RISK MANAGEMENT MAGAZINE

Vol. 18, Issue 1
January – April 2023

IN THIS ISSUE

Scientific journal recognized by ANVUR and AIDEA

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MAILED TO AIFIRM SUBSCRIBERS WHO ARE RESIDENT IN ITALY AND DULY REGISTERED

Journal printed on 29th April 2023

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The organization includes the managing editor, a joint manager and an Editorial Board and a Scientific Committee composed by academics.

The magazine promotes the diffusion of all content related to risk management topics, from regulatory aspects, to organizational and technical issues and all articles will be examined with interest through the Scientific Council.

The papers shall be presented in Microsoft Word format, font Times New Roman 10 and shall have between 5.000 and 12.000 words; tables and graphs are welcome.

The bibliography shall be written in APA format and shall accuratel specify the sources.

An Abstract in English is required (less than 200 words) highlighting the Key words.

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Supply Chain Finance techniques and risks

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Abstract

Supply Chain Finance is as a portfolio of financing and risk mitigation practices and techniques to optimize the management of the working capital and liquidity invested in supply chain processes and transactions. SCF techniques existing on the market can be divided into three categories: receivable purchase, advanced payable, and loans. These financing solutions are significantly 'event-driven', since they aim at satisfying the financial requirements of buyers and sellers, that are triggered by purchase orders, invoices, receivables, other claims, and related pre-shipment and post-shipment processes along the increasingly complex supply chains in which they are involved. Along the way from raw material procurement to production, sales and end-users, several source of risks can threaten the possibility of completing the transactions and the regular functioning of supply chain finance. Digitization can help in managing these risks, facilitating the control of the factors underlying them.

Key Words:

1) Introduction: a definition of Supply Chain Finance

Firms usually adopt three kinds of trade finance solutions: cash-in-advance, letters of credit, and open account (Foley et al., 2010; Schmidt-Eisenlohr, 2013; Antràs and Foley, 2015). Cash-in-advance trade finance terms impose importers to pay exporters before the shipment of the wares, thus requiring importers the need to secure finance. Letter-of-credit terms require banks to commit payments before exporters produce their goods and to pay exporters upon shipping, thus removing payment risk for exporters. Open account terms (trade credit) are predominant in current global business environments that are characterized by severe competitive conditions. In particular, they are commonly used in transactions between exporters from developed countries and importers from emerging markets, where documentary trade is interpreted as a lack of trust in the counterparty. In open account trade, importers are allowed to postpone the payment of exporters a certain time after receiving the wares. Thus, they are protected from the possibility that exporters may fail to deliver (performance risk). In addition, importers may resell the goods before paying exporters. On the other hand, they expose exporters to a high payment risk.

Several open account-based trade finance products and programs have been introduced through time, drawing deep attention from both the business community and scholars, as a new way of industry-finance integration and a new financing method compared to traditional ones. These instruments are commonly known as supply chain finance (SCF, hereinafter) and include a wide range of financing and risk mitigation techniques, connected to commercial relationships between companies.

After providing a definition of SCF and depicting the evolution of the literature on the topic, this paper describes the peculiarities of different SCF solutions and identify the risks that not only lead to the disruption of the whole supply chain but also cause substantial harm to the suppliers of funds. Therefore, they should be carefully analyzed and controlled to allow the SCF business to be realized safely. Digitization can help to manage these risks, facilitating the control of the factors underlying them.

