = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t$, $d\ln\big(S(t)\big) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t$, and by integrating the expression over time, we obtain:

$$\int_0^T d\ln\big(S(t)\big) = \int_0^T \left(\mu - \frac{\sigma^2}{2}\right)dt + \int_0^T \sigma dW_t \rightarrow \ln\left(\frac{S(T)}{S(0)}\right) = \left(\mu - \frac{\sigma^2}{2}\right)T + \sigma dW_T \rightarrow$$

$$S(T) = S(0)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma dW_T\right] \quad (51)$$

The above expression can be easily implemented in a vectorized way in a numerical processing software, such as R, for example.

$$S(T) = S(0)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma dW_T\right] \rightarrow S(T) = S(0)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon\sqrt{\Delta T}\right] \quad (52)$$

The formula allows to simulate, in a manner consistent with the BS framework, the value of the asset underlying an option at any point in time and in an efficient way in terms of computer time. This modeling methodology allows maximum flexibility in the definition of the pay-off $g(\cdot)$ even in the presence of exotic derivatives. As mentioned before, the problem with American option pricing is that such options can be exercised at any time until maturity, unlike a European option that can only be exercised at maturity. In this section we will use the following notation for the pay-off function: $P(S(t)) = \max(K - S(t), 0)$ and $P(S(t)) = \max(S(t) - K, 0)$ respectively for a put and for a call option. According to the Black-Scholes-Merton pricing framework, the underlying is modelled using the Geometric Brownian Motion (52).
For ease of notation, we consider the price of a put option, but the results can be extended to call options as well. The value at time 0 of a European option can be described as:

$$u(S,0) = E\big[\exp(-rT)P\big(S(T)\big)\big] \quad (53)$$

That is, the expected value of the discounted payoff at time T. In a similar way, the value of an American option at time 0 is given by:

$$u(S,0) = \sup_{t\in[0,T]} E\big[\exp(-rT)P\big(S(T)\big)\big] \quad (54)$$

i.e., the expected value of the discounted payoff at the time of exercise that yields the greatest payoff. This corresponds to the optimization problem of finding the optimal stopping time

$$t^* = \inf\{t \geq 0 | S(t) \leq b^*(t)\} \quad (55)$$

for some a-priori unknown exercise boundary b^* (Brandimarte, 2006). Thus, in order to price an American option, we need to find the optimal stopping time t^* and then estimate the expected value:

$$u(S,0) = E\big[\exp(-rt^*)P\big(S(t^*)\big)\big] \quad (56)$$

One of the most popular methods to solve this problem is called LSM, developed by Longstaff and Schwartz (Longstaff & Schwartz, 2001). This approach uses a dynamic programming approach to find the optimal stopping time and the Monte Carlo (i.e., numerical integration of Stochastic Differential Equation - SDE) to approximate the expected value. Dynamic programming is a general method for solving optimization problems by dividing them into smaller subproblems and combining their solution to solve the main problem (Kamien & Schwartz, 2012). In this case, this means that we divide the interval $[0, T]$ into a finite set of time points $\{0, t_1, t_2, \dots, t_N\}$ and, for each of these points, we decide if it is better to exercise than to hold on to the option. Starting from time T and working backwards to time 0, we update the stopping time each time we find a time where it is better to exercise, until we find the smallest time where exercise is better. Let $C\big(S(t_i)\big)$ denote the value of holding on to the option at time t_i i.e., the continuation value, and let the exercise value at time t_i be the payoff $P\big(S(t_i)\big)$. Then the dynamic programming algorithm to find the optimal stopping time can be summarized in the following pseudo-code:

> $t^* \leftarrow t_N$
> for t from t_{N-1} to t_1 do
> > if $C\big(S(t)\big) < P(S(t))$ then
> > > $t^* \leftarrow t$
> > else
> > > $t^* \leftarrow t^*$
> > end if
> end for

Using the same argument as in Equation (54), the continuation value at time t_i can be described in terms of conditional expectation:

$$C\big(S(t_i)\big) = E\big[\exp\big(-r(t^* - t_i)\big)P\big(S(t^*)\big)|S(t_i)\big] \quad (57)$$

Where t^* is the optimal stopping time in $\{t_{i+1}, \dots, t_N\}$. For ease of notation, we define the current payoff $\mathcal{P}$ as:

> for $t = t_N$:
> $\mathcal{P} = P(S(t))$
> from $t = t_{N-1}$ to $t = t_1$:
> if $C(S(t)) < P(S(t))$ then $\mathcal{P} = P(S(t))$, otherwise $\mathcal{P} = \exp(-r\Delta t)\,\mathcal{P}$

Where $\Delta t = t_{i+1} - t_i$.

Given this notation, Equation (57) becomes:

$$C(S_i(t)) = E[\exp(-r\Delta t)\mathcal{P}|S(t_i)] \quad (58)$$

To estimate this conditional expectation, the LSM method uses regular least squares regression (Huynh, Lai & Soumare, 2008).
This can be done since the conditional expectation is an element in L^2 space, which has an infinite countable orthonormal basis and thus all elements can be represented as a linear combination of a suitable set of basis functions.
So, to estimate this we need to choose a finite set of orthogonal basis functions, and project the discounted payoffs onto the space spanned by them. In the proposed implementation, the basis function chosen is the Laguerre polynomials, where the first four are defined as follows (Koornwinder, 2013):

$$\begin{cases} L_0 = 1 \\ L_1 = 1 - X \\ L_2 = \frac{1}{2}(2 - 4X + X^2) \\ L_3 = \frac{1}{6}(6 - 18X + 9X^2 - X^3) \end{cases} \quad (59)$$

Given a set of realized paths $S_i(t)$, $i = 1, \ldots, n$ that are in-the-money at time t, i.e. $P(S_i(t)) > 0$, and the payoffs $\mathcal{P}_i = \mathcal{P}(S_i(t))$, the conditional expectation in Equation (58) can be estimated as:

$$\hat{C}(S_i(t)) = \sum_{j=0}^{k} \hat{\beta}_j \, L_j(S_i(t)) \quad (60)$$

Where $L_0, \ldots, L_k$ are the first k Laguerre polynomials and $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are the estimated regression coefficients. The regression coefficients are obtained by regressing the discounted payoffs $y_i = \exp(-r\Delta t)\mathcal{P}_i$ against the current values $x_i = S_i(t)$ by regular least squares:

$$(\hat{\beta}_0, \ldots, \hat{\beta}_k)^T = (\boldsymbol{L}^T\boldsymbol{L})^{-1}\boldsymbol{L}^T(y_1, \ldots, y_n)^T \quad (61)$$

Where $\boldsymbol{L}_{i,j} = L_j(x_i)$, $i = 1, \ldots, n$ and $j = 0, \ldots, k$.

By approximating Equation (58) with Equation (60), we introduce an error in our estimation. In (Clement, Lamberton & Protter, 2002) it is shown that $\lim_{k\to\infty} \hat{C}(S(t)) = C(S(t))$.
Now that we have a method to estimate the continuation value, we can simulate a set of M realized paths $S_i(t)$, $t = 0, t_1, t_2, \ldots, t_N$ and $i = 1,2, \ldots, M$ and use the previous pseudo code to find the optimal stopping times t_i^* for all paths, and then estimate the expected value in Equation (56) using Monte Carlo:

$$\hat{u} = \frac{1}{M}\sum_{i=1}^{M} \exp(-rt_i^*)P(S(t_i^*)) \quad (62)$$

One way to speed up the algorithm is to use the discounted payoffs $\mathcal{P}_i$ in the Monte Carlo step instead of the optimal stopping times.
Since they are constructed and updated recursively in the same way as the stopping times, by the time we have gone from time $t = t_N$ and $t = t_1$ they will be:

$$\mathcal{P}_i = \exp(-r(t_i^* - t_1))P(S(t_i^*)) \quad (63)$$

Which means that:

$$\exp(-r\Delta t)\mathcal{P}_i = \exp(-rt_i^*)P(S(t_i^*)) \quad (64)$$

Thus Equation (62) becomes:

$$\hat{u} = \frac{1}{M}\sum_{i=1}^{M} \exp(-r\Delta t)\mathcal{P}_i \quad (65)$$

A pseudo code for the LSM algorithm is provided.

In each step only paths that are in-the-money are used since they are the only ones where the decision to exercise or continue is relevant.

> Initiate paths $S_i(t), t = 0, t_1, t_2, \ldots, t_N, i = 1,2, \ldots, M$

> Set $\mathcal{P}_i \leftarrow P(S_i(t_N))$ for all i
> for t from t_{N-1} to t_1 do
> > Find paths $\{i_1, i_2, \dots, i_n\}$ that are in the money: $P(S_i(t)) > 0$
> > Set $ITM\ paths \leftarrow \{i_1, i_2, \dots, i_n\}$
> > Set $x_i \leftarrow S_i(t)$ and $y_i \leftarrow \exp(-r\Delta t)\,\mathcal{P}_i$ for $i \in ITM\ paths$
> > Perform regression on x, y to obtain coefficients $\hat{\beta}_0, \dots, \hat{\beta}_k$
> > Estimate the continuation value $\hat{C}(S_i(t))$
> > Calculate the value of immediate exercise $P(S_i(t))$ for $i \in ITM\ paths$
> > for i from 1 to M do
> > > if $(i \in ITM\ paths)$ and $\left(P(S_i(t)) > \hat{C}(S_i(t))\right)$ then
> > > > $\mathcal{P}_i \leftarrow P(S_i(t))$
> > > else
> > > > $\mathcal{P}_i \leftarrow \exp(-r\Delta t)\,\mathcal{P}_i$
> > > end if
> > end for
> end for
> $Price \leftarrow \frac{1}{M}\sum_{i=1}^{M} \exp(-r\Delta t)\mathcal{P}_i$

4) Choice of the deterministic valuation model as a benchmark for the Monte Carlo method

The purpose of this paragraph is to select the best deterministic method to be compared with the results calculated with the Monte Carlo method. In order to carry out such validation of the methodologies described in paragraph 3, with the consequent selection of the best pricing approach, two different scenarios are considered:

CASE A – "Theoretical Case"
The entry parameters for the theoretical case are characteristic of a non-stressed market, as the risk-free rate is positive and equal to 6%, and in addition the option underlying pays a dividend, with a continuous dividend yield, or a rate of return calculated on a continuous basis, equal to 1%. Furthermore, the put option is ITM (in the money) as the strike price is greater than the spot. The parameters are as follows: $S = 40$ → Spot price; $K = 50$ → Strike price; $T = 2$ → Time to maturity (years); $r = 6\%$ → Risk-free rate; $q = 1\%$ → Continuous dividend yield; $b = r - q$ → Cost of carry; $\sigma = 25\%$ → Annualized volatility for the underlying

CASE B – "Market Case"
The second case, which we call "market case" for simplicity, deals with an American put option written on the S&P500 index. As in the previous case, the parameters for pricing the option are characteristic of a normal market condition, in which the risk-free rate is positive and equal to 1.949%, and the rate of return calculated on a continuous basis is equal to 1.466%. The ATM put option (At The Money) is valued with the market data as of 30/03/2022 (Source: Bloomberg®): $S = 4617.09$, $K = 4617.09$, $T = 1$, $r = 1.949\%, q = 1.466\%,$ and $\sigma = 20.526\%$.
Before testing the Monte Carlo method in a stressed market scenario, a benchmark deterministic model has to be identified to compare the results obtained between the deterministic and the stochastic techniques. To do this, it is necessary to determine which of the deterministic models has returned the best performance in terms of convergence level to the Black-Scholes model. The adopted approach for this purpose was to implement the closed Black-Scholes formula for pricing the European put option in the theoretical case, determining its price and sensitivity measures (Hull, 2015). The second step consisted in building a ranking model based on the magnitude of the percentage error regarding the prices, and on the mean and the standard deviation of the Greeks error, all of this compared to the results obtained with the B&S closed formula, for each of the deterministic models that have been implemented. The results obtained are reported in Table 1:

Price B&S	Delta B&S	Vega B&S	Rho B&S	Theta B&S	Gamma B&S
8.778847	-0.5568458	21.79769	-62.10536	0.2780668	0.02724711

Table 1. Price and Greeks of the European put option obtained with the Black-Scholes formula in the theoretical case

Where the price of the put option was calculated with the closed Generalized Black-Scholes formula (Black & Scholes, 1973), (Haug, 2007):

$$P = K\, e^{-rT}\, N(-d_2) - Se^{(b-r)T} N(-d_1) \quad (66)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}};\ d_2 = d_1 - \sigma\sqrt{T} \quad (67)$$

While the sensitivity measures were calculated with the exact formulas illustrated below (Haug, 2007):

$$Delta = e^{(b-r)T}(N(d_1) - 1) \ (68)\ Vega = Se^{(b-r)T} n(d_1)\sqrt{T} \ (69)\ Rho = -TKe^{-rT} N(-d_2) \ (70)$$

$$Theta = -\frac{(Se^{(b-r)T})n(d_1)\sigma}{2\sqrt{T}} + (b-r)Se^{(b-r)T}N(-d_1) + rKe^{-rT}N(-d_2) \quad (71) \qquad Gamma = \frac{n(d_1)e^{(b-r)T}}{S\sigma\sqrt{T}} \quad (72)$$

Once the price and the Greeks values of the European put option were determined with the Black-Scholes pricing framework, a dataset was drawn up with the differences, in absolute value and percentage value, between the price values calculated with the different deterministic methods. All the scenarios presented thereafter were conducted using 5,000 steps for the lattice models, the grid for the FDM is 5,000 by 5,000 in the underlying value/time to maturity discretization dimensions.

Ranking	Pricing Model	Price	Absolute Error	Error %
1°	Leisen-Reimer (LR)	8.778846	8.418940e-07	9.590029e-08
2°	Explict Finite Difference (FDM)	8.778863	1.612288e-05	1.836560e-06
3°	Tian (TIAN)	8.778895	4.806051e-05	5.474582e-06
4°	Trinomial (TRI)	8.778932	8.528244e-05	9.714538e-06
5°	Cox-Ross-Rubinstein (CRR)	8.778950	1.027063e-04	1.169929e-05
6°	Jarrow-Rudd (JR)	8.779062	2.153926e-04	2.453541e-05

Table 2. Ranking of deterministic models for the pricing of the European put option in the theoretical case

Subsequently, a further dataset was drawn up which contains the values of the mean and the standard deviation of the differences between the results obtained for the sensitivity measurements, calculated with the exact B&S formulas and those obtained with deterministic models, using the following numerical formulas: 2-sided finite difference for Delta, Vega and Rho, 1-sided finite difference for Theta, and Finite Central Difference for Gamma (Duffy, 2006).

Rank	Model	Delta Error	Vega Error	Rho Error	Theta Error	Gamma Error
1°	LR	0.16791 %	1.0335e-04 %	1.2587 e-06 %	0.025862 %	0.006015 %
2°	TRI	2.47082 %	1.9697e-04 %	1.0927 e-03 %	0.453552 %	8.595987 %
3°	JR	0. 13030 %	3.1874e-04 %	9.07064 e-02 %	0.795114 %	0.077334 %
4°	CRR	0.11538 %	2.6884e-04 %	2.27259 e-03 %	1.522928 %	0.174636 %
5°	FDM	6.07082 %	6.1439e-05 %	2.02134 e-03 %	2.11055 %	1.949889 %
6°	TIAN	0.223363%	1.5530e-04 %	4.41009 e-01 %	5.817020 %	0.327921%

Table 3. Ranking of deterministic models for estimating the Greeks of the European put option in the theoretical case

The results obtained from the comparison of the deterministic models with the Black-Scholes model have pinpointed the Leisen-Reimer binomial model as the most performing model, since the discrepancies generated by this pricing technique are significantly lower compared to the other implemented models. Given the outcome of the ranking model, the following tables show the comparison between the benchmark model and the LSM method for pricing the American put option in the theoretical case and in the market case. The number of paths simulated through the Monte Carlo technique is equal to 50,000 for the theoretical case and 100,000 for the market case. Price convergence was tested with 200 replications for both cases.

Theoretical Case	Price	Delta	Vega
LR	10.55684	-0.7526719	14.4264
LSM	10.54429	-0.7780889	13.9426

Table 4. Comparison of the price and the main Greeks between the LR Tree and the MC method - theoretical case.

Market Case	Price	Delta	Vega
LR	362.8773	-0.4482972	1805.850
LSM	362.2143	-0.4511456	1798.989

Table 5. Comparison of the price and the main Greeks between the LR Tree and the MC method - market case.

The standard deviation of the expected fair value for LSM is 0.005555 in the theoretical case and 0.5237 in the market case.

5) Definition of the price surfaces of a call option and of the Greeks under stressed market conditions

This paragraph shows the experimental results that define the valuation gap between a European call option and the corresponding American option written on equity with a zero pay-out and in the presence of negative interest rates, thus demonstrating the violation of the property of options according to which, under the assumption that the underlying equity pays no dividend, it will never be convenient to exercise an American call option prematurely, so such option will be priced like the European one (Hull, 2015). As already mentioned in the previous paragraph, the two pricing models involved in the stress-test are the binomial Leisen-Reimer technique, as a deterministic technique that has proved to be the most performing, and the Monte Carlo method of Longstaff-Schwartz, a stochastic technique whose simulation error is monitored based on the results of the LR model. This experimental phase was carried out according to the following scheme:
- Change of the option type in the two case studies, from put option to call option.
- Change in the scenario: the dividend yield parameter is equalized to zero and the risk-free interest rate is set as a parameter, covering values ranging from strongly negative to above zero values.
- Definition of the valuation gap surfaces and of the error of the estimation of the Greeks between the European call option and the corresponding American option with the Leisen-Reimer model.
- Definition of the price surfaces of the American call option with the Longstaff-Schwartz Monte Carlo method.
- Definition of the surfaces of the Greeks of the American call option with the Longstaff-Schwartz Monte Carlo method.
- Comparison between the methodological error introduced by the LR model and the experimental error of the LSM stochastic method.

The surfaces were calculated on the one hand, by setting the risk-free interest rate as a parameter, and on the other hand by setting the four fundamental input parameters for pricing the option. In particular, the following ranges of variation have been defined:
Risk-free rate $r \in [-10\% ; 2\%]$
Spot price $S \in [S - 50\% S ; S + 50\% S]$
Strike price $K \in [K - 50\% K ; K + 50\% K]$
Annualized volatility $\sigma \in [1\% ; 70\%]$
Time to maturity $T \in \left[\frac{1}{360} ; T\right]$
Regarding the granularity applied to the ranges of parameters used for calculating the different surfaces, the scheme was as follows:
For price surfaces:
- r step $25\ basis\ point = \frac{25}{10000}$
- S step $\frac{S}{100}$
- K step $\frac{K}{100}$
- σ step 1%
- T step $T * \frac{7}{360}$ for the theoretical case and $T * \frac{3.5}{360}$ for the market case.

For Greeks surfaces:
- r step $50\ basis\ point = \frac{50}{10000}$
- S step $\frac{S}{50}$
- K step $\frac{K}{50}$
- σ step 2%
- T step $T * \frac{14}{360}$ for the theoretical case and $T * \frac{7}{360}$ for the market case.

The granularity applied to the ranges of parameters for calculating the Greeks surfaces is reduced compared to that relating to the price surfaces. This choice is aimed at obtaining a suitable trade-off between the experimental test grid and the computational time, which is systematically greater for estimating the sensitivity measures, since it is calculated with numerical formulas, and the individual option pricing procedure is implemented at least twice for each measure.

5.1) Surfaces of the valuation gap between the European and American call option with the Leisen-Reimer model

In this sub-paragraph, the surfaces that determine the valuation gap between the European and the American call option are presented respectively, and those relating to the error in the estimation of sensitivity measures, both for the theoretical case and for the market case. The error is measured as a percentage according to the following formulas:
For the price surfaces → $\%\ Price\ Error = 100 * \frac{PAm - PEu}{PAm}$

For the Greeks surfaces → $\%\ Greek\ Error = 100 * \frac{GreekAm - GreekEu}{GreekAm}$

The surfaces are shown in a three-dimension box, based on the following orientation of the axes:

Horizontal axis 1 (x) → parameter r (risk-free rate)

Horizontal axis 2 (y) → variable parameter (S, K, σ, T)

Vertical axis (z) → Error ($\%\ Price\ Error, \%\ Greek\ Error$)

Furthermore, a surface coloring scheme is respected, based on the variable parameter used in the calculation:

$Spot\ Price\ (S)$ → Red

$Strike\ Price\ (K)$ → $LightBlue$

$Volatility\ (\sigma)$ → $Yellow$

$Time\ to\ maturity\ (T)$ → $Green$

In order not to burden the dissertation, all cases for the price of the considered derivative are reported, while for the surfaces of the Greeks, only those of the theoretical case are displayed.