The role of SCF can be better understood after a depiction of the relationship between physical and financial supply chains. The activities that are implemented in a physical supply chain allow to transform natural resources, raw materials and components into semi-finished and finished products or to perform services, that are moved from sellers to buyers, either in the national market or abroad. The transfer involves a network of organizations, people, activities, information, and resources. The purchase and sale of goods and services, as well as the payment for them, are facilitated by a range of corporate management practices and transactions, that are named 'financial supply chain management'. This includes for example the conclusion of contractual frameworks, the sending of purchase orders and invoices, the matching of goods sent and received to these, the control and monitoring of activities including cash collections, the deployment of supporting technology, the management of liquidity and working capital, the use of risk mitigation such as insurance and guarantees, and the management of payments and cash flow. SCF is a set of financial services that support the financial supply chain. For a long time, there have been significant dissonances regarding the definition of existing SCF solutions, among companies, financial institutions, and in different countries, due to the continuous operational and technological innovation (Tavecchia, 2018). In 2016, a large number of operators within the Global Supply Chain Finance Forum (GSCF) coined a shared definition - enhanced in 2021 - which refers to SCF as a range of tools aimed at optimizing the management of working capital and liquidity, to the benefit of the subjects (sellers and buyers of raw materials, semi-finished products, goods, or services) who cooperate along an entire production chain. The diffusion of the SCF involves the transition from a traditional approach, in which each firm relates individually with a lender, to a vision of the supply chain in which in the credit relationship the firm exploits its role within the supply chain and the relationships with other participants.

According to the comprehensive definition provided by the Global Supply Chain Finance Forum (2016), SCF is as a portfolio of financing and risk mitigation practices and techniques to optimize the management of the working capital and liquidity invested in supply chain processes and transactions. This comprehensive range of techniques and solutions is largely 'event-driven', as it is

aimed at addressing financial requirements of buyers and sellers, that are triggered by purchase orders, invoices, receivables, other claims, and related pre-shipment and post-shipment processes along the increasingly complex supply chains in which they are involved (European Commission, 2020).

Figure 1 shows how SCF intervenes to support the trade and financial flows along end-to-end business supply and distribution chains.
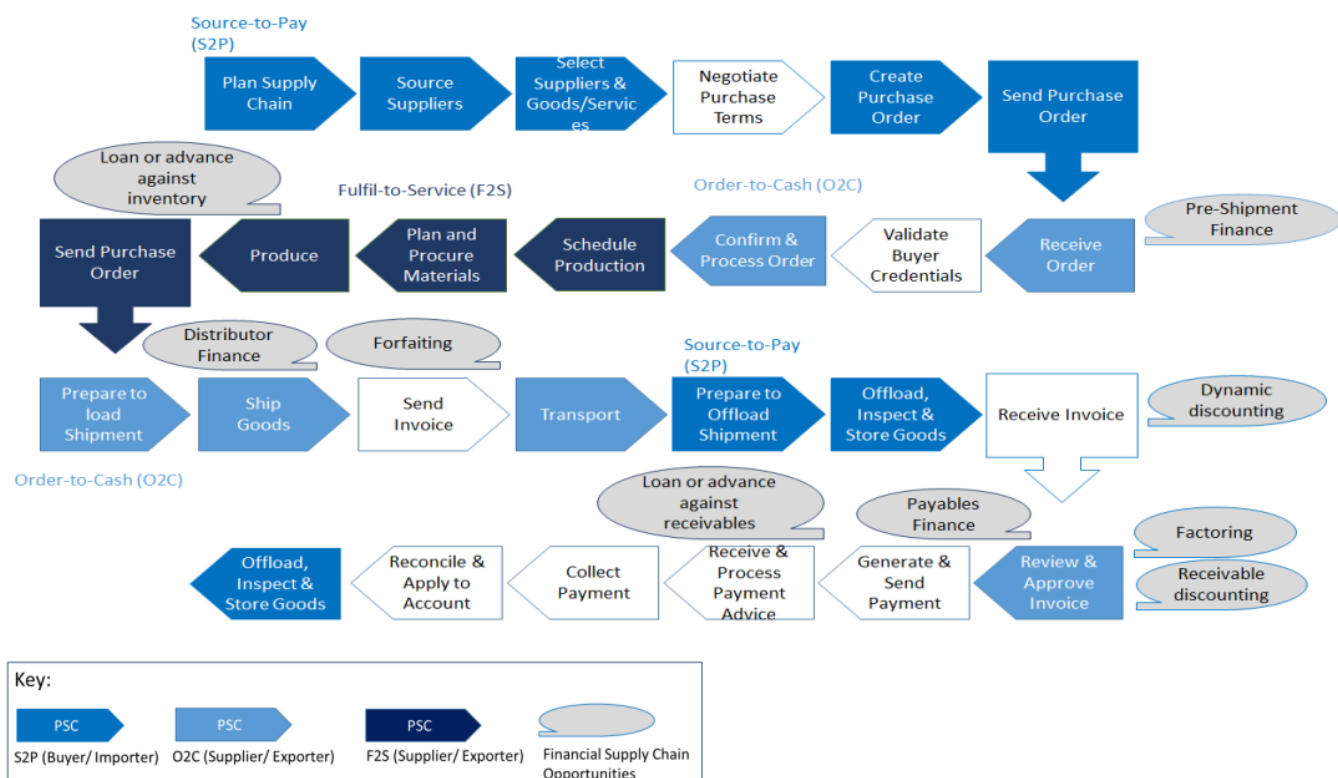
Figure 1. Financial Supply chain opportunities along the physical supply chain- Source: European Commission (2020).

GLOBAL WINNERS	
Best Supply Chain Finance Provider – Bank	Bank of America
Best Supply Chain Finance Provider – Non-Bank	Orbian
Best Customer Implementation	DBS
Best Dynamic Discounting Solution	Citi

REGIONAL WINNERS	
Africa	CIB
Asia-Pacific	ANZ
Caribbean	IDB Invest
Central & Eastern Europe	SupplierPlus
Latin America	BBVA
Middle East	First Abu Dhabi Bank
North America	Citi
Western Europe	MUFG

Figure 2. SCF Provider awards 2023 - Source: https://www.gfmag.com/magazine/february-2023

SCF volumes have grown significantly in recent years, reaching a total value of $1.31 trillion in 2020, according to the latest BCR (2021) World Supply Chain Finance Report. In Europe, the flourishing of these instruments is also the consequence of the economic recession following the global financial crisis of 2007 and in particular of the credit crunch, which led companies to explore new sources of financing. SCF operations have spread especially in the food and beverage, manufacturing, automotive, consumer goods, transport and logistics, construction and chemical and pharmaceutical sectors (European Commission, 2020). Credit intermediaries - especially banks and factoring companies - play a leading role in offering SCF solutions, thanks to the long-term relationships they develop with their corporate customers, and which allow them to play the role of partner integrated into the daily running of the business.

However, fintech vendors and other service and solution providers are increasingly active in the enablement of SCF. Global Finance Magazine ranking shows the most active SCF providers in 2023, both globally and regionally (Figure 2).

2) The evolution of the literature on Supply Chain Finance

The theoretical exploration of SCF by scholars has followed the development of SCF practices. The literature on SCF has undergone an evolution through four stages, according to the maturity and scope of activities of SCF programs and the participants or geographical scope involved in SCF (Song, 2019): finance-oriented; supply chain-oriented; network ecology-oriented; fintech-oriented.

Most of the early research on SCF is finance oriented and underlines the crucial role of financial institutions in providing innovative financial solutions. According to Camerinelli (2009), SCF is a set of products and services provided by financial institutions that facilitate the exchange of goods and information in the supply chain. Chen & Hu (2011) look at SCF as an innovative financial solution that uses funds to create value by bridging banks with fund-starved companies in supply chain and reducing the supply-demand mismatch in fund flow. Finance-oriented research also explores the scope of business involved in SCF techniques. Some studies view SCF activities as a short-term financing process, or as a financing solution generated based on accounts receivable and payable. Lamoureux and Evans (2011) see SCF as a fusion of technology and financial services that brings together global supply chain companies, suppliers, and financial institutions (especially technology service providers) who improve financial supply chain benefits by preventing damage cost shifting and improving the visualization, availability, delivery, and cost of funds of global supply chain participants.