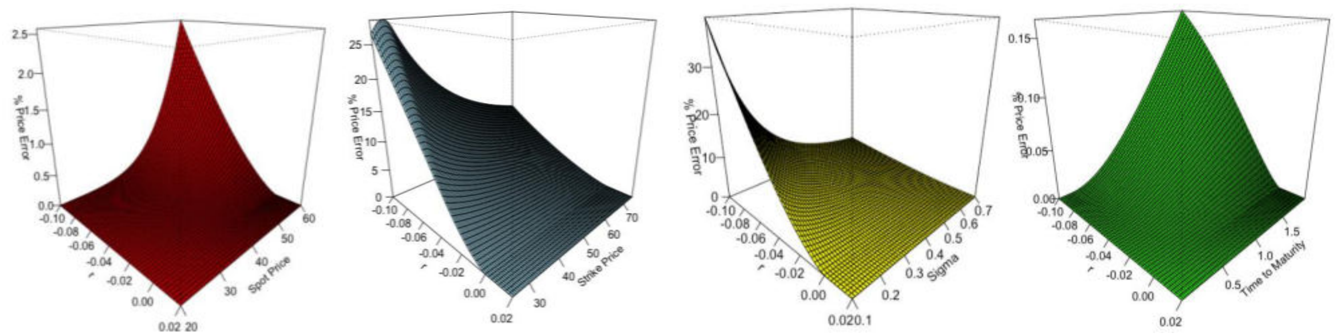


Figure 4. Gap in the fair value of the American-European LR in the theoretical case

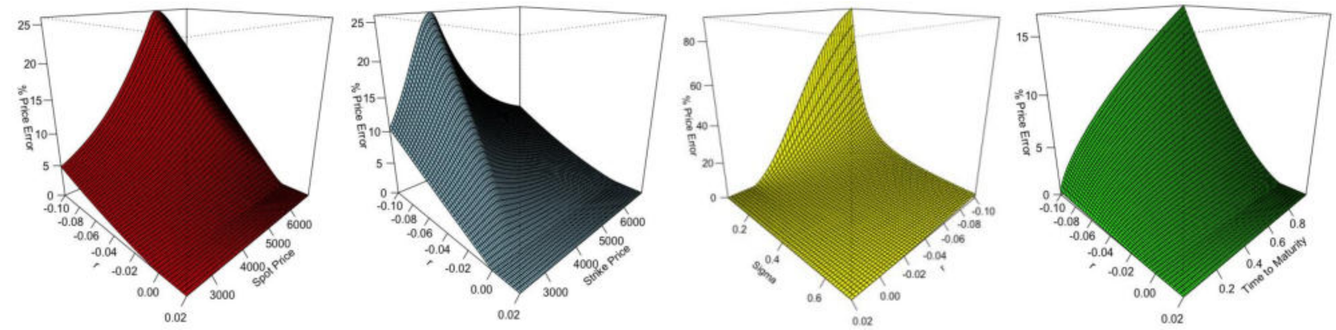


Figure 5. Gap in the fair value of the American-European LR in the market case

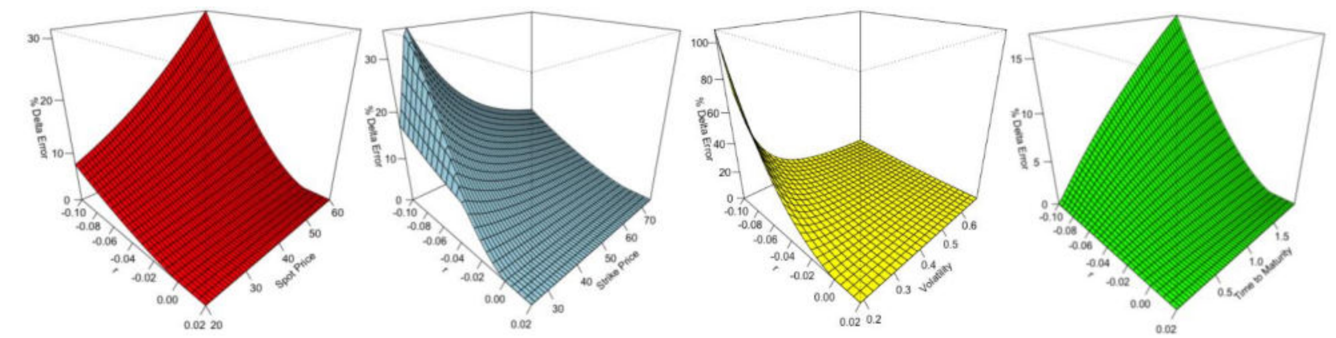


Figure 6. Gap in the estimate of the Delta of the American-European LR in the theoretical case

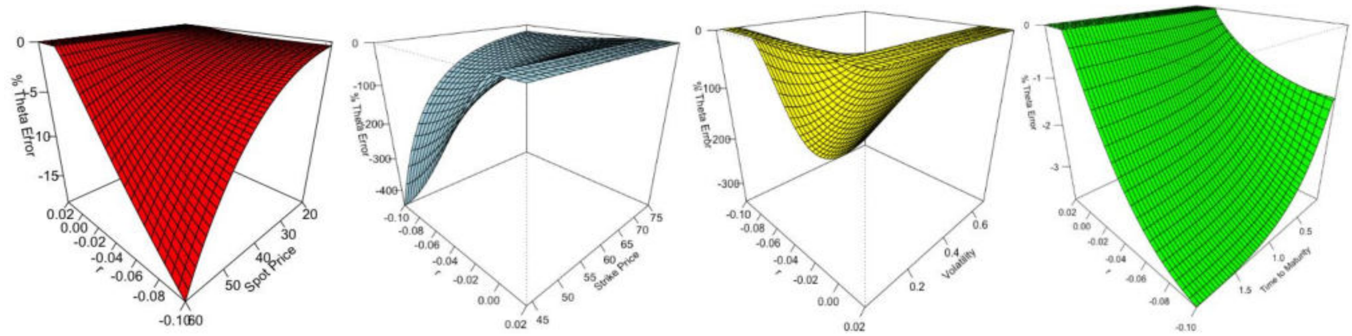


Figure 7. Gap in the estimate of the Theta of the American-European LR in the theoretical case

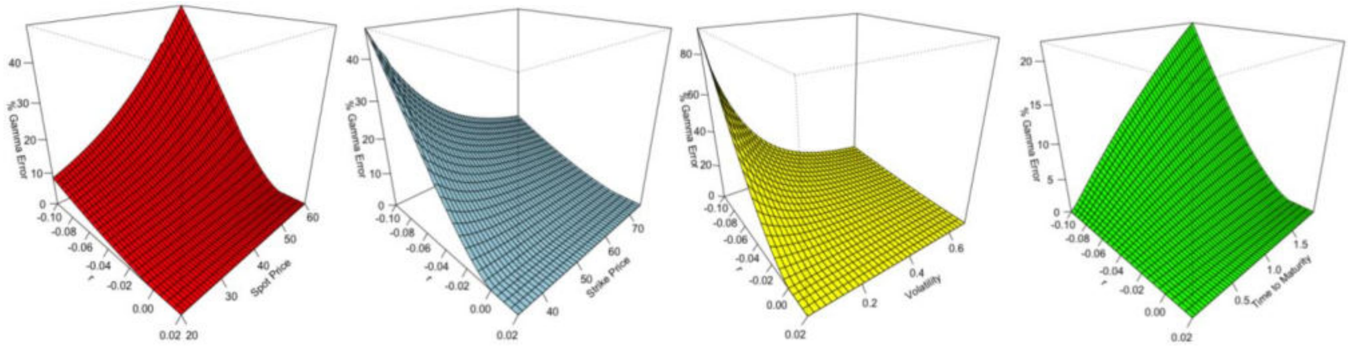


Figure 8. Gap in the estimate of the Gamma of the American-European LR in the theoretical case

5.2) Surfaces of the valuation gap between the European and the American call option with the Monte Carlo model.

Following the grid and color conventions used for the LR case, only the surfaces related to the theoretical and the market case for pricing are shown, in order not to burden the dissertation.

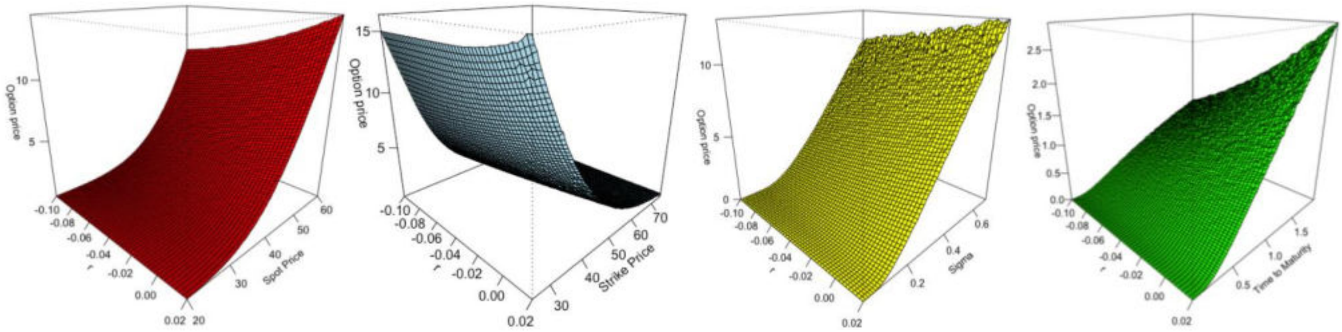


Figure 9. Estimate of the fair value of the Monte Carlo of the American option in the theoretical case

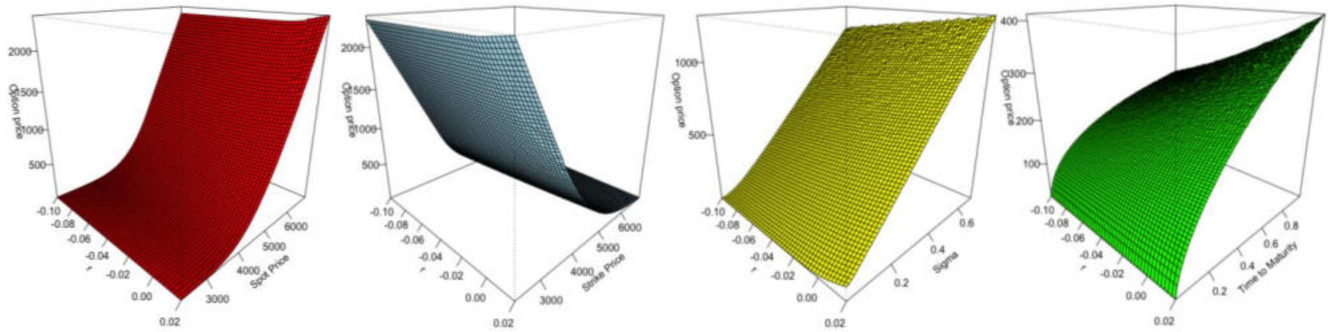


Figure 10. Estimate of the fair value of the Monte Carlo of the American option in the market case

5.3) Comparison between the methodological (LR) and the experimental error (LSM) in the pricing and estimation of the Greeks

This paragraph illustrates the comparison surfaces between the methodological error of the Leisen-Reimer model and the experimental error produced by the simulations of the Longstaff-Schwartz Monte Carlo method. The last phase of the experiment involves the comparison between the valuation gap of the prices calculated using the LR model and the size of the experimental error of the stochastic method. The purpose of this comparison is to analyze the behavior of the experimental error produced by the Monte Carlo simulations, and the extent of the discrepancy produced by such error with respect to the values of the LR benchmark, in extreme market conditions. In order to represent the extent of the Monte Carlo simulation error, with respect to the valuation gap of the Leisen-Reimer binomial model, a mask has to be applied to the error matrix containing the differences between the values of the European and the American call option: if the simulation error, resulting from the absolute value of the difference between the price of the American option calculated with the LSM and with the LR, is greater than the valuation gap produced by the deterministic model, then the resulting matrix highlights the size of the experimental error in absolute value, otherwise, that is, if the valuation gap of the LR is smaller than the experimental error, the matrix does not show such discrepancy. It should be remembered that for the Monte Carlo method only the sensitivity measures were estimated, and for them, the combination of the sensitivity of the numerical formulas and the model randomness allow to valuate with a relatively low error margin. The Greeks for which it was possible to obtain an acceptable and reasonably robust estimate were the Delta and the Vega. In order not to overly burden the dissertation, the comparison surfaces between the LR methodological error and the experimental error are shown below only for the case of pricing.

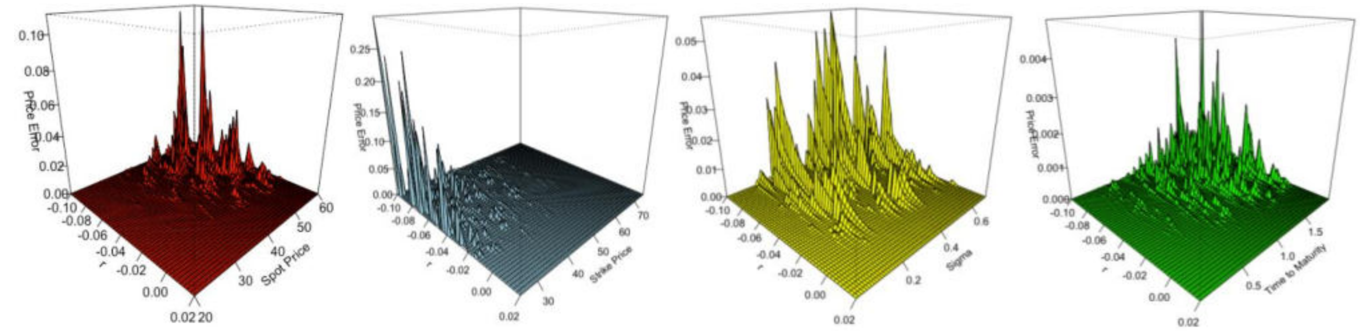


Figure 11. Comparison surface between methodological error (LR) and experimental error (LSM) for pricing the call option in the theoretical case

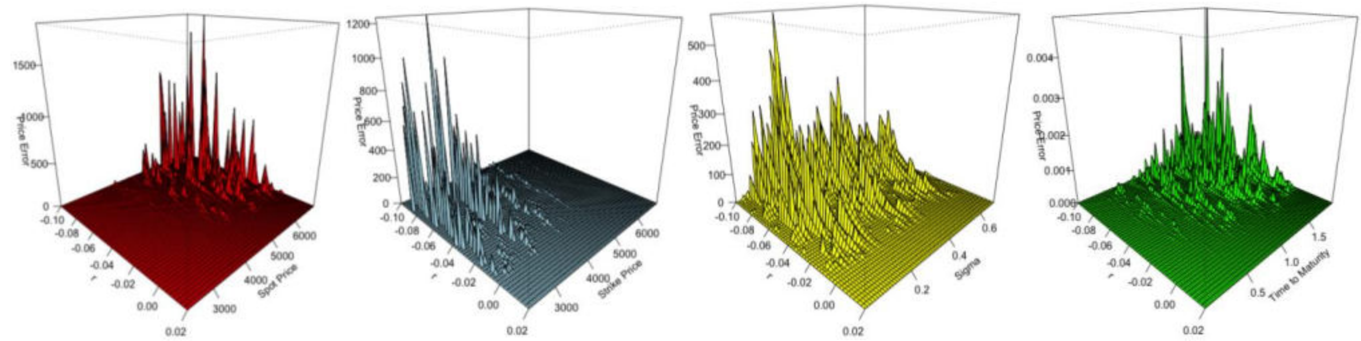


Figure 12. Comparison surface between methodological error (LR) and experimental error (LSM) for pricing the call option in the market case

6) Conclusions

The objective of the paper is to investigate the main problems that impact the pricing models and the sensitivity measures of American options written on shares that do not pay dividends, in the presence of negative interest rates. A literary review is conducted and the most popular lattice pricing methods are implemented as well as the Monte Carlo technique. The Leisen-Reimer binomial model proved to be the most performing deterministic methodology among those implemented in a non-stressed market context, this methodology was then tested in extreme market conditions, i.e., in the presence of negative interest rates and on a non-profitable underlying. Using the stress test, the valuation gap between the American and the European call option was determined. This discrepancy can be interpreted as the model risk deriving from the change in the valuation technique of the derivative, necessary it is impossible to use traditional techniques (quasi-closed formula), which are only applicable in ordinary market conditions (i.e. positive interest rates). The experimental results obtained with the Leisen-Reimer model were used as a benchmark for controlling the stability and performance of the Longstaff-Schwartz Monte Carlo in extreme scenarios. The concept underlying the Monte Carlo method is that being a stochastic methodology, by definition, it always produces different outputs, due to its random nature. The question which this work tries to answer is how the error generated by the Monte Carlo method "overlaps" the discrepancy relating to the model risk of the Leisen-Reimer.

If the error produced by the Monte Carlo method is lower than the model risk, then the same observations apply as for the LR: in a certain range of pricing parameters, the Monte Carlo method is stable and gives similar results to the LR. On the contrary, when the error generated by the stochastic methodology is higher, the model risk discrepancy is not so significant, therefore the outputs generated by Monte Carlo are considered unstable, in relative terms. Ultimately, if we observe the comparison surfaces between the methodological error and the experimental error, we observe that in many cases, except for rare out-of-scale peaks for the most extreme regions of the surfaces, the Monte Carlo method is reasonably stable, since the ranges of the error generated by the simulations are lower than the benchmark values. From the analysis of the surfaces, certain regions can be identified where a Monte Carlo instability occurs, caused by the combination of the following factors:

- for surfaces for which the spot price and the strike price are parameterized, extremely negative interest rate values, close to -10% and the deep in-the-money (ITM) option.
- for surfaces for which volatility is parameterized, extremely negative interest rate values and very low volatility values.
- for surfaces for which the time to maturity is parameterized, extremely negative interest rate values and the option close to maturity.

It should be highlighted that the Monte Carlo method, in the region of positive interest rates, always returns the results of the LR upon convergence. To conclude, we can state that the performance of the Monte Carlo method is effective for the interest rate intervals around zero, except for an extreme stress-test on the parameters S, K, σ, T. However, the further we delve into the regions of extremely negative interest rates, the greater the instability of the model. Considering the dynamic of the Euribor historical series from 2014 until today, and considering the negative values assumed by nominal interest rates, we can state that the Monte Carlo model ensures a reasonable reliability in the pricing of options written on equity, even in a context of moderately negative interest rates.

References

[1] Ansa (2014). “Bce taglia tassi, prima volta negativi su depositi”
Link: https://www.ansa.it/sito/notizie/economia/2014/06/04/attesa-per-mosse-draghi-bce-prova-a-riaccendere-inflazione
[2] Barone-Adesi, G., Whaley, R. E. (1987). “Efficient Analytic Approximation of American Option Values”. Journal of Finance 42, 301-320
[3] Bernanke, B. S. (2010). “Causes of the Recent Financial and Economic Crisis”. Federal Reserve Press Release.
Link: https://www.federalreserve.gov/newsevents/testimony/files/bernanke20100902a.pdf
[4] Bjerksund, P., Stensland, G. (1993). “Closed-Form Approximation of American Options”. Scandinavian Journal of Management 9, 87-99.
[5] Bjerksund, P., Stensland, G. (2002). “Closed-Form Valuation of American Options”. Norwegian School of Economics and Business Administration. Department of Finance and Management Science, Discussion Paper 9.
[6] Black, F. (1976). “The pricing of commodity contracts”. Journal of Financial Economics 3(1-2), 167-179.
[7] Black, F., Scholes M. (1973). “The pricing of options and corporate liabilities”, Journal of Political Economy 81(3), 637-654.
[8] Boyle, P.P. (1977). “Options: a Monte Carlo approach”. Journal of Financial Economics 4(3), 323-338.
[9] Boyle, P.P. (1986). “Option Valuation Using a Three-Jump Process”. International Options Journal 3, 7-12.
[10] Brandimarte, P. (2006). “Numerical methods in finance and economics.” Wiley Interscience.
[11] Brigo D., Mercurio F. (2006). “Interest Rate Models – Theory and Practice with Smile, Inflation and Credit”, Springer Finance
[12] Burro G., Giribone P. G., Ligato S., Mulas M., Querci F. (2017). “Negative interest rates effects on option pricing: Back to basics?”. International Journal of Financial Engineering 4(2)
[13] Cafferata A., Giribone P. G., Neffelli M., Resta M. (2019). “Yield curve estimation under extreme conditions: do RBF networks perform better?”. Chapter 22 in book: "Neural Advances in Processing Nonlinear Dynamic Signals". Springer
[14] Caligaris O., Giribone P. G., Neffelli M. (2017). “Ricostruzione di superfici di volatilità mediante l'utilizzo di reti neurali auto-associative: un caso studio basato sull'analisi non lineare delle componenti principali”. Risk Management Magazine 12(3), 11-18
[15] Cafferata A., Giribone P. G., Resta M. (2017). “The effects of negative nominal rates on the pricing of American calls: some theoretical and numerical insights”. Modern Economy 9(6).
[16] Clement, E., Lamberton D., Protter P. (2002). “An analysis of a least squares regression method for American option pricing”.Finance and Stochastic 6, 449-471.
[17] Clewelow, L., Strickland, C. (1998). “Implementing Derivatives Models”. John Wiley and Sons.
[18] Cox J. C., Ross S. A., Rubinstein M. (1979). “Option pricing: a simplified approach”. Journal of Financial Economics 7(3), 229-263
[19] Di Franco M., Polimeni F., Proietti M. (2002). “Opzioni e titoli strutturati”. Il Sole 24 Ore. Milano
[20] Duffy, D. J. (2006). “Finite difference methods in financial engineering”. Wiley Finance.
[21] Dunkley, E. (2017), “Ten years after Northern Rock: has UK banking changed?”. *Financial Times*
Link: https://www.ft.com/content/d05c3ea4-97bf-11e7-a652-cde3f882dd7b
[22] ECB (2014), “The ECB’s negative interest rate”
Link: https://www.ecb.europa.eu/ecb/educational/explainers/tell-me-more/html/why-negative-interest-rate.en.html
[23] Giribone, P. G. (2020). “Critical analysis of the most widespread methodologies for the simulation of the short rate dynamics under extreme market conditions”. Risk Management Magazine 11(2), 65-72.
[24] Giribone, P. G. (2017). “Il problema dei tassi d’interesse negativi nel pricing e nell’hedging”. Proceedings of the Conference: “Risk and Supervision”. ABI (Associazione Bancaria Italiana). Rome.
[25] Giribone P. G., Ligato S. (2016). “Flexible-forward pricing through Leisen-Reimer trees: implementation and performance comparison with traditional Markov chains”. International Journal of Financial Engineering 3(2)
[26] Giribone P. G., Ligato S. (2015). “Option pricing via Radial Basis Functions: Performance comparison with traditional numerical integration scheme and parameters choice for a reliable pricing”. International Journal of Financial Engineering 2(2)
[27] Giribone P. G., Ligato S., Mulas M. (2017). “The effects of negative interest rates on the estimation of option sensitivities: the impact of switching from a log-normal to a normal model”. International Journal of Financial Engineering 4(1)
[28] Giribone P. G., Raviola P. (2019). “Progettazione, validazione ed implementazione di un modello reticolare avanzato per il pricing di un Flexible Forward su valute”. Risk Management Magazine 14(3), 25-38
[29] Haug E. G. (2007). “The complete guide to Option Pricing Formulas”. McGraw-Hill
[30] Holt, J. (2009). “A Summary of the Primary Causes of the Housing Bubble and the resulting credit crisis: a non-technical paper”. The Journal of Business Inquiry 8(1)
[31] Hull J. C. (2015). “Options, Futures and other Derivatives”. Wiley
[32] Huynh, H. T., Lai V. S., Soumare I. (2008). “Stochastic simulation and applications in finance with Matlab programs”. Wiley.
[33] Jarrow R., Rudd A. (1993). “Option pricing”. Irwin Professional Publication
[34] Kamien, M., Schwartz, N. L. (2012). “Dynamic Optimization: the calculus of variations and optimal control in Economics and Management”. Dover Publications.
[35] Kloeden P. E., Platen E. (1992). “Numerical Solution of Stochastic Differential Equations”. Springer Verlag
[36] Koornwinder T. (2013). “Orthogonal polynomials, a short introduction”. Computer Algebra in Quantum Field Theory: Integration, Summation and Special Functions. Springer-Verlag.
[37] Lehman Brothers Holding Inc. (2008). “Lehman Brothers Holdings Inc. announces it intends to file Chapter 11”. Lehman Brothers Holding Inc. Press Release.
Link:https://www.sec.gov/Archives/edgar/data/806085/000110465908059632/a08-22764_4ex99d1.htm
[38] Leisen D. P., Reimer M. (1996), “Binomial models for option valuation – examining and improving convergence”. Applied Mathematical Finance 3(4), 319-346
[39] Longstaff, F. A., Schwartz, E. S. (2001). “Valuing American Options by Simulation: A Simple Least-Squares Approach”. The Review of Financial Studies 14(1), 113-147.
[40] Tian Y. (1993). “A modified lattice approach to option pricing”. Journal of Futures Markets 13(5), 563-577

The new supervisory outlier test (SOT) on net interest income (NII): empirical evidence from a sample of Italian banks

Domenico Curcio (University of Naples "Federico II"), Igor Gianfrancesco (University of Bari "Aldo Moro"), Annalisa Pansini (Prometeia) and Alina Preger (Prometeia)[1]

Article submitted to double-blind peer review, received on 27th September 2022 and accepted on 14th November 2022

Abstract

This paper contributes to prior literature and to the current debate concerning the prudential supervisory framework to measure interest rate risk in the banking book (IRRBB), which has been significantly changed on April 2016, when the Basel Committee on Banking Supervision (BCBS) published the latest update of its measurement standards. The consultation launched by the European Banking Authority (EBA) on December 2021, aiming at introducing the supervisory outlier test (SOT) on net interest income (NII), presents several issues and policy implications which could influence in the next future banks' asset and liability management strategies, their internal control systems, risk policies and procedures.
By analyzing a sample of 28 Italian commercial banks at the end of 2021, representing more than 70% of Italian baking system's total assets , we observe that the thresholds proposed by the EBA appear very strict and significantly depend on: i) the sample considered, ii) the lower bound applied to interest rates in the downward scenarios and iii) the current level of interest rates term structure. Our results suggest that the proposed values should be considered with caution as it seems that their potential impacts have not been thoroughly assessed. Further analyses are therefore necessary to guarantee greater robustness of the methodology used for the calibration of the thresholds, taking also into account a wider sample of banks and longer time series, as well as the correlation between the two approaches.

Key words: interest rate risk in the banking book, banking regulation, supervisory outlier test, net interest income.

JEL CODE: G21, G28, G32.

1. Introduction

The prudential regulation of interest rate risk in the banking book (IRRBB) has been going through a period of significant changes starting from April 2016, with the publication of the Basel Committee on Banking Supervision (BCBS)'s new standards, which replaced those issued in 2004. BCBS (2016) confirmed the second pillar classification of IRRBB, given its heterogeneous nature, and introduced new elements for its measurement and management. The new 2016 BCBS standards were implemented in the European Union in two phases. Firstly, through the update of the EBA guidelines issued in July 2018 (EBA/GL/2018/02) in those areas where the supervisor felt the need for a more practical approach. Particularly, the main innovations were:

i) the introduction of the new six interest rate shock scenarios, represented by a) parallel shock up; b) parallel shock down; c) steepener shock; d) flattener shock; e) short rates shock up; and f) short rates shock down;
ii) the removal of the non-negativity constraint, which has been replaced by a new floor of -100 basis points for the time bands up to 1 year and increasing by 5 basis points per year up to 0 for maturities beyond 20 years. The above-mentioned guidelines were introduced in Italy through the 32nd update of the Bank of Italy's Circular 285/2013 in April 2020.

Secondly, through the publication, in May 2019, of Directive (EU) 2019/878 and Regulation (EU) 2019/876 of the European Parliament and of the Council, that amended Directive 2013/36/EU and Regulation (EU) No 575/2013, introducing the remaining elements of the BCBS's 2016 standards in the European regulatory framework. Particularly, Directive (EU) 2019/878 gave the mandate to the EBA to issue specific Regulatory Technical Standards (RTS) and a further update of the guidelines, to provide a detailed regulation of the whole new IRRBB prudential framework. Subsequently, on December 2021, the EBA published the following three consultation documents:

i) EBA/CP/2021/36, a draft of RTS regarding the implementation of the supervisory outlier test (SOT) with reference to the economic value of equity (EVE) and net interest income (NII) approach. Particularly, in accordance with the article 98(5a) of Directive 2013/36/EU, the consultation document specifies the supervisory shock scenarios, the modelling and parametric assumptions for the SOT on EVE and on NII and provides a definition and calibration of the concept of "large decline" within the NII approach. Two different definitions of NII are proposed. It is worth noting the proposed modification of the EBA floor from -100 to -150 basis points for maturities up to 1 year, that increases by only 3 basis points per year (instead of the current 5 basis points). Consequentially, the new floor reaches the 0 level for maturities of 50 years and beyond (EBA, 2021a).
ii) EBA/CP/2021/37, a draft update of the guidelines currently in force. Particularly, based on article 84(6) of Directive 2013/36 /EU, the consultation document provides specific criteria: a) for the assessment and monitoring of credit spread risk from non-trading book activities (CSRBB) and b) to determine whether the internal systems implemented by institutions for the purpose of evaluating IRRBB are not satisfactory. Furthermore, it is extended to the full amount of non-financial deposits the five-year cap in terms of weighted average repricing date (EBA, 2021b).

---

[1] The authors thank Corrado Meglio (Deputy President of the Italian Association of Financial Risk Managers - AIFIRM) and all the participants to the AIFIRM Working group on IRRBB for the many useful discussions and suggestions. The views expressed are solely of the authors and so cannot be taken to represent those of University of Bari "Aldo Moro", University of Naples "Federico II" and Prometeia.
At the end of the double-blind review process, the EBA published the final reports of the consultation started on December 2021. With regards to the SOT on NII, the final report EBA/RTS/2022/10 confirms Metric 1, the "narrow NII" definition and the associated threshold proposed during the consultation. This makes the issues described in this article very topical and relevant from a business and risk management perspective. The Metric 2 has been discarded, as AIFIRM also suggested.

iii) EBA/CP/2021/38, a draft of RTS for a standardized methodology to evaluate IRRBB within both EVE and NII approach. Particularly, in accordance with article 84(5) of Directive 2013/36/EU, the consultation document defines a collection of procedural aspects and applicable assumptions for the standardized methodology and also describes the simplified standardized methodology for small and non-complex institutions, as defined in point 145 of Article 4(1) of Regulation (EU) n. 575/2013. The simplified methodology relies on various simplifications to reflect the generally less advanced capabilities of small and non-complex institutions and to satisfy, at the same time, the need for a methodology that is at least as conservative as the standardized one.

The present work aims to shed more light on the implications deriving from the definition and calibration of the "large decline" within the NII approach as defined by EBA/CP/2021/36. The contribution is placed on the research line that analyzes the robustness of the IRRBB prudential regulatory framework. To our knowledge, this is the first work dealing with the NII approach in a regulatory perspective. Most of the existing literature has focused on the drawbacks of the previous regulatory framework based on BCSB (2004), which only considered the EVE approach, by mainly testing its underlying assumptions.

As for papers published on international journals see Fiori and Iannotti (2007), Entrop et al. (2008), Entrop et al. (2009), Abdymomunov and Gerlach (2014), Cocozza et al. (2015) and Cerrone et al. (2017)[2]. As concerns papers published on Italian journals see Curcio and Gianfrancesco (2011, 2012), Partesotti and Preger (2015), Gianfrancesco (2016, 2017, 2018). A second research line concerning IRRBB is represented by those works that used evidence from the application of regulatory methodologies to study the determinants of risk exposure through specific econometric frameworks, such as Esposito et al. (2015), Chauldron (2018) and Hoffmann et al. (2019). Recently, a third research line has begun to analyze the implications deriving from the regulatory changes proposed by BCSB (2016), such as Curcio et al. (2022) and Cocozza et al. (2022).

AIFIRM gave particular attention to IRRBB issues by publishing three different position papers. The first, in 2015, following the participation to the consultation promoted by BCBS on the new regulatory standards in June 2015. Subsequently, in 2019 AIFIRM established a specific working group on IRRBB, whose works led in February 2021 to the release of a second position paper regarding the implications of the new supervisory regulatory framework based on BCBS (2016). Finally, the third position paper was published in April 2022, following the response that the IRRBB working group gave to the EBA consultation launched last December on the RTS and the guidelines update listed above.

By analyzing a sample of 28 Italian commercial banks at the end of 2021, representing more than 70% of Italian baking system's total assets, our evidence shows that the calibration methodology proposed by EBA (2021a) is significantly dependent on the sample the agency took into account, as well as on the lower bound applied and/or the level of interest rates term structure. Generally, the proposed thresholds seem to be too strict and therefore should be considered with caution. We support the need of further analyses to guarantee a greater robustness of the methodology used to calibrate the thresholds, also account for a wider sample of banks to capture different business models, as well as longer time series to consider banks behavior under different interest rate environments. The correlation between the two different the EVE and NII approaches, and the results they produce, also represent an important driver to consider. Finally, caution in calibration should be suggested also by the current context of restrictive monetary policy carried out by the European Central Bank, and the expected introduction of the new accounting framework for macro fair value hedge, which will pose further constraints in managing IRRBB.

The rest of the paper is organized as follows. Section 2 provides an overview of the new SOT declined by EBA (2021a), focusing on the two different metrics proposed for the NII approach. Section 3 presents the results obtained by applying the methodological framework described in the previous paragraph 2 to a sample of Italian commercial banks, by also making a comparison with the thresholds proposed by EBA (2021a). Finally, in Section 4 we present our main conclusions and the related policy implications.

2. The new supervisory Outlier Tests (SOTs)

2.1. An overview

The consultation on IRRBB supervisory outlier tests (EBA/CP/2021/36) has fulfilled the mandate required in Article 98 (5a) of Directive (EU) 2019/878 of 20 May 2019 to define regulatory technical standards for: i) the regulatory shock scenarios; ii) the modeling and parametric assumptions underlying the SOT in the context of both EVE and NII approach; and iii) the definition and calibration of the concept of "large decline" associated with NII. It is important to highlight that the article 98 of Directive 2013/36/EU sets at point a) the threshold at 15% of Tier 1 within the EVE approach. The same article in the following point b) refers only to the concept of "large decline" in the NII approach without any further specifications. Therefore, there is no threshold set for the NI approach, and its definition is mandated to EBA.

If a bank exceeds the aforementioned thresholds, the competent authorities may use the powers referred to in Article 104 of Directive 2013/36/EU which include the request for a capital increase and the limitation of business activities deemed excessively risky. In addition, as indicated in paragraph 5 of article 98 of Directive 2013/36/EU, the competent authorities themselves could specify further modeling and parametric assumptions, in addition to those in force. However, the same paragraph 5 of Article 98 establishes that the competent authorities are not required to exercise the above-mentioned supervisory powers if they believe, based on the assessments made, that the management of the risk by the same bank is adequate and that the bank itself is not excessively exposed. As regards the EVE approach, the consultation document EBA/CP/2021/36 refers to the indications suggested in BCBS (2016) and partly reported in the guidelines issued in 2018. Particularly, the consultation document aims to confirm:

i) the modeling and parametric assumptions reported from point a) to j) and at point m) of Article 4 of the draft RTS. In this context, it is important to highlight the possibility, reported at point i), to exclude commercial margins and other spread

[2] See Appendix A for a short description of these papers.

components from the calculation of the economic value and the adoption of the assumption, reported at point j), of a "run-off balance sheet", according to which maturing existing balance sheet items are not replaced. It is the so-called static balance sheet assumption;

ii) the six scenarios of changes in interest rates, already included in EBA (2018), represented by a) parallel shock up; b) parallel shock down; c) steepener shock; d) flattener shock; e) short rates shock up; and f) short rates shock down, which remain, in fact, unchanged in terms of both size and sign associated to the different time-bands of the regulatory maturity ladder.

Compared to the current regulatory framework, it is worth noting the revision of the floor to post shock interest rates defined by point k), which now starts from a level of -150 basis points and then increases by 3 basis points per year up to the 0 level for maturities of 50 years and more. It has been also updated the criterion defined in point i) used to aggregate currencies to calculate banks' risk exposure. Particularly, positive changes can be weighted by a factor of 80%, previously set at 50%, only in the case of an exchange rate mechanism (ERM) with a formally agreed fluctuation band narrower than the standard band of ± 15 basis points to offset losses in euro currency. In the other cases it is confirmed a weighting factor equal to 50%.

Despite EBA (2021a) retains appropriate the recalibration of the lower bound on the basis of a AAA bond yield analysis reported in the accompanying documents of the consultation, AIFIRM (2022) believes that it is too low and that the current bound should be maintained. Particularly, under the new proposal, the bound would be set to a level far below any level that interest rates have reached in the past. Besides, the EBA Guidelines currently in force already envisage the possibility to adapt the lower bound in case of interest rates curves were lower than the bound. AIFIRM (2022) also pointed out that the impacts on IRRBB metrics of the proposed new lower bound has never been properly tested with previous QIS or IRRBB dedicated supervisory exercises. As a matter of fact, the QIS data used to support the calibration of SOT assumptions only include metrics calculated according to the current bound and the unconstrained negative scenarios. Moreover, the EBA itself admits that only a limited number of banks participating in the QIS exercise filled in the IRRBB sections and the data provided had some quality issues.

In addition, a specific provision was introduced in point n), referring to interest rate sensitive products that are linked to inflation or other market factors, which requires the application of prudent assumptions in assessing the related risks. Particularly, these assumptions must be based on the current/last observed value, on the forecast of a renowned economic research institute or on other largely accepted market practices and must be generally dependent on the scenario. Finally, it is no longer reported the provision referred to the previous point o), relating to the treatment of retail and non-financial wholesale deposits without any specific repricing dates (non-maturity deposits), which should be constrained to a maximum average of 5 years. This issue is now treated within the new updated version of guidelines (EBA/CP/2021/37) at point 111, which also proposes applying the five-year cap to the full amount of non-maturity deposits.

With regard to the NII approach, the SOT proposed in the consultation document EBA/CP/2021/36/ is based on:

i) the application of only two parallel scenarios with respect to the six scenarios considered within the EVE approach;

ii) interest income and expenses over a 1-year time horizon, regardless of the maturity and the accounting treatment of the relevant interest rate sensitive non-trading instruments;

iii) the same assumptions applied in the EVE approach except for those referred to in points (i) and (j), respectively concerning the hypothesis of a run-off balance sheet and the treatment of commercial margins and other components. Under the NII approach, it is required to apply the assumption of constant balance sheet and to always include commercial margins in the NII projections.

Therefore, within NII approach it is required the application of the EBA floor previously described, as well as the new method of aggregation between the currencies introduced for the EVE approach. Compared to this latter, it is important to highlight the use of the "constant balance sheet" hypothesis according to which maturing positions are replaced with comparable characteristics regarding the volume, maturity and features (e.g., caps/floors). Furthermore, commercial margins of the new business should be based on those related to recently bought or sold products with similar characteristics. In the case of instruments with observable market prices it is required to use recent and not historical market spreads. As pointed by the EBA itself, the reason underlying this choice is that historical/original commercial margins might no longer be representative under current market conditions.

In this context EBA (2021a) also underlines that using a one-year horizon and a constant balance sheet assumption in the NII SOT seems a better balance between accuracy, reliability of estimates and operational feasibility for its management, instead of three years and a dynamic balance sheet, for example. In addition, EBA (2021a) also introduces a wider definition of NII, which includes not only the impact on interest rate income and expenses, as reported in the previous point ii) (the so-called narrow NII), but also the effects related to changes in market value of non-trading financial instruments with a maturity of more than one year accounted at fair value, either shown in the profit and loss account or directly in equity via other comprehensive income. The inclusion of such items in the NII will be decided by the EBA itself as part of the consultation. Finally, two different measurement metrics with related thresholds are proposed, described in detail in the next paragraphs, to identify the so-called "large decline" for the purpose of the SOT for NII.

AIFIRM (2022) agrees on the proposal of a one-year time horizon and the constant balance sheet assumption since, as pointed out by EBA (2021a) itself, it is consistent with banks' internal practices. AIFIRM (2022) recognizes that the proposal to consider current commercial margins aims at obtaining more precise NII projections, although observing that it implies the use of a simulation approach, thus potentially requiring the evolution of current models and operational processes. However, it should be noted that for managerial purposes different assumptions might be considered for commercial margins when banks use NII projections for planning process. Therefore, the NII projections carried out for risk measurement might still not be aligned with the internal ones.

Finally, regarding the choice to include changes in market value in the calculation of NII SOT, AIFIRM (2022) highlights that accounting approaches are not consistent across the European Union. Therefore, the above-mentioned inclusion may reduce comparability, which is one of supervisors' objectives. Besides, a wider NII metric would differ both from the current managerial NII metrics and from the accounting measures of NII. Hence, AIFIRM (2022) states a preference to exclude market value changes from the NII SOT calculation. Nonetheless, in case the regulatory choice will include this component, AIFIRM (2022) suggests that more detailed instructions for the calculation should be provided in order to avoid unintended double counting of the impact of NII and EVE of fair value instruments over the first year.

It should be clarified that the changes in fair value should be calculated at the end of the NII time horizon. Finally, AIFIRM (2022) noted that the issue of double counting arises also in the disposal set in the RTS for the standardized methodology, where the computation of market value changes is explicitly calculated at the beginning of the time horizon.