The authors also argue that the activities triggering SCF are mainly events that occur in the trade process, such as order receiving, shipping, issuing bills, and due payments. More and Basu (2013) define SCF as the management, planning, and control of all processes related to transaction activities and funds among all supply chain stakeholders in a bid to improve working capital for all parties. They distinguish SCF services into three stages based on the activity process, namely pre-shipment financing, in-transit financing, and post-shipment financing. Other finance-oriented studies consider SCF as a buyer-driven solution concerning working capital. For example, Wuttke et al. (2013b) view SCF as an automation solution that allows buyers to make flexible and clear payments based on the entire supply chain.

The second evolution stage of the literature on SCF includes supply chain-oriented research that considers SCF to be anchored in supply chain operations. As such, it includes solutions that lessen working capital requirements through inventory optimization throughout the supply chain, or transfer working capital to supply chain participants who have access to funds at lower cost. Pfohl and Gomm (2009) consider SCF as a way of financial optimization and integration with clients, suppliers, and service providers to increase value for all the participating members of the supply chain. Hoberg et al. (2017) investigate whether inventory management is impacted by capital constraints and whether companies take into account the cost of capital when making inventory decisions. Their results show that financially constrained companies have higher inventories, nevertheless capital constraints are not taken into account in supply chain inventory management and decision-making. Differently from finance-oriented research, supply chain-oriented studies state that even though financial institutions are actively involved in SCF, they are not always the dominant actors. The main players who promote and develop the supply chain financial services are the organizers of supply chain operations, including producers, distributors or third-party or fourth-party logistics providers.

More recently, the in-depth analysis of SCF practices has allowed scholars to point out some limits of supply-chain-oriented research. First, these studies pay no attention to the investigation of the conditions, scenarios, and innovative models for the development of SCF from a whole supply chain perspective (Caniato et al., 2016), limiting their research field to financial services originated by certain links of operations (e.g., inventory transfers) or assets (e.g., collateral and pledges). Moreover, the role of professional platform companies or service providers has not been explained enough by supply-chain focused research (Song et al., 2018). Thus, research on SCF has gradually entered the third stage of its evolution path, that is network ecology-oriented research.

Network ecology-oriented research show three main features. First of all, it tries to understand whether specific supply chain contexts ease the origination of SCF, that is, which contextual factors bring about SCF. For example, Caniato et al. (2016) suggest that the successful execution of SCF solutions depend on the degree of cooperation between enterprises in supply chain, the bargaining power, the degree of digitalization, and the appeal of financial services. Song et al. (2016) examine whether financing is influenced by SMEs' supply chain network characteristics. They find that weak relations play a crucial role for SMEs' access to financing. Another feature of network ecology-oriented studies is distinctive in respect to prior research, namely the investigation of the role of supply chain integration and the degree of cooperation among supply chain participants for financing services. Wandfuh et al. (2016) argue that successful implementation of SCF is conditioned to the concurrent occurrence of the elements: the strategic integration of corporate finance within an enterprise – through the realization of synergies between supply chain business department and finance department – and the synergies between suppliers and buyers, based on information sharing and finance integration between these two participants. Wuttke et al. (2016) added that financing performance is subject to further factors.

Specifically, the payment terms and purchase volumes that are negotiated between buyers and sellers in the course of business are able to affect the timing and the effects of SCF. Large purchase volumes and long payment cycles increase the need for early adoption of supply chain financial services.

The progress of network ecology-oriented research has been characterized also by the recognition of driving role of professional service providers in SCF, which has led to the formal introduction of the concept of 'financial service provider' in theoretical studies. Silvestro and Lustrato (2014) are the first scholars to suggest the enabling role of financial service providers – namely banks – in supply chain integration. Martin and Hofmann (2017) argue that different player may exercise the function of a financial service provider: either a traditional bank, or an innovative financial company, or a technology provider. The authors stress the crucial role of financial service providers in clearing up the mismatches existing between different supply chain participants and between financial institutions and commercial banks. Song et al. (2018) compare the differences between traditional commercial banks and new platform-based financial service providers in the process of providing supply chain financial services for pre-trade, mid-trade, and post-trade risk control. They find that traditional commercial banks are less able to control pre-trade, mid-trade, and post-trade risks by monitoring the whole process of supply chain operations through trading information, networks, and processes. On the contrary, platform-based financial service providers are more capable to perform this task, taking advantage from their experience as companies that directly engage in or organize supply chain operations.