2.2. The two different metrics in the SOT on NII

The NII SOT aims to identify the decline of an institution's income which would jeopardize its normal business operations, due to its non-trading book's IRRBB exposure. It is the so-called "large decline". To do that two steps need to be followed: firstly, to determine the metric for measuring a decline of the NII; secondly, to calibrate the threshold associated with the chosen metric. A breach of the threshold would indicate the presence of a "large decline" in the NII, following the application of the supervisory shock scenarios. In the consultation paper EBA/CP/2021/36 two different metrics are proposed, described below in this sub-paragraph. The next sub-paragraph 3.3 deals with the calibration of the thresholds.

The first metric proposed by EBA (2021a) is based on the ratio between change in NII following the application of the parallel scenarios and the Tier 1 capital. In symbols:

$$\frac{NII_{shock} - NII_{baseline}}{Tier\ 1\ Capital} < [threshold] \quad (1.)$$

Where NII_{shock} is the (minimum) level of forecasted NII following the application of the parallel scenarios and $NII_{baseline}$ is the level of forecasted NII in the baseline scenario.

This is a capital related metric consistent with the idea behind the EVE SOT that relates the losses in EVE to the Tier 1 Capital. According to EBA itself it seems to be the most manageable option from an operational perspective. Even if it measures the losses of NII, it does not consider other non-NII related elements in the assessment of the sustainability of the business operations. In other words, it makes more straightforward to calculate the indicator, but it does not allow to assess whether the post-shock NII can sustain normal business operations.

The second metric proposed by EBA (2021a) is the ratio between the value of the NII following the application of the parallel scenarios and the NII obtained via the baseline scenario. Both NII measures are adjusted to consider the administrative expenses component multiplied by a coefficient α, given by the ratio of the bank's interest margin to the total banking operating income. In symbols:

$$\frac{NII_{shock} - \alpha \cdot administrative\ expenses}{NII_{baseline} - \alpha \cdot administrative\ expenses} - 1 < [threshold] \quad (2.)$$

where administrative expenses shall be taken from FINREP, as the amount of administrative expenses reported in column 0010 of row 0360 of the template F02.00 on the "Statement of profit or loss" and referred to the latest end-year value. The parameter α is calculated as follows:

$$\alpha = \frac{NII_{hist}}{Operating\ income} \ (3.)$$

where NII_{hist} is the latest year-end historical NII from FINREP, calculated for this purpose as the difference between the amount of "Interest Income" and "Interest expenses" as reported in column 0010 of rows 0010 and 0090 respectively, of the template F02.00 in the "Statement of profit or loss" table. In case EBA should decide in favor of the broader definition of NII, the values related to column 0010 of rows 0287 and 0290 of the same template should be added. Operating income shall be taken from FINREP as the amount of "Total operating income" reported in column 0010 of row 0355 of the template F02.00 on the "Statement of profit or loss" and referring to the latest end-year value. This is a cost related metric that takes also into account the part of the administrative expenses that can be attributable to the interest margin. The parameter α represents the share of NII in the operating income of the institution and is used as a criterion to measure the part of the administrative expenses attributable to the NII. According to EBA (2021a), this approach has the main advantage to consider both the business model and the cost-structure of a bank in assessing the continuity of the business operations. However, it builds on a strong assumption for the definition of the administrative expenses to be considered. Furthermore, it should be noted that the parameter α is time specific and should be updated yearly, although it should not be expected to vary significantly from year to year.

AIFIRM (2022) states a preference for the first metric (metric 1), since it believes it is simpler to manage by the risk-taking center, easier to communicate to top management and more aligned to EVE metric, making the two measures able to provide better integrated information. In this regard AIFIRM (2021) remarks that during the RAF process, the limits setting should ensure coherence between EVE and NII limits, so that managing IRRBB may impact both measures without any other noisy factor. Having the thresholds set with the same metric would help this process. On the opposite, the second metric (metric 2) could be strongly influenced by extraordinary events (for example, merger and acquisition, pandemic events, etc…) and multi-annual budget planning which are, in most of the cases, not related to the specific risk factors. For these reasons, AIFIRM (2021) believes that this formulation would add an undesired volatility to the indicator and moreover it would reduce the comparability across banks.

2.3. The calibration of the thresholds

The calibration of the thresholds for the definition of the so-called "large decline" in the SOT on NII was carried out based on the EBA QIS as of December 2020, where dedicated EU-specific IRRBB worksheets have been included in the Basel III monitoring exercise. It is important to highlight, as pointed out by EBA itself, that: i) less than half of participating banks provided data on IRRBB; ii) the exercise did not consider the lower bound EBA under two different time horizons respectively of 1 and 3 years; and iii) the QIS data do not show strongly different results if fair value changes are included or not. Therefore, based on these data, the final calibration was not much influenced by the different NII definitions adopted. The calibration of the thresholds was carried out based on the principle, stated in the consolidated version of BCBS standards at point 31.83, that supervisors may also implement additional outlier or materiality tests, provided these tests are applied through their jurisdiction in the same form. These additional tests could use a different capital measure or capture the bank's IRRBB relative to earnings. However, the relative threshold for defining an outlier bank should be at least as stringent as 15% of Tier 1 capital. Therefore, the SOT on NII is expected to be at least as severe as the SOT on EVE.

Based on this principle, EBA decided to set the threshold to a level so that the number of banks becoming outliers according to the SOT on EVE is at least equal to the number of outlier banks according to the SOT on NII. Consequently, based on the QIS data provided by a sample of 53 banks, the percentile of the distribution of the EVE changes associated with a risk indicator equal to or above the regulatory threshold of 15% was calculated.

To do that, EBA considered for each bank the most penalizing scenario, i.e., the scenario of changes in interest rates among the six required by the supervisory prudential regulatory framework, which leads to the highest reduction in the economic value. The percentile obtained through the application of this criterion was equal to 0.086. Therefore, this percentile has been applied to the four distributions of changes in NII, considering the most penalizing scenario between the two regulatory parallel scenarios and taking into account both the two different metrics and the two definitions of NII previously described. The four thresholds calculated and proposed by EBA (2021a) are reported in the following Table 1 where the number of banks that provided data within the QIS exercise for each specific metric combination are in the brackets.

Table 1. Thresholds in the SOT on NII (1 year time horizon)

Metric	Definition of NII	
	Narrow	Wider
1	-2,5% (46)	-3,0% (37)
2	-35,0% (38)	-30,0% (33)

Note: Metric 1 is based on the ratio between change in NII following the application of the parallel scenarios and the Tier 1 capital as reported in the equation (1.). Metric 2 is the ratio between the value of the NII following the application of the parallel scenarios and the NII obtained via the baseline scenario as reported in the equation (2.), including the adjustment for the administrative expenses component as defined in equation (2.) and (3.). Narrow NII is determined as interest income minus interest expenses. Wider NII is determined as interest income minus interest expenses including fair value changes. In brackets is indicated the number of banks that provided data within QIS exercise.
The final draft RTS on IRRBB supervisory outlier tests (SOT) pubblished on 20 october 2022 confirms Metric 1, the "narrow NII" definition and the associated threshold proposed during the consultation equals to -2,5%. The metric 2 has been discarded.

In the case of metric 1 and the narrow definition of NII, the threshold corresponding to the 0.086 percentile of the distribution of changes in NII calculated on a sample of 46 banks is equal to -2.5%. This threshold is equal to -3,0% if the delta NII includes market values changes in addition to interest income and expenses, according to the 37 banks that provided data used for this specific analysis. Similar considerations can be made for the metric 2.

In conclusion, following the analysis implemented on the basis of QIS data, EBA shall still make two decisions for the definition of the SOT on NII, in terms of the metric and of the definition of NII, in order to finally specify the concept of a large decline in NII and, consequently, the threshold that identifies outlier banks.

3. The case of Italian banks

In this paragraph, we provide a benchmark analysis carried out on a sample of 28 Italian commercial banks with data referred to the end of 2021, to support discussion about the NII thresholds calibration, taking into account both the floor currently in force and the new one proposed by EBA (2021a). Our sample represents more than 70% of Italian banks' total assets. NII metrics are computed according to the narrow definition as the difference between interest income and expenses, thus not considering fair value changes. Our results are shown in the following Table 2, where the level of risk indicators for both the EVE and NII approach is reported, starting from the bank with the highest risk indicator and gradually all the others, down to the one with the lowest indicator. The level of the risk indicators has been obtained for each bank through the application of the criterion of the most penalizing scenario both in the EVE and NII approach. A negative sign in the indicator represents a decrease in economic value.

Table 2. Benchmark analysis on SOT metrics.

	NII Metric 1 (A)	NII Metric 2 (B)	NII Metric 1 new lower bound (C)	NII Metric 2 new lower bound (D)	EVE (E)	EVE new lower bound (F)
1	-17,9%	-154,2%	-9,9%	-304,1%	-23,1%	-30,1%
2	-8,5%	-132,4%	-8,2%	-91,7%	-19,7%	-23,1%
3	-5,5%	-107,4%	-8,1%	-80,8%	-16,5%	-19,8%
4	**-4,2%**	**-37,0%**	-6,3%	-67,7%	**-16,5%**	-19,5%
5	-4,1%	-36,2%	-4,8%	-67,0%	-14,9%	-17,8%
6	-2,9%	-33,8%	-4,7%	-59,1%	-14,0%	-17,0%
7	-2,4%	-33,7%	-4,5%	-55,3%	-13,5%	-16,9%
8	-2,3%	-31,6%	-4,4%	-54,2%	-12,1%	-16,2%
9	-2,3%	-27,2%	**-3,5%**	**-45,6%**	-11,9%	**-15,5%**
10	-2,3%	-27,0%	-3,2%	-41,8%	-11,5%	-15,0%
11	-1,8%	-20,8%	-3,0%	-39,0%	-11,1%	-13,5%
12	-1,8%	-18,1%	-2,9%	-33,2%	-10,9%	-12,6%
13	-1,6%	-17,9%	-2,9%	-33,1%	-10,2%	-11,7%
14	-1,6%	-17,1%	-2,6%	-25,7%	-9,5%	-9,1%
15	-1,4%	-16,2%	-2,5%	-24,9%	-8,8%	-9,0%
16	-1,4%	-15,6%	-2,3%	-24,1%	-8,5%	-8,7%
17	-1,3%	-12,8%	-1,7%	-24,0%	-7,6%	-7,9%
18	-1,3%	-12,5%	-1,4%	-16,4%	-7,5%	-7,5%
19	-1,2%	-12,3%	-1,3%	-10,6%	-7,5%	-7,5%
20	-1,1%	-12,2%	-1,2%	-10,0%	-6,7%	-5,0%
21	-1,0%	-11,4%	-0,9%	-9,7%	-5,6%	-3,8%
22	-0,9%	-9,2%	2,1%	16,3%	-4,5%	-3,5%
23	-0,9%	-8,8%	n.a.	n.a.	-4,5%	3,9%
24	-0,6%	-7,4%	n.a.	n.a.	-3,8%	n.a.
25	-0,5%	-5,0%	n.a.	n.a.	-3,3%	n.a.
26	-0,5%	-4,5%	n.a.	n.a.	-2,8%	n.a.
27	-0,2%	-2,3%	n.a.	n.a.	-1,9%	n.a.
28	0,6%	4,6%	n.a.	n.a.	-1,4%	n.a.
N.umber of banks	28	28	22	22	28	23
Number of outlier	6	5	15	11	4	9
Outlier %	21,4%	17,8%	68,2%	50,0%	14,3%	39,1%
Average risk indicator	-2,5%	-29,3%	-3,6%	-50,1%	-9,6%	-12,5%
Median risk indicator	-1,5%	-16,6%	-3,0%	-36,1%	-9,1%	-12,6%

Legend and explanatory notes

(A)	NII Metric 1	Is the change in Net Interest Income (NII) over one year, corresponding to the worst impact under the two parallel regulatory shocks. The NII is calculated as the difference between interest income and expenses, excluding market value changes of positions accounted for at fair value. It corresponds to Metric 1 referred to in Article 6 of the EBA/CP/2021/36: (NII.shock − NII.baseline)/Tier1 For the negative shock scenario, the current lower bound envisaged in par.115(k) of EBA/GL/2018/03 is applied
(B)	NII Metric 2	Is the change in Net Interest Income (NII) over one year, corresponding to the worst impact under the two parallel regulatory shocks. The NII is calculated as the difference between interest income and expenses, excluding market value changes of positions accounted for at fair value. It corresponds to Metric 2 referred to in Article 6 of the EBA/CP/2021/36: (NII.shock − alfa*Admin.Expenses) / (NII.baseline − alfa*Admin.Expenses) − 1. For the negative shock scenario, the current lower bound envisaged in par.115(k) of EBA/GL/2018/03 is applied
(C)	NII Metric 1 new lower bound	Is the change in Net Interest Income (NII) over one year, corresponding to the worst impact under the two parallel regulatory shocks. The NII is calculated as the difference between interest income and expenses, excluding market value changes of positions accounted for at fair value. It corresponds to Metric 1 referred to in Article 6 of the EBA/CP/2021/36: [NII(shock) - NII(baseline)]/Tier1. For the negative shock scenario, the new proposed lower bound envisaged in Article 4(k) of EBA/CP/2021/36 is applied
(D)	NII Metric 2 new lower bound	Is the change in Net Interest Income (NII) over one year, corresponding to the worst impact under the two parallel regulatory shocks. The NII is calculated as the difference between interest income and expenses, excluding market value changes of positions accounted for at fair value. It corresponds to Metric 2 referred to in Article 6 of the EBA/CP/2021/36: (NII.shock − alfa*Admin.Expenses) / (NII.baseline − alfa*Admin.Expenses) − 1. For the negative shock scenario, the new proposed lower bound envisaged in Article 4(k) of EBA/CP/2021/36 is applied
(E)	EVE	Change in the economic value of equity / Tier1 capital, corresponding to the worst impact under the six regulatory shocks. It is the measure referred to in the current regulatory framework, i.e. par. 114 and 115 of EBA/GL/2018/02
(F)	EVE new lower bound	Change in the economic value of equity / Tier1 capital, corresponding to the worst impact under the 6 regulatory shocks. It is the measure referred to in the current regulatory framework, i.e., par. 114 and 115 of EBA/GL/2018/03, except for the application of the lower bound. Here the new proposed lower bound envisaged in Article 4(k) of EBA/CP/2021/36 is applied

The application of the regulatory thresholds defined by EBA (2021a) for the NII SOT leads to 6 and 5 outliers (highlighted in gray) for respectively Metric 1 and Metric 2. If we applied to our sample banks the calibration methodology proposed by EBA (2021a), based on the number of outliers observed for the EVE SOT under the current lower bound, which is equal to 4 (see Column 6 of Table 2), the NII threshold should be set at -4,2% (shown in bold in column 2) in case of Metric 1, instead of the proposed -2,5%, and at -37,0% (shown in bold in column 3) for Metric 2, instead of the proposed -35,0%. The two above mentioned thresholds have been calculated with reference to the number of outlier banks in the EVE approach (equals to 4) corresponding to the 0,1429 percentile of the distribution, which is higher than that defined by EBA (2021a), equal to 0,086. Referring to Metric 1, the 2 banks (number 5 and 6) that present a risk indicator equal respectively to -4,1% and -2,9% would be considered outliers according to the regulatory threshold, but not outliers according to the threshold calculated on the basis of our sample. As for Metric 2, only bank number 5 would have a different treatment depending on the threshold used. These results show that the calibration is significantly dependent on the sample considered.

The switch to the new proposed lower bound of post shock interest rates leads to an increase of the share of outlier banks over the total number of banks goes from 14,3% (4 out of 28) to 39,2% (9 out of 23) in the case of EVE SOT, while for NII SOT the outlier percentage would rise from 21,4% (6 out of 28) to 68,2% (15 out of 22) and from 17,8% (5 out of 28) to 50,0% (11 out of 22) for metric 1 and metric 2 respectively. The switch also determines an increase in the average risk exposure of our sample banks both in EVE (from -9,6% to -12,5%) and in NII (from -2,5% to -3,6% for Metric 1 and from -29,3% to -50,1% for Metric 2).

The increase in the average exposure is attributable to banks exposed to downward scenarios. The application of wider negative changes in interest rates, as a consequence of lowering the floor, leads, *ceteris paribus*, to a greater reduction in economic value and in interest margins and therefore to a higher level (in absolute value) of the corresponding risk indicators. The impacts of the introduction of the new proposed lower bound appear to be extremely material both for EVE and NII measures under negative shock scenarios, with significant implications for IRRBB management and strategy.

The application of the new proposed floor to our sample banks would lead to a reduction of the thresholds for NII metrics under metric 1, which should be set at -3,5% (shown in bold in column 4). This value is lower than the one obtained in our sample under the floor currently in force, equals to -4,2%, but would still be higher than the regulatory one. In the case of metric 2 we would find a new threshold of -45,6% (shown in bold in column 5), which is higher than both the value obtained in our sample with the floor currently in force (equals to -37,0%) and the regulatory one. The two thresholds are calculated considering a number of outlier banks in the EVE approach equal to 9 (shown in bold in column 7). Thus, these results show that the setting of threshold also depends on the floor applied.

We also find that this approach to thresholds calibration leads to a counterintuitive result, as the threshold on NII SOT calculated for metric 1 decrease from -4,2% to -3,5% despite a greater risk exposure by banks. This suggests the need for further analysis to guarantee a greater robustness of the methodology to be used for the calibration of the thresholds in the NII approach. We note that this counterintuitive result is not observed for metric 2, where we observe an increase in the threshold on NII SOT from -37,0% to -45,6% following the higher risk exposure driven by the application of the new proposed lower bound.

In this context, it is important to underline that the effects deriving from the introduction of the proposed lower bound could also be determined by an increase in the term structure of interest rates, that would lead to an increase in the size of the shock applied, even under the current lower bound. Therefore, banks exposed to a decrease in interest rates could be characterized by an increase in their IRRBB exposure in a rising interest rates environment such as the current one.

From a methodological perspective, several factors could influence the calibration of the thresholds, in addition to the ones already mentioned (the choice of the reference sample, the level of the lower bound and the level of the term structure of interest rates), for instance: the correlation between the EVE and NII approach, the different number of scenarios considered under the two approaches as well as the different time horizon considered. These factors interact with each other. For example, the change in the lower bound impacts differently on the two measures (EVE and NII) depending on the related time horizon and slope of the term structure of interest rates.

It is important to highlight that the greater average risk exposure and the higher number of outlier banks registered in both the EVE and NII approaches following the transition to the new proposed floor could depend also on the asset and liability management strategies implemented by banks in the low interest rates environment that characterized the recent past. Therefore, the results obtained should be assessed with caution and, at the same time, suggest using longer series of historical observations to calibrate thresholds, so to capture banks' behaviors in different interest rates environments, including those that did not require any floor. This should be in line with the prudential regulatory framework in force. For example, the calibration of the new six interest rates scenarios by BCBS (2016) was based on interest rates time series ranging from 2000 to 2015. Or, referring to the treatment of non-maturity deposits, EBA (2021a) requires a ten-year observation period for modelling the stable / non-stable part of deposits. Hence, a wider data set with longer series of historical observations seems to be necessary to apply the proposed calibration methodology based on outliers' distribution.

4. Conclusions and policy implications

This paper contributes to the current debate concerning the revision of the prudential supervisory framework dealing with IRRBB, which has gone through significant changes since April 2016, when BCBS published its latest standards. The recent consultation issued by EBA on December 2021 aiming at introducing the SOT on NII presents several issues, which could influence in the next future banks' asset and liability management strategies, as well as their internal control systems, risk policies and procedures.

By analyzing internal data referred to a sample of 28 Italian commercial banks, observed at the end of 2021 and representing more than 70% of the Italian banking system in terms of total assets, this paper results suggest that the thresholds proposed by EBA (2021a) appear to be very strict and also depending significantly on the sample considered, as well as on the lower bound applied. Therefore, the proposed values should be considered with caution as their potential impacts deserve to be thoroughly assessed. Particularly, the switch to the new floor would lead to a significantly high number of outlier banks, as well as to an increase of the banks average detected risk exposure, both under the EVE and the NII approach. In fact, the application of wider negative changes in downward scenarios following the lowering of the floor would lead, at equal conditions, to a greater reduction in economic value and net interest income, thus increasing the level (in absolute value) of the corresponding risk indicators.

Overall, the above-mentioned regulatory changes might cause limit breaches, triggering managerial actions to adjust the IRRBB position of a bank, causing potential negative impacts on profitability. Similarly, in terms of market disclosure, the publication of higher risk indicators in Pillar III reports could also have possible consequent reputational effects, thus suggesting a thorough assessment of potential implications on the market.

We suggest the need for further analysis in order to guarantee a greater robustness of the methodology used for the calibration of the thresholds, possibly taking into account a wider sample of banks to capture different business models and longer time series aiming at considering banks' behaviors under different interest rate environments. The correlation between the results for EVE and the setting of the threshold for the NII SOT also represents an important driver to be carefully assessed, also considering the different assumptions underneath the two approaches (e.g., shocks scenarios and time horizons).

References

- Abdymomunov, A. & Gerlach, J. (2014). Stress testing interest rate risk exposure. *Journal of Banking & Finance*, *49*, 287-301.
- AIFIRM (2015). *Risposta al documento di consultazione Interest Rate Risk in the banking book del Comitato di Basilea sulla vigilanza bancaria* (edit by Curcio D. & Gianfrancesco I.). Position Paper n.4, September.
- AIFIRM (2021). *Rischio di tasso di interesse del portafoglio bancario (IRRBB): evoluzione normativa ed implicazioni gestionali* (edit by Curcio D., Gianfrancesco I., Meglio C., Pansini A., Preger A. & Trentini S.). Position Paper n. 25, February.
- AIFIRM (2022). *Risposta alle consultazioni EBA sul rischio di tasso di interesse del portafoglio bancario* (edit by Curcio D., Gianfrancesco I., Meglio C., Pansini A. & Preger A.). Position Paper n.34, April.
- Basel Committee on Banking Supervision (2004). Principles for the management and supervision of interest rate risk. *Bank for International Settlements*, July.
- Basel Committee on Banking Supervision (2015). Interest rate risk in the banking book. *Bank for International Settlements*. Consultative document, June.
- Basel Committee on Banking Supervision (2016). Interest rate risk in the banking book. *Bank for International Settlements*, April.
- Cerrone, R., Cocozza, R., Curcio, D. & Gianfrancesco, I. (2017). Does prudential regulation contribute to effective measurement and management of interest rate risk? Evidence from Italian banks. *Journal of Financial Stability*, *30*, 126-138.
- Chaudron, R. F. (2018). Bank's interest rate risk and profitability in a prolonged environment of low interest rates. *Journal of Banking & Finance*, *89*, 94-104.
- Cocozza, R., Curcio, D. & Gianfrancesco, I. (2015). Non-maturity deposits and banks' exposure to interest rate risk: Issues arising from the Basel regulatory framework. *Journal of Risk*, *17*(5).
- Cocozza R., Curcio D., Gianfrancesco I. & Onorato G. (2022). *Interest rate risk in the banking book and internal capital: what implications from the new supervisory regulatory framework?*. Paper presented at the International Finance and Banking Society (IFABS) Conference held at the University of Naples Federico II from 7 to 9 September 2022.
- Curcio D. & Gianfrancesco I. (2011). La misurazione del rischio di tasso di interesse del portafoglio bancario in Basilea 2: quali possibili criticità nella ricerca di nuove best practices?. *Newsletter AIFIRM*, Anno 6, Numero 1.
- Curcio D. & Gianfrancesco I. (2012). Il rischio di tasso di interesse del banking book: profili applicativi. *Banche e Banchieri*, n.2.
- Curcio D., Gianfrancesco I., Modina M. & Onorato G. (2022). La disciplina del rischio di tasso di interesse del portafoglio bancario: evoluzione e impatti sulle prassi. *Bancaria,* n. 5.
- Entrop, O., Memmel, C., Wilkens, M. & Zeisler, A. (2008). Analyzing the interest rate risk of banks using time series of accounting-based data: Evidence from Germany. *Deutsche Bundesbank Discussion Series 2: Banking and Financial Studies*, No.1/2008.
- Entrop, O., Wilkens, M. & Zeisler, A. (2009). Quantifying the interest rate risk of banks: assumptions do matter. *European financial management*, *15*(5), 1001-1018.
- Esposito, L., Nobili, A. & Ropele, T. (2015). The management of interest rate risk during the crisis: evidence from Italian banks. *Journal of Banking & Finance*, *59*, 486-504.
- European Banking Authority (2018). Guidelines on the management of interest rate risk arising from non-trading book activities. Final Report, July.
- European Banking Authority (2021a). *Draft Regulatory Technical Standards specifying supervisory shock scenarios, common modelling, and parametric assumptions and what constitutes a large decline for the calculation of economic value of equity and of the net interest income in accordance with Article 98(5a) of Directive 2013/36/UE*, December.
- European Banking Authority (2021b). *Draft Guidelines issued on the basis of Article 84(6) of Directive 2013/36/EU specifying aspects of the identification, evaluation, management and mitigation of the risks arising from potential changes in interest rates and of the assessment and monitoring of credit spread risk, of institutions' non trading-book activities*, December.
- European Banking Authority (2021c). *Draft Regulatory Technical Standards specifying standardized and simplified standardized methodologies to evaluate the risks arising from potential changes in interest rates that affect both the economic value of equity and the net interest income of an institution's non-trading book activities in accordance with 84(5) of Directive 2013/36/EU*, December.
- Fiori, R. & Iannotti, S. (2007). Scenario-based principal component value-at-risk when the underlying risk factors are skewed and heavy-tailed: an application to Italian banks' interest rate risk exposure. *The Journal of Risk*, *9*(3), 63.
- Gianfrancesco I. (2016). L'esposizione al rischio di tasso di interesse del portafoglio bancario: quali implicazioni per le strategie di Asset & Liability Management? *Newsletter AIFIRM*, Anno 11, Numero 3-4.
- Gianfrancesco I. (2017). L'applicazione dei nuovi scenari di variazione dei tassi di interesse proposti dal Comitato di Basilea: quali implicazioni per le banche italiane? *Risk Management Magazine,* Anno 12, Numero 3.
- Gianfrancesco I. (2018). La misurazione dell'esposizione al rischio di tasso di interesse del portafoglio bancario: quali implicazioni in sede ICAAP a seguito della recente introduzione dell'approccio del margine di interesse nel quadro normativo di vigilanza prudenziale? *Risk Management Magazine*, Anno 13, Numero 3.
- Hoffmann, P., Langfield, S., Pierobon, F. & Vuillemey, G. (2019). Who bears interest rate risk?. *The Review of Financial Studies*, 32(8), 2921-2954.
- Partesotti A., & Preger A. (2015). Risposta di Prometeia alla consultazione "Interest Rate Risk in the Banking Book" del Basel Committee on Banking Supervision. *Newsletter AIFIRM*, Anno 10, Numero 4.

Appendix A: review of literature

Table 3: Papers published in international journals

Authors	Description
Fiori and Iannotti (2007)	The authors develop a *Value at Risk (VaR)* methodology based on a principal component Monte Carlo simulation. By analyzing a sample of the 18 major Italian banks, they show that their results are consistent with the findings obtained through the parallel scenario of +/-200 bp if the regulatory duration coefficients are calibrated on the basis of current market data at the evaluation date.
Entrop et al. (2008)	The authors develop the so-called times-series accounting based model (TAM) to estimate the distribution of bank's assets and liabilities within each time band of regulatory maturity ladder. Referring to a sample of German banks, the authors show that TAM is able to explain the cross-sectional variation in bank's interest rate risk better that the regulatory methodology proposed by BCBS (2014) and provides results that are more line with those obtained by banks' internal models.
Entrop et al. (2009)	The authors analyze how bank's risk exposure changes if some of main assumption underlying the regulatory model are modified. By considering the aggregated German universal banking systems, the authors find that bank's risk exposure depends significantly on the assumption underlying the regulatory framework. Therefore, they warn that results coming from the regulatory framework should be treated with caution if used for supervisory and risk management purpose.
Abymomuvov and Gerlach (2014)	The authors propose a new methodology for generating yield-curve scenarios for stress testing bank's exposure to interest rate risk based on Nielson-Siegel (1997) yield curve model. By considering an aggregated bank's balance sheet based on Call Report data from a sample of large United States banks, they show that their methodology produces scenarios with a wider variety of slopes and shapes than others generated by internal methods commonly used in industry and proposed in literature including the regulatory methodologies of +/-200bp parallel shift.
Cocozza et al. (2015)	The authors develop a behavioral model to allot non-maturity deposits in the time bands of maturity ladder. By considering a sample of 30 Italian commercial banks the authors show that different criteria to allocate non-maturity deposits could impact not only on the size of the risk indicator but also on the nature of risk exposure. The authors also discover the presence of risk-neutral banks in a low interest rates environment, i.e., banks that appear to experience an increase in their equity economic value whether interest rates decrease or increase under the parallel shifts method.
Cerrone et al. (2017)	The authors show how banks might adapt internal measurement systems based on simulation technique to face the risk-neutrality phenomenon detected by Cocozza et al. (2015). They also develop a back-testing procedure, modifying the original framework proposed by Lopez (1996) in the IRRBB perspective, to test the consistency of regulatory and simulation methodologies results with the actual bank risk exposure. By considering a representative sample of 130 Italian banks between 2006 and 2013 the authors show that simulation techniques perform better those regulatory methodologies.
Esposito et al. (2015)	The authors measure interest rate risk using the duration gap approach proposed by the BCBS (2004). Based on a representative sample of 68 Italian intermediaries observed from the second half of 2008 to the first half of 2012, they show that Italian banking system has a limited exposure to interest rate risk. Italian banks have managed this risk by using changes in their balance-sheet exposure and interest rate derivatives as substitute. They also show the relationship with other risks such as credit and liquidity risk.

Chaudron (2018)	The author investigate how bank risk position changes over time by analyzing the interest-rate risk position of 42 Dutch banks during the period 2008 – mid-2015. The empirical evidence obtained show that interest-rate risk positions are negatively related to on-balance sheet leverage, exhibit a U-shaped relation with solvability, and do not vary systematically with the size of the banks. Finally, banks that received government help during the crisis took on greater interest rate risk.
Hoffmann et al. (2019)	The authors study the allocation of interest rate risk for a sample of 104 banks from 18 euro area countries. Their results show that banks' exposure is limited on aggregate but there is a considerable heterogeneity across individual institutions. In contrast to conventional wisdom roughly half of the banks benefit from an increase in interest rates in terms of both EVE and NII. Finally, hedging via interest rate swaps eliminate only 25% of on-balance sheet exposure arising from deposit taking and lending activities.

Modello LGSR forward looking

David Cavallini (Risk Management, BU Governance – CABEL Industry S.p.a.) and Francesco Letizia (Internship at Cabel Industry S.p.a.)

Article submitted to double-blind peer review, received on 22nd September 2022 and accepted on 13th November 2022

Abstract

In this work, we propose a hierarchical model to introduce *Forward-Looking* effects on the Loss Given Default Rate (LGDR) estimate, as required by IFRS9. The Framework consists of two modules: a SURTS satellite model (*Seemingly Unrelated Regressions Model Time Series*), which analyses the dynamics of the systemic LGSR (*bad loans LGDR*) and a set of selected macroeconomic factors, and a *Beta Inflated-(0,1)* model which estimates the LGSR for the single entity. The basic hypotheses for the construction of the hierarchical model will also be illustrated, underlining how this approach is particularly relevant for LSIs (*Less Significant Institutions*). The theoretical aspects are followed by an application on a series released by the Bank of Italy, presenting the LGDR estimation process on an archive of closed bad loans by a set of banks belonging to the CABEL (ICT Service Provide) network. By way of example, we illustrate the forecast results for the three-year period 2022-2024 for the systemic LGDR. Other aspects related to the construction of LGDR models are addressed, such as the segmentation of the portfolios and the selection of individual attributes. In particular, we introduce the NPL *vintage* as an explanatory variable in the LGDR model, outlining the interconnections with the effects of macroeconomic projections.

1. Introduction

The financial crisis of 2008 raised the need to promptly account the quality deterioration of loans portfolio, to align share funds with credit risk. The *International Financial Reporting Standard (IFRS9)* required operators to switch to a lifetime perspective in the accounting of the expected losses for those assets characterized by a significant increase in credit risk (SICR). Furthermore, to reduce the *shortfall* of the provisions, was introduced the necessity to assess the probability distribution of losses from a *forward-looking* perspective.

From the first adoption of the new accounting standard, more or less complex methodologies have been proposed, which underly on different hypotheses. This phenomenon has highlighted an already known aspect in the *risk management* area, linked to model risk and to comparison of results. However, a prevalent school has been consolidated, widespread both in institutional environments and among consulting private company (Deepak Parmani & Analytics.), which envisages the use of macroeconomic factors as a fundamental element to consider the *forward-looking* component in risk parameters.

Among the various attempts, it is worth mentioning (Joubert, et al., 2021), which proposes to include macroeconomic factors as explanatory variables in the LGDR model. This approach, which we call *direct*, seems reasonably applicable to systemic banks as major contributors to national data, but we believe it is not as suitable for *Less Significant Institutions* (LSIs). After having defined LGDR in Section 2, in Section 3.1, we set out in detail the reasons for our proposal, which is embodied in the formulation of a hierarchical model, using basic concepts of conditional independence.

Following the economic-financial laws related to credit recovery a *satellite* model is implemented, which establishes a relationship between the systemic *Recovery Rate* (RR) and macroeconomic variables. In Subsection 3.1, we illustrate the methodological aspects of the *Seemingly Unrelated Regressions Model Time Series* (Harvey, 1989 and Hamilton, 1994).

Empirically, the distribution of LGDR of a banking portfolio is bimodal with points of discontinuity at the extremes {0,1}, with consistent forms of asymmetry. In this context, *a Beta-Inflated* distribution has been adopted, considered in the literature as a reference density for the estimation of econometric LGDR models (Grzybowska & Karwański, 2020) and (Joubert, et al., 2021). In Subsection 3.3, we illustrate a methodological synthesis of the *Generalized Additive Models for Location, Scale and Shape (GAMLSS)* proposed by (Rigby & Stasinopoulos, 2005), which represents the foundation of the adopted regression model. In a direct approach, in addition to the individual characteristics of NPL, the explanatory variables also consider a set of macroeconomic factors that are empirically significant to explain the RR. The proposed hierarchical model condenses the trend of the economic cycle into a single factor, the systemic LGDR, which represents the only macroeconomic variable considered in the set of regressors. This approach has the advantage of limiting the effects of multicollinearity but, above all, it brings cognitive benefits to the estimation process by facilitating the interpretation of the results.

Once the macroeconomic scenarios that reflect the prospects of the national economy have been defined, the projections of the LGDR are drawn up. In Subsection 3.4 it is shown that, with respect to the specified model, it is not possible to obtain a closed form of the forecasts in terms of expected value. Even though the Monte Carlo method would allow to obtain an adequate approximation in probabilistic terms. Here, we have adopted the *plug-in* technique which considerably reduces the computational efforts.

In Section 3 we present the results obtained from the application to Italian data, related to NPL closed through internal credit recovery (the ones not sold on the market). The system of equations reflects the segmentation that we believe to be minimal in the context of LGDR models, deriving from the intersection of two attributes: type of counterparty (firms/households) and the presence or absence of collateral (secured/unsecured).

Maintaining the level of confidentiality of the data of the banks belonging to the CABEL[1] network, in Section 4 we illustrate the results of the application to a historical archive of losses. From the series of publications of Bank of Italy, Financial Stability and Supervisory Notes (Fischetto, et al., 2021), we show the quite relevant role played by the duration of the recovery process on RRs. In fact, the Italian average recovery rate for NPL closed in the first year is about twice the one on those closed in more than five years. In other words, an inverse relationship exists between recovery rates and duration. This empirical evidence is reflected by regulation in the *Calendar Provisioning* framework introduced by the European Central Bank's (ECB) "Impaired Credit Guidelines" in March

[1] CABEL (ICT Service Provide, Center for Banking and Leasing Assistance) company dedicated to providing services in the banking industry. (www.cabel.it).

2017[2], which establishes progressive minimum levels of prudential provisioning (*Prudential Backstop*) that are based on the NPL vintage[3]. Given the relevant role of this attribute, the duration of recovery processes was introduced as an explanatory variable in the regression model. The study ends presenting in Section 6 some final remarks and future developments.
As detailed above, the following section outlines the basic concepts of LGDR calculation with a brief mention of the regulatory and normative framework. The section on conclusions is devoted to expressing the benefits of the proposal, emphasizing that the model is not an exhaustive solution to the issue addressed. Methodological aspects are subject to improve.

2. Definition of LGDR

Despite the entry into force of the new definition of *default*[4] that banks must adhere to as of January 1, 2021, it is still relevant to split the life cycle of impaired loans (NPLs) into two main stages. The first stage is the administrative states of *Past-Due* and *Unlikely to Pay* (UTP), while the next step is the transition to non-performing. Although in banking practice legal actions and out-of-court settlements are also undertaken for UTP, the recovery of the loan intended in the strict sense is fundamentally to be ascribed to the bad debt status. This distinction is also inevitably reflected in the methodological aspects related to the estimation of the LGDR parameter, which consists of two basic modules: the estimated LGDR model on NPL (LGSR) and the estimation of the probability of transition from *Past-Due* and *Unlikely to Pay* states to non-performing status, which is considered an absorbing state (*Danger Rate*).
This paper is framed in the context of estimating distressed LGSR models following the *Workout approach*, which is based on the measurement of cash flows from the recovery process, appropriately discounted. In formal terms, LGSR is calculated as the complement with respect the RR i.e., $\text{LGS} = 1 - \text{RR}$. Given a schedule $\text{t} = (t_1, t_2, t_3, \dots, t_n)$, representing the n relevant time events along the duration of recovery process, we define:

$$\text{RR} = \frac{1}{EAD} \sum_{i=1}^{n} \frac{R(t_i) - C(t_i)}{[1 + r(t_i)]^{t_i}} \quad (2.1)$$

where:

$\text{R}(t_i)$ is the nominal value of the gross amount recovered as at t_i;
$\text{C}\,(t_i)$ is the nominal value of the cost (direct/indirect) related to the recovery procedure incurred at instant t_i;
$\text{r}(t_i)$ rate curve for the discounting observed at the time of bad status entry;
EAD is the initial nominal value of the exposure at the time of bad status entry;
t is the the duration of the recovery procedure.

The choice of interest rate maturity curves, which are useful for discounting cash flows, depends on the scope of application. From a regulatory perspective, using historical rates, the discount factor must consider a risk premium (e.g., by estimating a CAPM - *Capital Asset Pricing Model*). The new accounting standards (IFRS9), on the other hand, require the use of the *Effective Interest Rate* (EIR) calculated at the origination of the loan. For further discussion of aspects related to the selection of discount rates for calculating LGDR we refer to (Gibilaro & Mattarocci, 2006).
The basic principle underpinning the regulatory approach is the principle of *prudence*: the Basel Committee expects LGDR to be estimated during a *downturn* in the credit cycle. The drafting of the new IFRS9 accounting standards follows the *point in time* (PIT) philosophy and, therefore, the LGDR risk parameter must be constructed taking into account expected risk factors (LGDR *forward looking*). In addition, to avoid *double accounting*, costs are derecognized in the calculation of RRs in equation 2.1. For a comprehensive analysis of the main differences emerging between the two regulatory and accounting approaches, please refer to AIFIRM Position Paper No. 8/2016.
The priority source for feeding the data structure is the historical data contained in banks' internal archives, which form the information base for building internal models and for reporting regarding losses (LD matrix[5]). The detail in supervisory reporting often does not provide sufficient information for the segmentation (see subsection 3.2). Therefore, anagraphics additions are required.
Alternatively, following Bank of Italy publications (Ciocchetta, et al., 2017), LGDR can be calculated by accessing data from the Centrale Rischi (CR). The information available in that repository is not sufficient for a detailed calculation as expressed in equation 2.1, but it is possible, through reasonable working assumptions, to achieve an adequate approximation.

3. LGSR model in forward-looking perspective

The inclusion of macroeconomic factors within LGSR models can be tackled by following two alternative approaches.
The first, which we call *direct*, involves relating the entity's LGSR to macroeconomic variables by following *model selection* techniques and/or referring to concepts from a well-defined economic theory. In practical application, problems may be encountered both in the construction phase and in the final interpretation of the selected model; in some cases, parameter estimates may assume a sign that is in contrast with the economic laws of credit recovery. In general, there may be a variety of causes for such difficulties, which can be traced to multicollinearity phenomena in macroeconomic variables and/or the use of metrics that tend to select models affected by *overfitting*. Since credit recovery duration often have long to consider a completely economic cycle, the analyst may encounter the additional difficulty of defining the instant of observation of the macroeconomic factor with respect to the date of entry and closure of bad status[6].

[2] Then supplemented in March 2018 with the Addendum to the Impaired Credit Guidelines and in April 2019 with the publication of Regulation (EU) 2019/630 of the European Parliament and of the Council (the Regulation 2019/630).
[3] Length of permanence in NPL (Non-Performing Loan) status.
[4] European issuance of guidelines on the application of the definition of default pursuant to Article 178 of Regulation (EU) No. 575/2013 (EBA/GL/2016/07) and regulatory technical standards on the materiality threshold for credit obligations in arrears and related Delegated Regulation (EU) 171/2018 of the European Commission of October 19, 2017 (EBA/RTS/2016/06).
[5] Circular No. 284 of June 18, 2013 - First Update of Bank of Italy
[6] Financial Duration seems to be the most congenial instant to take as a reference point for the detection of macroeconomic factors.

Despite the technical complexities associated with model construction, the attempt to identify a relationship between macroeconomic factors and micro phenomena inherent individual items (or portfolio) may be a sustainable solution for systemic banks, as they contribute to the national data on a consistent basis. In our view, *Less Significant Institutions* (LSIs) deserves special attention. By their nature, the portfolios of these institutions are characterized by a narrow, or at least uneven, spatial distribution across the country, as well as by significant idiosyncratic differences in the share of loans allocated to various economic sectors of customers and amount classes. The limited contribution to systemic data of small and medium-sized banks, combined with the limited availability of data from these institutions, makes estimates of LGSR bank elasticities with respect to macroeconomic factors insensitive and often inconsistent in statistical terms. In our view, these considerations assume substantial and sufficient relevance to suggest that a direct approach is not suggested for LSIs.

Here, we have therefore opted for an approach we call *indirect*, which involves the construction of a *hierarchical* model. Below, we summarize the steps followed in constructing the model:

- *Step 1*: A satellite model to estimate the relationships between systemic LGSR and macroeconomic factors following the economic laws of credit recovery. As detailed in subsection 3.1, the model is implemented through the specification of a *Seemingly Unrelated Regressions Model Time Series* (SURTS), constructed following the minimal segmentation that is used in practice by analysts. Specifically, the system of equations reflects the segmentation resulting from the intersection of two dimensions: counterpart (firms/families) and presence or absence of collateral (secured/unsecured);
- *Step 2*: Segmentation of the historical data matrix of closed NPL with internal process of credit recover, based on the bank's *Business Model*. As emphasized in subsection 3.2, the characteristics used to construct the OLAP (*On-Line Analytical Processing*) cube can vary from bank to bank and depends on the information availability (number of items for cell);
- *Step 3*: For each OLAP cell estimate a *Beta Inflated-(0,1) Regression* by considering the systemic LGSR among the explanatory variables. The set of covariates can also consider other individual attributes that are typically continuous, such as, for example, the duration of the recovery process, the collateral coverage rate (LTV - *Loan to Value*), etc.

The LGSR model estimation procedure as shown by *Steps 1*, *Step 2*, and *Step 3* has an immediate derivation in probabilistic terms, with important conditional independence assumptions[7].

We denote by x_t the vector of macroeconomic factors, s_t is the vector of the systemic LGSR with $\mathrm{t} = 1, 2, \ldots, \mathrm{T}$. The systemic data is organized in a design matrix $\mathrm{X} = [x_t]_{t=1}^{T}$ and a matrix of response variables $\mathrm{S} = [s_t]_{t=1}^{T}$. For single bank, the internal data are substantiated by two sets of explanatory variables: d_j attributes, which allow the j-th observation to be ranked in an OLAP cell, and z_j, a set of individual characteristics inherent the counterparty or/and the NPL positions being recovered. The symbol l_j is used for indicating the LGSR for j-th NPL closed with internal recovery process, to be calculated according with the methodology outlined in Section 2. Hence, the database held by the bank consists of the triplet: $\mathrm{L} = [l_j]_{j=1}^{N}$, $\mathrm{D} = [d_j]_{j=1}^{N}$ e $\mathrm{Z} = [z_j]_{j=1}^{N}$, where N is the number of observations in the train sample.

Taking advantage of the well-known rules of probability calculus, the joint density of the response variables $\mathrm{S} = \mathrm{s}$, $\mathrm{L} = \mathrm{l}$ conditional on the set of exogenous $(\mathrm{X}, \mathrm{D}, \mathrm{Z})$ can be decomposed as follows:

$$\mathrm{P}(\mathrm{s}, \mathrm{l}|\mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) = \mathrm{P}(\mathrm{l}|\mathrm{s}, \mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) \cdot P(\mathrm{s}|\mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) \quad (3.1)$$

where θ is the set of parameters to be estimated.

The underlying probabilistic assumptions that justify the indirect approach for the construction of the LGSR hierarchical model can be summarized basically in two points:

- (*Hypothesis 1*) $\mathrm{P}(\mathrm{s}|\mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) = \mathrm{P}(\mathrm{s}|\mathrm{X}; \theta)$: systemic LGSR is not affected by the data of single financial institution. The evidence on macroeconomic factors encapsulates all the information needed to explain the loan loss trends of the entire banking system; the recovery action of a single bank brings no relevant cognitive to the national phenomena. In our view, this assumption is commensurately correct for LSIs but may fail for institutions that are the backbone engine of the financial system;
- (*Hypothesis 2*) $\mathrm{P}(\mathrm{l}|\mathrm{s}, \mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) = \mathrm{P}(\mathrm{l}|\mathrm{s}, \mathrm{D}, \mathrm{Z}; \theta)$: conditional on the systemic LGSR, the credit losses of a bank are independent of macroeconomic factors, $\mathrm{L} \perp \mathrm{X}|\mathrm{S} = \mathrm{s}$[8]. By considering the systemic LGSR, knowledge of macroeconomic factors adds no information in terms of the economic cycle in modelling credit losses of a single financial institution.

Thus 3.1 can be written as follows:

$$\mathrm{P}(\mathrm{s}, \mathrm{l}|\mathrm{X}, \mathrm{D}, \mathrm{Z}; \theta) = \mathrm{P}(\mathrm{l}|\mathrm{s}, \mathrm{D}, \mathrm{Z}; \theta) \cdot P(\mathrm{s}|\mathrm{X}; \theta) \quad (3.2)$$

Furthermore, based on probabilistic assumptions, it is quite natural to assume that the parameter vector θ can be partitioned into two sub-vectors: β and γ such that:

$$\mathrm{P}\,(\mathrm{l} \mid \mathrm{s}, \mathrm{D}, \mathrm{Z}; \theta) = \mathrm{P}\,(\mathrm{l} \mid \mathrm{s}, \mathrm{D}, \mathrm{Z};\ \beta)$$
$$\mathrm{P}\,(\mathrm{s} \mid \mathrm{X};\ \theta) = \mathrm{P}\,(\mathrm{s} \mid \mathrm{X};\ \gamma). \quad (3.3)$$

Therefore, given 3.3, from 3.2 we have that:

$$\mathrm{P}\,(\mathrm{s},\ \mathrm{l} \mid \mathrm{X}, \mathrm{D}, \mathrm{Z};\ \theta) = \mathrm{P}\,(\mathrm{l} \mid \mathrm{s}, \mathrm{D}, \mathrm{Z};\ \beta) \cdot \mathrm{P}(\mathrm{s}|\mathrm{X}; \gamma). \quad (3.4)$$

[7] For a definition of conditional independence and *Graphical Models*, see Whittaker (1990).
[8] The symbol ⊥ is used for indicating the independence among random variable Whittaker (1990).

Having the observations on the endogenous and exogenous variables, it is immediate to see that the likelihood function, equation 3.4, enjoys the property of separability in the parameters. This ensures that γ and β can be estimated separately according to the likelihood principle. Specifically, from Step 1 (*satellite model*) we obtain the estimation of γ, while Steps 2 and 3 are devoted to the estimation of β.
The indirect approach represents a generalization, because a response variable, systemic LGSR, is introduced into the model, which participates as exogenous variables in the bank LGSR model and takes the role of endogenous in the satellite model. The two methodologies are related; in fact, by marginalizing 3.4 with respect to s, we obtain by reduction the direct approach model:

$$\mathrm{P}\,(\mathrm{l}\,|\,\mathrm{X, D, Z;}\ \theta) \ = \int_{\mathcal{S}} \mathrm{P}\,(\mathrm{s,\ l}\,|\,\mathrm{X, D, Z;}\ \theta)\ \mathrm{ds}\ (3.5)$$

where $\mathcal{S}$ is the sample space of S.
The two working approaches have differences not only in terms of construction and estimation but also in the final application i.e., forecasting. Once the macroeconomic scenarios that allow defining the expectations on the explanatory variables X are available, for each OLAP cell and conditional on the individual characteristics of the institution's portfolio it is possible to obtain the expected bank-level LGSR. Since in the direct approach, the relationship between bank LGSR and macroeconomic factors is estimated following a probabilistic approach, Equation 3.5, the forecasting step in terms of expected value become immediate (except for complex functional forms). For the indirect method is not the same thing. In fact, as discussed in detail in subsection 3.4, it is not possible to produce closed-form forecasts, but it is necessary to implement a Monte Carlo solution or to propose an approximation by *plug-ins*. The latter solution, which will be followed to expose the empirical results, has a reduced computational complexity compared to estimation by simulation.

3.1 Satellite Model: SURTS

In econometrics, the SUR (*Seemingly Unrelated Regressions Model*), proposed by Zellner (1962), is a generalization of the linear model and consists of a series of equations each of which has its own dependent variable and a set of explanatory variables.
Denoting by k the number of equations and by m the number of explanatory variables, which define the dimensions of the vectors $r_t = 1_k - s_t$ (systemic RR) and x_t, respectively, the SUR model has the following linear structure:

$$r_t = \Lambda\, x_t + \epsilon_t\ (3.6)$$

where Λ is the parameters matrix of k rows and m columns and ϵ_t is the vector of k stochastic errors.
Typically, each regressor does not appear in all equations, so the matrix Λ occurs in a sparse form. Some cells of that matrix are set equal to zero, which indicates the exclusion of a regressor from an equation. Therefore, Λ is subject to the following linear constraints:

$$\Lambda \ = \ \textstyle\sum_{i=1}^{q} G_j\ \gamma_j\ (3.7)$$

where $\gamma\ = (\gamma_1, \gamma_2, \dots, \gamma_k)$ is the vector of q free parameters and the G_j are sparse matrices[9].
The SUR model can be equivalently reformulated as follows:

$$r_t \ = \ X_t\, \lambda + \epsilon_t\ (3.8)$$

where:

$$\mathrm{X}_t = (x_t^T \otimes I_k)$$
$$\lambda \ = \mathrm{vec}\ (\Lambda) \ = \ \textstyle\sum_{i=1}^{q} g_j\ \gamma_j\ (3.9)$$

with $g_j \ = \ \mathrm{vec}\,(G_j)$.[10]

Regarding stochastic errors ϵ_t, we assume that they follow a *Multivariate Autoregressive State-Space* (MARSS) process proposed by Holmes (2021), namely:

$$\epsilon_t = \eta_t + \nu_t$$
$$\eta_t = \Gamma\, \eta_{t-1} + \xi_t\ (3.10)$$

where ν_t and ξ_t are mutually independent Normal random vectors with zero mean and variance/covariance matrix Σ_ν and Σ_ξ respectively. In 3.10, Γ represents the not necessarily symmetric square matrix of autoregressive parameters. For the process to be stationary, the k eigenvalues of Γ must be in modulus less than 1. By tending $\Sigma_\nu \to 0$, that is, defining ν_t as a degenerate Normal on the zero mean, Equation 3.10 specifies stochastic VAR (*Vector Autoregressive*) disturbances of order 1.
In the context of state-space models, see (Harvey, 1989) and (Hamilton, 1994), the model consisting of equations 3.6 (*Measurement Equation*) and 3.10 (*State Equation*) goes by the name of *Seemingly Unrelated Regressions Model Time Series* (SURTS).
Maximum likelihood (ML) estimation of the parameters β, Γ and the matrices of variances/covariances is then obtained using the *Expectation Maximization* (EM) algorithm proposed by Dempster (1977). For the formal aspects inherent the application of EM to the

[9] The linear constraints defined in 2.7 represent a generalization with respect to our analysis, which focuses on including or excluding a regressor in a given equation. Each matrix G_j is specified sparse with only one value being 1 and everything else being 0. Specifically, if the (i, p) element of a G_j is equal to 1 it implies that the i-th explanatory variable is included in the p-th equation. To make the system identifiable all the matrices G_j must be different from each other.
[10] By $\otimes$ we denote the Kronecker product, while vec is the matrix vectorization operator.

class of MARSS models we refer to Holmes (2013). In this work the numerical part of computation was developed in *R on Cran* with the MARSS package version 3.11.3 (Holmes, et al., 2020 and Holmes, et al., 2021).
The system of equations 3.8 consists of four equations $k = 4$ i.e., the vector r_t has the following elements:

r_t^{cr} recovery rate for loans to firms covered by collateral;
r_t^{cn} recovery rate for loans to firms not covered by collateral;
r_t^{rr} recovery rate for loans to households covered by collateral;
r_t^{rn} recovery rate for households not covered by collateral.

Since 2017, Bank of Italy (Financial Stability and Supervision Notes) has been implementing a series of publications in which the trend of recovery rates on NPL for the Italian banks is in terms of a weighted average. Regarding the calculation methodology, see Ciocchetta (2017), and concerning the annual data update, see Fischetto (2017), Fischetto (2018), Fischetto (2019), Fischetto (2020) and Fischetto (2021). The appendices to these publications show time series not only for recovery rates, but also of other quantities of stock and flow: in particular, the number and volume of closed NPL. From the aforementioned Tables we extracted the RRs not subject to divestment in order to construct the time series of the vector endogenous to the system of equations 3.8. For the design matrix X, we refer to Section 4, which is devoted to data and exposition of the results of model estimation.
From a methodological point of view, another aspect to consider concerns the structure of the matrix Γ. To be able to parsimoniously specify a model that succeeds in capturing state-space correlations, we opted to impose Γ as diagonal. Each latent factor η_{it} with $i = 1, 2, \dots, \kappa$, follows a stationary AR (*Autoregressive*) process of order 1. Moreover, specifying unconstrained and full Σ_ξ allows us to estimate the spatial correlations of the latent factors that contribute to the covariation of the response variables. As for Σ_ν, we impose that it is diagonal i.e., the elements of the random vector ν_t are independent from each other.

3.2 Segmentation (OLAP)

Customer segmentation for loss analysis is closely related to the institution's *Business Model* and from its internal organization. Despite some subjective elements in application, seems widespread practice to consider at least two dimensions: type of counterparty (firms/families) and presence of collateral (secured/unsecured).
In this context, basic segmentation is a minimum and strictly necessary requirement because it allows the systemic LGSR to be assigned to each closed NPL in the train sample. A natural criterion is to proceed to assign the systemic LGSR based on the closed date of the distress loan.
To have an adequate stratification, additional attributes that reasonably influence the credit recovery process can be added. As pointed out by Resti and Sironi (2021), the attributes to be considered can basically be grouped into two categories. The first refers to attributes inherent to the exposure such as, for example, the promptness and liquidity of collateral, the face value to be recovered (EAD), the presence or absence of personal guarantees (sureties issued by third parties and/or consortia), etc. The second category encapsulates the characteristics of the debtor, the sector of economic activity in which the company operates, the presence or absence of *forbearance*, territorial court of jurisdiction, etc.
Having established the dimensions for segmentation, the NPLs are classified: each distress loan, based on observed attributes, is uniquely placed in a single cell of the multidimensional table. Then, the train sample is organized into an OLAP cube: each partition (also called *LGDR grade*) contains several closed NPL from which we observe several measures that are typically continuous variables such as LGSR, EAD, duration of recovery process, etc.
Expert judgment guiding the choice of attributes defining the dimensions of the OLAP table must necessarily be followed by careful data analysis. First, the final decision in adopting a multidimensional structure must be conditioned on the number of items for cell, to ensure the consistency of the estimates of the LGSR models. Another aspect to be checked with appropriate statistical tests concerns the level of separation introduced by segmentation. The main objective is to construct groups that share similar characteristics and exhibit sufficiently small residual variance (i.e., within-grade), but also where a significant explained variance is manifested (i.e., between-cell).
In practice, may also arise the need to discretize a quantitative variable by changing its nature to qualitative ordinal to be able to consider that characteristic among the dimensions of the OLAP cube. A data processing is used to transform a measure into a dimension. Such an operation decreases the degrees of freedom, reducing the impact of volatility and limiting the noise. Most importantly, it improves the interpretability of the results. Some drawbacks appear. The definition of the discrete variable is affected by choice of breakpoints with the possibility of information loss in terms of entropy. Increasing the size of the OLAP cube is always constrained by an appropriate data analysis, but mostly by the number of items for grade that considerably affect the consistency of the LGSR model estimates.
Once the dimensions of the OLAP cube have been outlined, careful maintenance work is empirically required to make the segmentation adherent to one's banking real world. Once the process of constructing the OLAP cube on the train sample has been concluded, it is necessary to proceed a constant testing on existing portfolio, i.e., open NPL has been consistent with the historical data composing the train sample in terms of value and number of items.
In this work we adopt the minimum segmentation required and, at the same time, we maintain the duration of the recovery process as a continuous variable that appear as an explanatory variable of the regressions.

3.3 LGSR Model: Beta Inflated-(0,1)

Given a generic OLAP cell, let n be the number of observations. We assume that observations on the response variable l_j (bank LGSR) with $j = 1, 2, \dots, n$, are generated independently by the probability distributions $f(l_j|\theta_j)$ where θ_j is the vector of p incidental parameters. Each element of θ_j is then additively linked to the vector of explanatory variables $\boldsymbol{w}_j$ by specification of an appropriate monotone function (*link function*), namely:

$$g_i(\theta)_{ji} = \boldsymbol{w}_j^T \beta_i \text{ with } i = 1,2,\dots,q \quad (3.11)$$

The above model goes under the name *Generalized Additive Models for Location, Scale and Shape* (GAMLSS) proposed by Rigby and Stasinopoulos (2005).
As pointed out by Resti and Sironi (2021), the distribution of LGSR for a single financial institution takes a U-shape, i.e., bimodal, with points accumulating on the extremes of the [0, 1] interval. Typically, for exposures secured by residential real estate, recovery rates tend to be high (close to 1) while for all other lines not backed by any collateral, total losses are often experienced (LGSR = 1).
A reduced residual variance is expected for each cell since all items share similar attributes. Typically, in some grades, such as those secured by real estate, the LGSR distribution will exhibit left skewness with accumulation points on the zero in proportion more pronounced than unity. By contrast, for groups that are characterized by unsecured positions, the LGSR distribution will be skewed to the right with an excess of values close to 1. Figure 1 shows two histograms illustrating the described phenomenon by way of example.

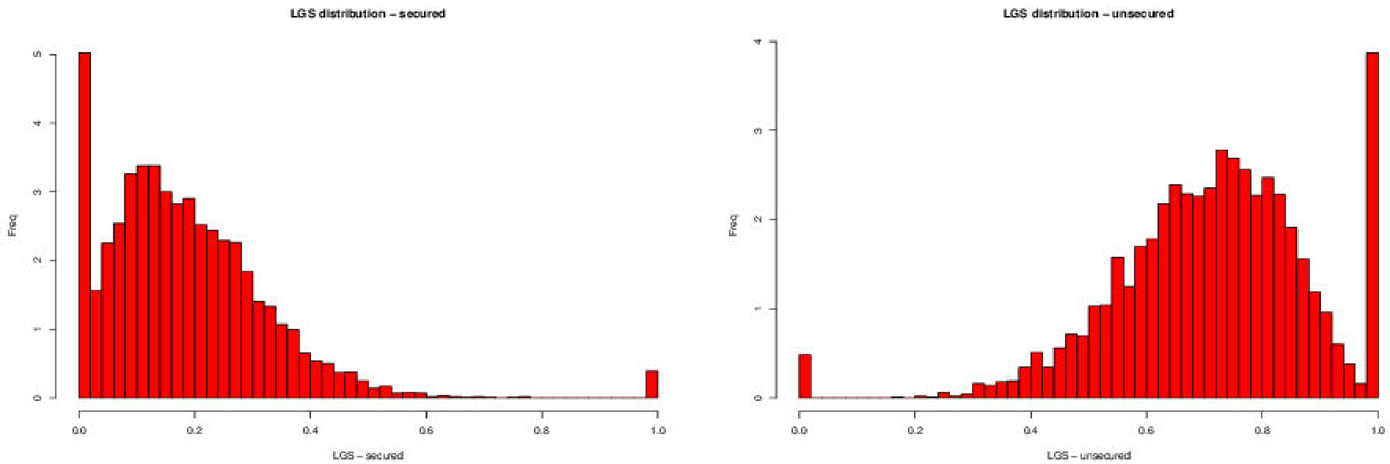


Figure 1: LGSR Distribution for secured and unsecured NPLs (Source: processing on own data)

Based on these empirical considerations, as proposed by Hossain (2016b) (see also Stasinopoulos (2017) and Resti and Sironi (2021)), the *Beta Inflated - (0, 1)* distribution seems to be one among the most suitable ones for constructing a model of LGSR. The use of such a probability distribution succeeds in meeting the two main application needs:

- consider, in inferential terms, the fact that there are two points of discontinuity in 0 and 1;
- the need to capture the forms of skewness and kurtosis typical in the LGSR.

The *Beta Inflated - (0, 1)* distribution is defined as follows:

$$\mathrm{f}(l_j|\theta_j) = \mathrm{f}(x) = \begin{cases} \eta_{0j} & l_j = 0 \\ (1 - \eta_{0j} - \eta_{1j})\, \mathbb{B}\,(l_j \mid \alpha_j, \pi_j), & l_j \in (0,1) \\ \eta_{1j} & l_j = 1 \end{cases} \quad (3.12)$$

where $\theta_j = (\alpha_j, \pi_j, \eta_{0j}, \eta_{1j})$, $\mathbb{B}\,(l_j \mid \alpha_j, \pi_j)$ is the standard Beta density with shape parameters (α_j, π_j) both positive, while $\eta_{0j}, \eta_{1j} \in (0, 1)$ refer to the probabilities on the discontinuity points 0 and 1 with $\eta_{0j} + \eta_{1j} < 1$, respectively. To facilitate the estimation process, it is practical to define monotonic reparameterization:

$$\eta_{0j} = \frac{\delta_{0j}}{1+\delta_{0j}+\delta_{1j}},\ \eta_{1j} = \frac{\delta_{1j}}{1+\delta_{0j}+\delta_{1j}},\ \alpha_j = \frac{\mu_j\,(1-\sigma_j^2)}{\sigma_j^2},\ \pi_j = \frac{(1-\mu_j)\,(1-\sigma_j^2)}{\sigma_j^2} \quad (3.13)$$

where δ_{0j}, δ_{1j} are defined on the positive real axis, $\mu_j = \alpha_j\,(\alpha_j + \pi_j)^{-1}$ is the expected value of standard Beta, and $\sigma_j^2 = (1 + \alpha_j + \pi_j)^{-1}$. We note that the parameter of the mean $\mu_j \in (0, 1)$, with $\sigma_j^2 \in R^+$.[11]
Regarding the specification of link functions, we will proceed as follows:

$$\log(\delta_{0j}) = \boldsymbol{w}_j^T\beta_0,\ \log(\delta_{1j}) = \boldsymbol{w}_j^T\beta_1,\ \mathrm{logit}(\mu_j) = \boldsymbol{w}_j^T\beta_\mu,\ \mathrm{logit}(\sigma_j) = \boldsymbol{w}_j^T\beta_\sigma \quad (3.14)$$

where logit(.) is the Logit[12] transformation e $\beta = (\beta_0, \beta_1, \beta_\mu, \beta_\sigma)$ is the vector of regression coefficients to be estimated.
The vector of explanatory variables $\boldsymbol{w}_j$ in 3.14 appears in all parameter transformations. This approach seems to be entirely general; in fact, in the strictly application domain, some regressors might be useful in explaining the accumulation of observations on

[11] The variance of the Beta distribution according to the reparameterization in 2.13 is equal to $\sigma_j^2\,\mu_j\,(1 - \mu_j)$ i.e., it depends on the mean μ_j and σ_j^2 representing the variance amplification parameter. In fact, with respect to μ_j the variance function has its maximum value equal to $\sigma_j^2\,/\,4$.
[12] For every $x \in (0,1)$, $logit\,(x) = log\,(x) - log\,(1-x)$.

discontinuity points, others might be more suitable for modelling the phenomenon in terms of levels, variability and symmetry within the (0, 1) support.
The direct approach requires macroeconomic factors to be considered in the set of explanatory variables i.e., $\boldsymbol{w}_j$ consists of $\boldsymbol{z}_j$ and $\boldsymbol{X}_j$ or appropriate selection. Moving from a *Long List* to a *Short List* of regressors can be done by referring to expert judgments or by setting up *Model Selection*.
The alternative, consisting of the hierarchical model, considers as a single factor the systemic LGSR that is assigned to each item based on the closing time i.e., $\boldsymbol{w}_j = (\boldsymbol{z}_j, \boldsymbol{s}_j)$. Considering a single synthetic indicator as a thermometer of economic trends makes it possible to streamline the set of explanatory variables by reducing the effects of multicollinearity.
Maximum Likelihood (ML) method is used for estimating the β parameters. As demonstrated in (Hossain, et al., 2016b), the likelihood for the *Beta Inflated - (0, 1)* model enjoys the separability property in the parameters (β_0, β_1) and $(\beta_\mu, \beta_\sigma)$. Specifically, (β_0, β_1) are estimated by means of a *Multi-Logit* model and $(\beta_\mu, \beta_\sigma)$ are obtained by proceeding to estimate a *Beta* regression for only those observations that lie in the interval (0, 1). The statistical computations shown in this paper were developed in R on Cran, GAMLSS package version 5.4-1. For further technical details we refer to (Rigby, et al., 2021).
Once the parameters are estimated $\hat{\beta}$ for a generic OLAP cell $\star$, conditional on the individual characteristics $\boldsymbol{Z} = \tilde{\boldsymbol{z}}$ and the prediction on the systemic LGSR $\boldsymbol{S} = \tilde{\boldsymbol{s}}$, it is possible to construct the probability distribution of the bank LGSR in that grade, which we denote by $L^\star |\, \tilde{\boldsymbol{z}}, \tilde{\boldsymbol{s}}$. Given the assumptions made about the data generating process, the predictive distribution is also *Beta Inflated - (0, 1)* with expected value:

$$E\left[L^\star \middle| \tilde{\boldsymbol{z}}, \tilde{\boldsymbol{s}}, \hat{\beta}\right] = \hat{\eta}_1 + (1 - \hat{\eta}_0 - \hat{\eta}_1)\,\hat{\mu}. \quad (3.15)$$

The conditional estimate of LGSR for the generic $\star$ cell thus consists of two elements. The first, $\hat{\eta}_1$, represents the estimated probability of the LGSR=1 event: this component tends to increase or decrease the expected loss based on the percentage of positions that empirically closed the recovery process without registering any repayment flow. The second component $\hat{\mu}$ is the expected value of estimated losses from NPL closed for which an LGSR was observed in the support (0, 1). Specifically, $\hat{\mu}$, which is weighted by the corresponding empirical percentage $(1 - \hat{\eta}_0 - \hat{\eta}_1)$, is the expected value of a standard Beta distribution that will be located further to the left or further to the right than the simple mean according to the symmetry and kurtosis of the distribution observed in the grade.
The *Beta Inflated - (0, 1)* regression belongs to the class of nonlinear models; therefore, the sensitivity of the expected loss with respect to a generic explanatory variable does not have an immediate interpretation. In fact, the partial derivative of 3.15 with respect to any element of $\boldsymbol{w}$ is not constant, as is the case of simple linear models, but has a rather complex formulation that depends on both parameter estimates and on all exogenous variables. Therefore, all measures of sensitivity, such as partial derivatives, semi-elasticity, and elasticity, are strictly conditioned by the value that the vector $\boldsymbol{w}$ assumes, which will have to be evaluated numerically.

3.4 Forward Looking Projections

Once *Steps 1-3* have been completed, we can proceed to the last step inherent the application of the model for forecasting. Having one or more macroeconomic scenarios, for each OLAP cell, conditional on individual attributes, the best forecast of the LGSR is expressed in terms of expected value

$$E\left[L^\star_{T+h} \middle| \mathcal{I}_{T+\hbar}; \hat{\theta}\right] \quad (3.16)$$

where $\mathcal{I}_{T+\hbar}$ with $h = 1, 2, 3, \ldots$ is the *Information Set* at instant $T + h$ and $\hat{\theta}$ is the parameters estimate from the train sample.
The number of years with respect to which forecasts are made are chosen according to the depth of macroeconomic scenarios[13]. Since the model is specified considering serial correlations, the information set at $T + h$ $(\mathcal{I}_{T+\hbar})$ is formed by the train sample and the forecasts on exogenous variables until the $T + h$ time.
By using the standard rule of probabilistic calculus, the expected value, 4316 can be rewritten as follows:

$$E\left[L^\star_{T+h} \middle| \mathcal{I}_{T+\hbar}; \hat{\theta}\right] = E\left[E\left(L^\star_{T+h} \,\middle|\, S^\star_{T+h}, \mathcal{I}_{T+h}; \hat{\theta}\right) \,\middle|\, \mathcal{I}_{T+\hbar}; \hat{\theta}\right] \quad (3.17)$$

where the first expected value on the right side of the equation is calculated with respect to the predictive probability distribution of the systemic LGSR (satellite model), and the second expected value is estimated using the density of the bank LGSR conditional on the systemic LGSR equation 3.15.
It is not possible to obtain a closed-form solution of 3.17, since the required integral does not have an analytical solution. Therefore, either numerical integration techniques have been used or estimation by simulation is required. In the context of a Monte Carlo study, one generates random numbers $\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_v$ of replications from the predictive probability distribution of the systemic LGSR (satellite model) and then proceeds to aggregation by using mean, i.e.:

$$E\left[L^\star_{T+h} \middle| \mathcal{I}_{T+\hbar}; \hat{\theta}\right] = \frac{1}{v}\sum_{i=1}^{v} E\left[L^\star \middle| \tilde{\boldsymbol{Z}}, \tilde{\boldsymbol{S}}, \hat{\beta}\right] \quad (3.18)$$

where the expected value within the sum coincides with 3.15. It can be shown that 3.18 is an unbiased and consistent estimator of 3.17.
The proposed implementation of estimation by simulation is certainly an attractive solution because the estimator satisfies good asymptotic properties but may be computationally expensive. The required effort is strictly dependent on the number of replications **v** to which the accuracy of the predictions is related.
An alternative way is to proceed by *plug-in*, which consists of setting **v** = 1 and substituting in 3.15 the prediction coming from the satellite model in term of conditional expected value. This solution has the advantage of being computationally feasible and at the

[13] Typically, government authorities proceed to publish scenarios that at most cover a three-year time frame.

same time sufficiently indicative of the trend of the phenomenon. The use of point forecasts, as opposed to exploring the sample space by simulation, means that the volatility of the phenomenon around the mean is not considered. For purely computational reasons, but aware that this is an approximation, we adopt *plug-in* prediction in this paper.

4 Satellite Model – Estimates and Forecast

Chosen an appropriate econometric methodology (subsection 3.1), the satellite model specification consists of selecting the set of explanatory variables.-An initial univariate exploratory analysis is conducted by using Bayesian approach[14], but the resulting model is built according to causal laws by referring to economic-financial mechanisms related to credit recovery. From a long list of macroeconomics factor the following are selected (short list):

PIL: Annual percentage change in Gross Domestic Product
RISPARM: Marginal propensity of saving
TAXOCC: Unemployment Rate
PREIMM: Annual percentage change in the real estate price index
PRESIMPR: Year-on-year growth rates of loans granted to non-financial firms
PRESFAM: Year-on-year growth rates of loans granted to households
TAXIMPR: Interest rates charged to non-financial firms
TAXABIT: Interest rates charged for home purchases (households)

The time series of the variables PIL, TAXOCC, PREIMM, PRESIMPR, and PRESFAM are retrieved from the publication inherent to the Bank of Italy's Coefficient of Countercyclical Capital Reserve (2022) and the data collection inherent to Italian household savings is downloaded from EUROSTAT (seasonally adjusted and calendared nasq-10-nf-tr series). As for interest rates TAXIMPR and TAXABIT are acquired from the *Statistical Database* (BSD) of Bank of Italy[15].
The time series 31/03/2006 - 31/12/2021 have quarterly frequency, except for loan growth rates (PRESIMPR | PRESFAM) and the unemployment rate (TAXOCC) are published monthly. From this series we have made an aggregate by quarterly moving averaging. Dependent variables evidence, as indicated in subsection 3.1, have been observed as annual averages (2006-2020). We adopted, as a working hypothesis, that the RR of each segment remains constant within each year by applying a *smoothing* through a moving average of order 4. In Table 1 we show the estimation of the regression coefficient[16]. For the NPL covered by collateral a year-on-year increase of about 1 percent in GDP growth, the model estimates, with a lag of two quarters, an increase in recovery rates on firms of 0.2208% on secured and about 0.1339% on unsecured. As mentioned in other work, see Belotti and Crook (2012), the rates practiced by the banks have a negative effect on RRs. Specifically, a 1 percent change in rates charged to firms for term loans results a RR reduction of about 0.8252 percent for secured and 2.4232% for unsecured with two quarters lag. As for households, a 1% increase of rates on housing loans reduces RRs for secured by about 3.0700% and 4.2793% for unsecured, with a two-quarter lag. As pointed out by Bonaccorsi Di Patti and Gascarino (2020), default rates are related to bank loan growth rates according to an inverse relationship, i.e., an expansion of credit tends to reduce the default rates. Lower inflows of assets to impairment states have a positive effect on recovery rates; in fact, following a 1 percent increase in loan growth, an improvement of 0.3294% is estimated for RRs of collateral-backed and 0.3047% for unsecured. Referring to the household, the improvement for unsecured would be 0.0842% with a lag of two quarters. The performance of the housing market plays a significant role in the credit recovery phase, especially for that part of the portfolio covered by collateral. In fact, a 1% increase in the general real estate price index, sign of a market with expanding demand, tends to increase RR by about 0.3379% for firms and 0.2212% for households. Finally, for the household sector, labour market factors (unemployment rate - TAXOCC) and marginal propensity to save affect recovery rates. A 1% increase in the unemployment rate reduces the RRs of secured positions by 1.6963% and by 1.0380% for unsecured positions. An increased marginal propensity to save by households of 1% manifests with two quarters delay in a 0.9094% improvement in recovery rates for NPLs covered by collateral and a 0.7468% for the unsecured ones.
For completeness, in Table 2 we illustrate a brief diagnostic. The first column shows the estimates of the diagonal matrix of Γ i.e., autoregressive parameters of order 1 combined with the latent factors. Being in modulus less than 1, we infer that these processes are stationary (immediate consequence of the EM estimation algorithm). This consideration allows us to estimate the matrix of long-run variances/covariances of the response variables, according to the following formula:

$$\lim_{t\to\infty} vec[V(Y_t)] = (I_{k^2} - \Gamma \otimes \Gamma)^{-1}\text{vec}(\Sigma_\nu) + \text{vec}(\Sigma_\xi) \quad (4.1)$$

The second and third columns show the sample and long-run standard deviations estimated by the model. In general, we observe that the estimated variability is higher than observed, especially for the equations related to firms.

As shown in Table 3, the long-run correlations estimated by the model are also found to be higher than the sample correlations. In general, the satellite model tends to over-estimate the log run variances/covariance matrix. This consideration, in combination with the fact that the model fits the data rather egregiously (fifth column of Table 2), does not in general raise any particular concerns. The diagnosis does not seem to be a reason for invalidation, especially for the use for which it was constructed. Indeed, in case the satellite model is used to generate random simulations, once the macroeconomic scenarios are specified, the synthetic samples will be such that they will explore in probability a larger region of the sample space.

[14] The *Bayesian Model Selection* techniques used are similar to those proposed by Bonaccorsi Di Patti and Gascarino (2020) as part of the construction of the dynamic model to explain default trends in the Italian banking system.
[15] Specifically, TAXABIT corresponds to the series "Harmonized interest rates - home purchase loans - flows" while TAXIMPR "Harmonized interest rates - nonc/c loans - nonfinancial companies - flows".
[16] The *s next to the coefficient estimates refer to their significance: (***) 99%, (**) 95% and (*) 90%.

As previously explained in subsection 3.4, defined one or more macroeconomic scenarios, the satellite model makes it possible to produce forecasts of systemic LGSR. By way of example, let us consider the scenario published by the Bank of Italy, dated January 21, 2022, regarding forecasts of the performance of the Italian economy over the three-year period 2022-2024. Table 4 shows the forecasts in terms of values and percentage change from the previous year of the macroeconomic factors used as exogenous in the satellite model.

Equation	Explanatory variables	LAG	Estimates
Firms - secured	INTERCEPT PIL PREIMM log(TAXIMPR) PRESIMPR	- 2 - - -	32.2500 (***) 0.2208 (**) 0.3379 (***) -0.8252 (**) 0.3294 (**)
Firms – unsecured	INTERCEPT PIL PRESIMPR log(TAXIMPR)	- 2 - 2	25.3097 (***) 0.1339 (**) 0.3047 (**) -2.4232 (**)
Householders - secured	INTERCEPT PREIMM TAXOCC RISPARM TAXABIT	- - 1 2 2	64.4106 (***) 0.2212 (**) -1.6963 (**) 0.9094 (**) -3.0700 (**)
Householders – unsecured	INTERCEPT TAXOCC TAXABIT PRESFAM RISPARM	- 1 2 4 2	48.3380 (***) -1.0380 (**) -4.2793 (**) 0.0842 (**) 0.7468 (**)

Table 1: Parameters Estimation of Satellite Model (Source: own computations on macroeconomic data)

Equation	Γ	Std. Obs.	Std. Estim.	R^2
Firms - secured	0.98	2.308	5.306	0.97
Firms – unsecured	0.96	2.735	4.725	0.96
Householders - secured	0.97	9.444	13.299	0.98
Householders – unsecured	0.94	7.751	8.911	0.97

Table 2: Diagnostics of Satellite Model (Source: own computations on macroeconomic data)

Sample	Firms Secured	Firms Unsecured	Hous. Secured	Hous. Unsecured
Firms Secured	1	-	-	-
Firms Unsecured	0.524	1	-	-
Hous. Secured	0.734	0.605	1	-
Hous. Unsecured	0.616	0.676	0.889	1
Estimates	Firms Secured	Firms Unsecured	Hous. Secured	Hous. Unsecured
Firms Secured	1	-	-	-
Firms Unsecured	0.702	1	-	-
Hous. Secured	0.742	0.731	1	-
Hous. Unsecured	0.645	0.735	0.689	1

Table 3: Linear Correlation Indexes Response Variables (Source own computations on macroeconomic data)

The growth of the Italian economy recorded in 2021 (by about 6 percent in terms of GDP) is also confirmed in the three-year period 2022-2024, projecting an increase in GDP and a reduction in the unemployment rate. Despite some inflationary tensions due to rising energy prices, consumption would grow robustly reaching pre-pandemic values in 2024 with a one-year lag in GDP growth. The increase in consumption would be mainly attributable to household spending on durable goods. During the pandemic period a sharp reduction in consummation was observed and consequently a considerable increase in the marginal propensity to save (15% in 2020, 11% approximately 2020 versus 8% in 2019). Growth prospects in terms of GDP and domestic demand would be followed by a reduction in savings which, relative to disposable incomes, is estimated to return to pre-crisis values at the end of 2024.
From industry studies, the outlook for the house market is positive due to change in household[17] preferences and tax breaks from government authorities (e.g., Bonus-110). A stationary general trend is expected in real estate prices with small upward changes, in line with recovery confidence: the scenario depicted in Table 4 has as its underlying assumption an increase in the general index of real estate prices of about 1%, constant in the years 2022-2023.

[17] The analysis conducted by Nomisma points out that for many households there has arisen a need to replace their first home to improve the inadequacies found during lock down periods. Demand for home purchases is mainly heading to the suburbs: a phenomenon that has been taking place for some time of suburbanization towards the search for higher quality spaces and lifestyles.

Variable	2022	2023	2024
PIL	3.8	2.5	1.7
TAXOCC	9.0	8.9	8.7
PREIMM	1	1	1
RISPARM	10	9	8
PRESFAM	3	2.7	2.4
PRESIMPR	1.5	1.3	1.2
TAXIMPR	1.2	1.3	1.4
TAXABIT	1.5	1.6	1.65

Table 4: Macroeconomic Scenario (source: economic bulletin no. 1, 2022, bank of Italy)

The scenario on economic trends[18], assumes that monetary and financial conditions remain favourable despite a slight increase in nominal rates, also to be attributed to inflationary tensions. Rates charged to firms would stand at around 1.4% at the end of the three-year period, versus 1.13% in 2021. The cost of credit for the purchase of the first home, indicative rates for household loans, would stand at around 1.65% in 2024 (at the end of 2021 it is about 1.42%)[19]. Expectations for growth in the Italian economic system are based on the assumption that government policies, particularly the *National Recovery and Resilience Plan* (NRP), will be implemented in a timely and effective manner: private investment would be financed through access to credit, which would cause an increase in terms of growth of 1.5% in 2022 and then settle at 2018 values of around 1.2%. As for the growth in private sector lending, thanks to positive outlooks on the housing market and the fact that about 80% of new home purchases are made through access to credit, it would thirst at about 3% in 2022 versus 3.6% in 2019 and then align with pre-crisis value (2.4% at 2018).

Variable	2019	2020	2021*	2022	2023	2024
Firms - secured	47.89	45.12	43.81	43.60	42.23	41.30
Firms – unsecured	35.43	35.88	33.40	32.91	31.40	30.21
Householders - secured	48.40	63.74	58.93	56.88	56.88	54.14
Householders – unsecured	32.54	48.29	43.82	42.23	42.23	39.75

Table 5: Forecast on Recovery Rates of the Italian System (Source: own computations on macroeconomic data)

In Table 5 we illustrate the satellite model's forecasts of recovery rates for non-disposable NPL in the Italian banking economy. For comparative purposes, we also report the final data for the years 2019 and 2020 while for 2021, which is marked with an asterisk, the values are estimated with the observed covariate.
As also shown in the graphs in Figure 2, after an up and down trend in the first part of the historical series, recovery rates show a downward trend. For the household sector, the series appears to have a cyclical component around the trend, while for firms the recovery rates have a linear downward trend.
The baseline macroeconomic scenario, Table 4, combined with the sign of the regression coefficients allows us to appreciate which elements contribute to increasing/decreasing recovery rates over the 2022-2024 forecast horizon. The growth expectation of the Italian economy is associated with a reduction in the unemployment rate, which is a contributing element to the improved recovery rates for the household segment. A positive contribution to RRs for secured NPL comes from the upward forecast of the general real estate price index. On the other hand, for the household, the contraction in the marginal propensity to consume, the increase in interest rates and the weakening of loan growth in the banking sector compared to the 2020-2021 period contribute to the reduction. The recovery rates of the household presenting a peak in 2020, having a downward trend even if they remain above those observed in the period before the economic crisis. As for the firms, the increase in interest rates and the reduction in terms of growth compared to the year 2021 of gross domestic product and bank loans contribute to a reduction in recovery rates. In the midst of the health emergency (year 2020), a reduction in recovery rates was recorded, and this trend is confirmed downward in terms of expectation in the three-year period 2022-2024.

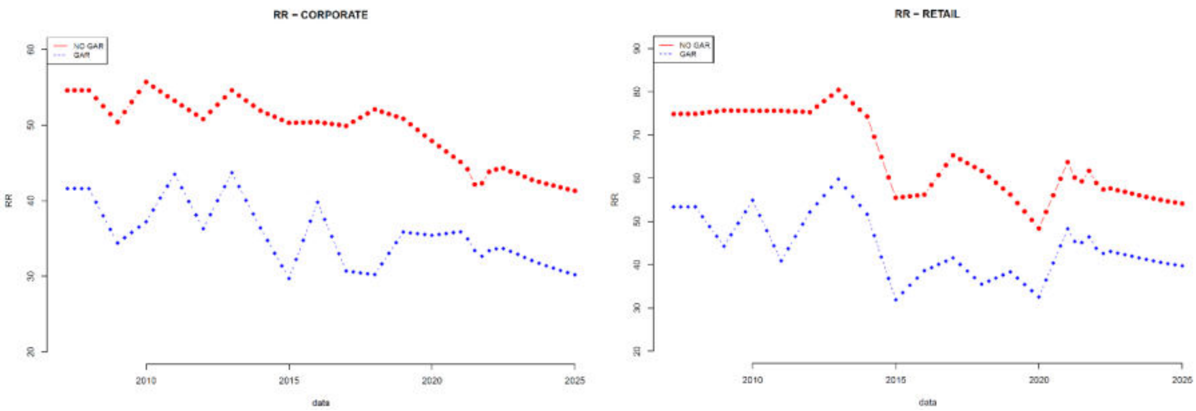


Figure 2: RR Estimates for Positions not Subject to Disposal (source: Bank of Italy (Financial Stability and Supervision Notes and forecasting results by the satellite model)

[18] Bank of Italy, 2022. The Countercyclical Capital Buffer (CCyB) rate. Macroprudential policy decisions of the Bank of Italy.
[19] The interest rate evolutions (TAXIMPR and TAXABIT) presented in Table were obtained by estimating an ECM model on historical data assuming a long-run relationship with the 10-year BTP yield rates and the 3-month EURIBOR.

In conclusion, we would like to point out how the results were obtained as a mere illustrative exercise trying to adhere as closely as possible to the Italian real economy. The set goal of tracing the process of model estimation and use in forecasting seems to have been satisfactorily achieved.

5 LGSR Internal Model

In this section we illustrate the results of the LGSR model, presented in section 3.3, estimated on a data archive of about 5,000 non-performing positions closed in the past 10 years (2011-2021), extracted from a sample of banks from the CABEL network.
The data were processed before estimating the model, so to limit any arbitrage phenomena that banks unconsciously engaged in the allocation of payment flows for unsecured. Specifically, all unsecured NPLs or covered by personal guarantees at the time of the subject's entry into default are aggregated into a single one. The LGS of those items is calculated as a weighted average of the individual LGSR whose weights are represented by the exposures. Respecting the related indirect costs incurred by the bank in the recovery phase, as described in Section 3, were not considered in the calculation of LGSR. The EIRR rates of each transaction are used to discount the nominal cash flows and expenses.
Table 6 shows the percentages of data for which an LGSR value is observed within the open $(0,1)$ range. Complement to 100 represents the fraction of the data on which the LGSR collapses to 1 or 0: LGSR=1 cases are the NPL on which the recovery process had no efficacy whatsoever and in the LGSR=0 cases the bank was able to fully repossess the non-performing exposure. The existence of discontinuity points, which justifies the specification of *inflated* distributions (see section 3.3), shows different intensities in the *grades*; in particular, the phenomenon is more pronounced for unsecured NPLs (about 50 percent for firms and 40 percent for households). The introduction of additional state variables that may be an appropriate proxy for the bank's attention to the recovery process (such as, for example, EAD at the time of entry into non-performing loans), would allow us to circumscribe the intensity of the phenomenon to those segments on which the institution is pessimistic in terms of expectations about debt repayment.
Considering *forward-looking* effects in LGSR model as our main objective, we preferred not to introduce further dimensions of analysis beyond besides the minimal ones: firms/households and secured/unsecured. Each institution can introduce additional stratum variables to build a segmentation that better represents its *Business Model* and, more importantly, its recovery process.

Warranties	Firms	Householders
Secured	57.21	62.14
Unsecured	52.48	38.22

TABLE 6: PERCENTAGE OF LGSR VALUES OBSERVED IN THE INTERVAL $(0,1)$ (SOURCE: OWN DATA).

As explained in Section 3.3, the hierarchical LGSR model involves defining a set of individual attributes (z-vector) to go along with the corresponding systemic LGSR. Aware it does not represent an exhaustive solution, we introduce the duration of recovery as the only continuously characteristic, which nonetheless assumes a rather relevant role in explaining RR trends (Fischetto (2021)). The specified model contemplates that the observed maturity on the closed bad loans enters as an explanatory variable in all the equations in 3.13: in essence, we consider that the duration influences the probabilities of the total ineffectiveness of the recovery action as well as the complete return (parameters δ_0 and δ_1 of equation 3.13, respectively). At the same time, it helps explain the level and volatility of all other items with LGSR within the range $(0,1)$. Empirical results confirm that systemic LGSR is not relevant in explaining the failure or complete recovery of credit. These phenomena should be related purely to the type of recovery and strategies of the institution. For these reasons, the systemic LGSR enters as a regressor in the equations related to μ and σ in Equation 3.13 i.e., explain the trend in the levels and variability of the LGSR distribution for NPLs closed in the interval $(0,1)$.
To maintain the necessary confidentiality of the results, parameter estimates as well as absolute values of prospective projections of loss rates are not published. In our opinion, the reader can observe the consistency of the model by viewing dimensionless numbers, from which the main trends of the phenomenon can be appreciated.
In Table 7 and Table 8 we report the index numbers with 2021 basis of the LGSR *Forward-Looking* estimates for the three-year period 2022-2024, in accordance with some values of duration of the recovery process and in the presence or absence of collateral. The forecasts are obtained following the plug-in methodology: we refer to subsection 3.4 for the reasons for using this method as an alternative to a more correct implementation of a Monte Carlo technique. Index numbers are interpreted according to a comparative key: for instance, a non-performing position with a counterparty belonging to the secured household segment, which is covered by collateral, is estimated to close with an LGSR of 1.0444 times larger in 2023 than in 2021 in case the total duration of the process is around 5 years. Two main reflections are necessary. First, we note that all indices are greater than 1 and have a monotonic increase: that is, the LGSR that considers the systemic perspectives, see Table 5, has an upward expectation. This statement is full agreement with the satellite model results shown in Section 5: an estimated progressive reduction in systemic RRs is reflected in a worsening of the LGSR of each individual bank. A further aspect to note relates to the inverse relationship between percentage increases in LGSR and duration required for closure. This phenomenon is justified by the fact that LGSR values are increasing with respect to the duration of the NPL closed: with higher values of expected losses in 2021, therefore, smaller prospective percentage changes are expected. This rationale is also fully extendable to the fact that, with the same duration, lower LGSR percentage changes are generally observed for secured rather than for unsecured ones.

Years - Duration	Secured			Unsecured		
	1 Year	2 Year	5 Year	1 Year	2 Year	5 Year
2022	1.0085	1.0078	1.0075	1.0020	1.0019	1.0020
2023	1.0642	1.0600	1.0562	1.0081	1.0081	1.0079
2024	1.1031	1.0953	1.0886	1.0129	1.0127	1.0126

TABLE 7: FORWARD-LOOKING LGSR BY DURATION -NON-FINANCIAL FIRMS (SOURCE: OWN DATA)

Years - Duration	Secured			Unsecured		
	1 Year	2 Year	5 Year	1 Year	2 Year	5 Year
2022	1.0252	1.0253	1.0249	1.0070	1.0070	1.0071
2023	1.0450	1.0449	1.0444	1.0139	1.0139	1.0139
2024	1.0650	1.0602	1.0595	1.0192	1.0192	1.0195

TABLE 8: FORWARD-LOOKING LGSR BY DURATION – HOUSEHOLDERS (SOURCE: OWN DATA)

Vintage	Firms		Householders	
	Sec.	Unsec.	Sec.	Unsec.
1 (2022)	1.0256	1.0213	1.0572	1.0130
2 (2023)	1.1494	1.0477	1.1108	1.0261
3 (2024)	1.2346	1.0733	1.1622	1.0379
4 (2025)	1.2781	1.0946	1.1984	1.0442
5 (2026)	1.3219	1.1161	1.2350	1.0507
6 (2027)	1.3654	1.1380	1.2730	1.0571
7 (2028)	1.4086	1.1601	1.3119	1.0635
8 (2029)	1.4515	1.1824	1.3512	1.0700
9 (2030)	1.4932	1.2047	1.3919	1.0766
10 (2031)	1.5342	1.2270	1.4330	1.0830

TABLE 9: LGSR VINTAGE CURVES WITH FORWARD-LOOKING EFFECT (SOURCE: OWN DATA)

Lastly, Table 9 shows the index numbers of the vintage curves with *forward-looking* effect, having 2021 as the base and zero duration. In essence, these curves can be interpreted as the evolution of LGSR with respect to the economy's expectations for non-performing loans originated in 2021. A position belonging to the firm, backed by collateral and with the expectation of closure at 5 years (i.e., 2026), is estimated to have an expected loss 1.3219 times larger than it would be with respect to the 2021 closure event. For the three-year period 2022-2024, LGSR projections consider two elements: the effect of macroeconomic scenarios using the satellite model and the expected duration of NPL closure. From the fourth year onward (i.e., 2025), LGSR estimates were obtained by holding constant the macroeconomic scenario of the third year (2024). The estimated evolution of the vintage curves reflects the considerations mentioned above: namely, increasing monotonicity with respect to the expected time of position closure and larger percentage development for positions covered by collateral. We reiterate that this evidence is a natural consequence of the fact that secured NPL have lower LGSR values than those without collateral.

6 Conclusions

This paper proposes a methodological framework for estimating LGSR from a *forward-looking* perspective. In literature and practice, the use of direct models involving the inclusion of macroeconomic factors as explanatory variables in regressions seems widespread. Here we explored the alternative consisting of implementing a hierarchical model by leveraging conditional probability concepts. The *framework* consists of two modules: the module involving the estimation of a satellite model on the macroeconomic data and the second involving the estimation of the bank's LGSR model.

The choice of an indirect approach involves greater complexity in both estimation and forecasting, for which a closed form is generally not possible. However, this approach has several advantages that we believe to be crucial: on the one hand, the plausible applicability even to LSIs, which tend to have a low contribution to systemic data; on the other hand, the limitation of the effects of multicollinearity on the volatility of estimates.

The proposal is not intended to be definitive but represents a feasible alternative with consistent methodological basis in economic theory supported by established concepts of probability calculus. Two points of potential improvement are highlighted. The first concerns the approximation of forecasts using the *plug-in* technique, which by its nature is unable to capture the volatility of the phenomenon around the mean and detect the presence of any concentration bubbles. One possible resolution to this problem involves the implementation of a Monte Carlo simulation, which requires significant computational effort but ensures consistent estimates. A second element to be introduced is the correlation within and between segments of the classification adopted by the bank through the implementation of mixed-effects models. Regression models, such as the one proposed in subsection 3.3, have an underlying data-generating process with assumptions of independence among observations. In general, it seems implausible to assume stochastic independence among NPLs classified in each segment, i.e., the assumption of no correlation between recovery actions in a same grade and between grade is not a worth hypothesis.

In general, credit risk management models assume stochastic independence between the default event and the random variable "losses" as the basic assumption. However, a set of factors that simultaneously affect both the PD and LGDR parameters may coexist. As pointed out by Resti and Sironi (2021), the factors that could be detected have a macroeconomic nature such as, for example, the value of interest rates but also other indicators that represent the thermometer of the economic cycle. Also worth mentioning is the presence of possible knock-on and sectoral effects. The authors themselves formally illustrate in Merton's model the relationship between PD and RR. In this context, an interesting line of research lies in the attempt to integrate satellite models that are used to propagate macroeconomic effects on PD and LGDR risk parameters. Another element to be tested is to integrate the individual institution's LGSR, Danger Rate and PD models by leveraging the well-established Merton-style model theory. There are several literature contributions that consider default rate as exogenous in LGD model (see Höcht et al. (2022), Wang et al. (2020), Bruce et al. (2010), Khieu, H. D., Mullineaux et al. (2012) and Zhang (2009)), but at the same time doesn't seem to be any explorative study that estimates the correlations between PD and LGD risk parameters. In such context, the default rates assume the role of endogenous variables in a system of equations and, therefore, can be considered as determinants of bank loan recovery rates.

References

- AIFIRM, 2016. *Il principio contabile IFRS 9 in banca: la prospettiva del Risk Manager*. Position Paper N. 8.
- Bank of Italy, 2022. *Coefficiente della riserva di capitale anticiclica (countercyclical capital buffer, CCyB).* s.l.:Decisioni di politica macroprudenziale della Banca d'Italia.
- Belotti, A. & Crook, J., 2011. *Loss given default models incorporating macroeconomic variables for cards*. International Journal of Forecasting,. In: Vol. 28, no. 1 a cura di s.l.:International Journal of Forecasting, Vol. 28, no. 1, pp. 171-182.
- Bonaccorsi Di Patti, E. & Gascarino, G., 2020. *Modelling the dynamics of nonperforming loans in Italy.* Note di Stabilità Finanziaria e Vigilanza N.19 a cura di s.l.:Banca d'Italia.
- Bruche, M. & Gonzalez-Aguado C., 2010. *Recovery rates, default probabilities, and the credit cycle.* Journal of Banking & Finance, Vol. 24, issue 4, pp. 754-764
- Ciocchetta, F., Conti, F.M., De Luca, R., Guida, I., Rendina, A. & Santini, G., 2017. *I tassi di recupero delle sofferenze.* In: Note di stabilità finanziaria e vigilanza N.7. s.l.:Banca d'Italia.
- Deepak Parmani, Y. P. & Analytics., w. f. M., 2017. *Forward-looking Perspective on Impairments using Expected Credit Loss.* s.l.:Whitepaper for Moody's Analytics.
- Dempster, A., Laird, N. & Rubin, D., 1977. *Maximum Likelihood from Incomplete Data via the EM Algorithm.* Journal of the Royal Statistical Society, Series B, 39(1), pp. 1-38.
- Fischetto, L.; Guida, I.; Rendina, A. & Santini, G., 2017. *I tassi di recupero delle sofferenze nel 2016*. Note di Stabilità Finanziaria e Vigilanza, Banca d'Italia*,* Volume 11.
- Fischetto, L.; Guida, I.; Rendina, A.; Santini, G. & Scotto di Carlo, M., 2018. *I tassi di recupero delle sofferenze nel 2017*. Note di Stabilità Finanziaria e Vigilanza, Banca d'Italia*,* Issue 13.
- Fischetto, L.; Guida, I.; Rendina, A.; Santini, G. & Scotto di Carlo, M., 2019. *I tassi di recupero delle sofferenze nel 2018.* Note di Stabilità Finanziaria e Vigilanza, Banca d'Italia, Issue 18.
- Fischetto, L.; Guida, I.; Rendina, A.; Santini, G. & Scotto di Carlo, M., 2020. *I tassi di recupero delle sofferenze nel 2019.* Note di Stabilità Finanziaria e Vigilanza, Banca d'Italia, Issue 23.
- Fischetto, L.; Guida, I.; Rendina, A.; Santini, G. & Scotto di Carlo, M., 2021. *I tassi di recupero delle sofferenze nel 2020*. Note di Stabilità Finanziaria e Vigilanza, Banca d'Italia, Issue 27.
- Gibilaro, L. & Mattarocci, G., 2006. *La selezione del tasso di attualizzazione nella stima della Loss Given Default: un'applicazione al mercato italiano* (working paper).
- Grzybowska, U. & Karwański, M., 2020. *Application of machine learning method under IFRS 9 approach to LGD modeling.* In: s.l.:Acta Physica Polonica A, pp. 116-122.
- Hamilton, J. D., 1994. *Time Series Analysis.* s.l.:Princeton University Press.
- Harvey, A. C., 1989. *Forecasting, Structural Time Series Models and the Kalman Filter.* s.l.:Cambridge University Press.
- Holmes, E. E., 2013. *Derivation of an EM algorithm for constrained and unconstrained multivariate autoregressive state-space (MARSS) models.* s.l.:arXiv:1302.3919 .
- Holmes, E. E., Ward, E. J., Scheuerell, M. D. & Wills, K., 2020. *MARSS: Multivariate Autoregressive State-Space Modeling.*. R package version 3.11.4 a cura di s.l.:s.n.
- Holmes, E. E., Ward, E. J. & Wills, K., 2021. *MARSS: Multivariate Autoregressive State-space Models for Analyzing Time-series Data.* The R Journal, 4(1), pp. 11-19.
- Hossain, A., Rigby, R. A., Stasinopoulos, M. D. & Enea, M., 2016b. *A flexible approach for modelling a proportion response variable: Loss given default.* In: s.l.:In Proceedings of the 31th International Workshop on Statistical Modelling, pp. 127-132.
- Höcht, S., Min, A., Wieczorek, J. & Zagst R. (2022). *Explaining Aggregated Recovery Rates*. Risks, MDPI, vol. 10(1) pages 1-30.
- Joubert, M., Verster, T., Raubenheimer, H. & Schutte, W. D., 2021. *Adapting the Default Weighted Survival Analysis Modelling Approach to Model IFRS 9 LGD.* s.l.:Centre for Business Mathematics and Informatics, North-West University, Potchefstroom 2531, South Africa.
- Khieu, H. D., Mullineaux D. J. & Yi, H. C., (2012). *The determinants of bank loan recovery rates*. Journal of Banking & Finance, Vol. 36, issue 4, pp. 923-933.
- Resti, A. & Sironi, A., 2021. *Rischio e valore nelle banche. Misura, regolamentazione, gestione.* s.l.:Egea.
- Rigby, R. A. & Stasinopoulos, D. M., 2005. *Generalized additive models for location, scale and shape.* In: Applied Statistics, 54. s.l.:Journal of the Royal Statistical Society.
- Rigby, R. A., Stasinopoulos, M. D., Heller, G. Z. & De Bastiani, F., 2021. *Distributions for Modeling Location, Scale, and Shape Using GAMLSS in R.* s.l.:Chapman and Hall/CRC.
- Stasinopoulos, M., Enea, M., Rigby, R. A. & Hossain A., 2017. *Inflated distributions on the interval [0, 1].* s.l.:Working Paper at https://www.gamlss.com/the-books/.
- Wang H., Forbes C.S., Fenech J.P. & Vaz J., 2020. *The determinants of bank loan recovery rates in good times and bad - New Evidence.* Journal of Economic Behavior & Organization, vol. 177, pp. 875-897.
- Whittaker, J., 1990. *Graphical Models in Applied Multivariate Statistics.* Chichester: J. Wiley and Sons.
- Zellner, A., 1962. *An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias.* In: s.l.:Journal of the American Statistical Association, Vol. 57, No. 298, pp. 348-368.
- Zhang, Z., 2009. *Recovery Rates and Macroeconomic Conditions: The Role of Loan Covenants*. AFA 2010 Atlanta Meetings Paper, Available at SSRN: https://ssrn.com/abstract=1346163 or http://dx.doi.org/10.2139/ssrn.1346163

Risk Management Magazine
Volume 17, Issue 3, September – December 2022

Direttore Responsabile (Editor in Chief)
Maurizio Vallino

Condirettore (Managing Editor)
Corrado Meglio

Ownership, Newsroom and Secretariat:
Associazione Italiana Financial Industry Risk Managers (AIFIRM), Via Sile 18, 20139 Milano

Registration number at Court of Milan n. 629 dated 10th September 2004

ISSN Print 2612-3665 – **ISSN Online** 2724-2153
DOI 10.47473/2016rrm

E-mail: risk.management.magazine@aifirm.it; Tel. +39 389 6946315

Printing
Algraphy S.n.c. - Passo Ponte Carrega 62-62r 16141 Genova


(*)
(*)
CON QUESTI PREZZI SULLA TERRA
NON CI VEDRANNO ANCORA
PER UN BEL PO'!!