The recognition of the driving role played by financial technology (fintech)[1] in SCF has led research on this topic to the current fourth evolution stage. Technologies help alleviate information friction that refers to incentive misalignment caused by information asymmetry between two parties of an economic or financial transaction, improving both information collection and information processing. On one hand, information collection technologies allow gathering additional new data, through digitization and automation, biometrics and identity management, and blockchain. Digitization enables lenders to have huge amounts of data ready to be analyzed. Advanced analytics and artificial intelligence help them in this task, drastically cutting the time needed to review documents and extract data. Biometrics and identity management allow lenders to recognize micro, small and medium enterprises and link their business history with their current status faster and more efficiently.

On the other hand, information processing technologies turn the enormous amounts of data lenders own into useful information in lending decisions. Therefore, not only are technologies able to collect more useful data, but they also enable lenders to extract more and better information, thereby increasing loan process efficiency. Thanks to digital transformation promoted by the implementation of digital technologies, financial technology (fintech) companies have become a new participant that empowers SCF (Moretto and Caniato, 2021). Several scholars have shown the unique advantages of using technologies in the context of SCF (Yan, 2017; Omran et al., 2017; Lee et al., 2019; Lee et al., 2021; Song et al., 2022; Soni et al., 2022; Le, 2022). Even if fintech companies may neither take part to specific transactions nor coordinate business operations, they can be embedded in the supply chain information network, thereby promoting the effective and smooth flow of signals between small and medium enterprises and lenders, improving the efficiency of processing information, and easing the lenders' decision- making process.
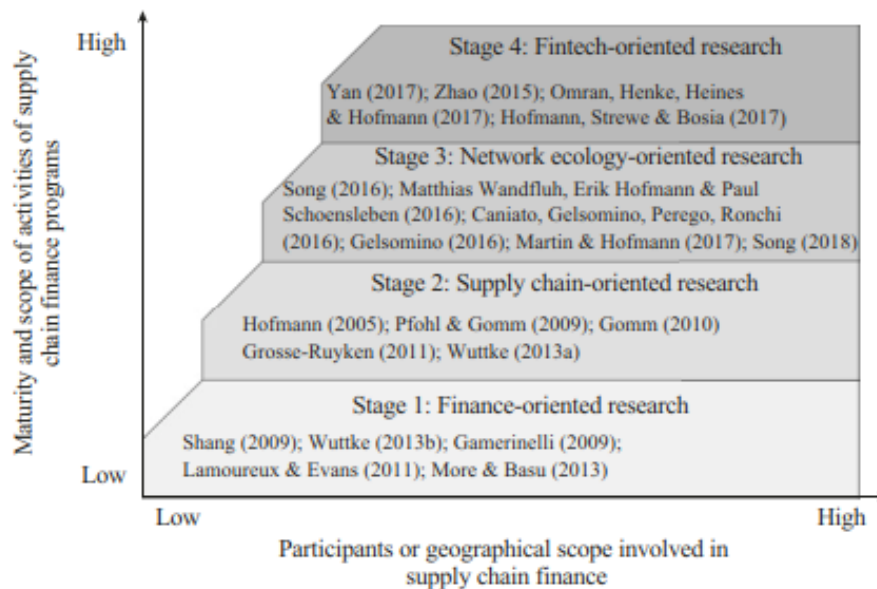
Figure 3. The evolution stages of SCF theories - Source: Song (2019).

3) Supply Chain Finance techniques

According to the definition provided by the Global Supply Chain Finance Forum (2021a), SCF techniques existing on the market can be divided into three categories: receivable purchase, advanced payable, and loans.
Hereafter, a depiction of the peculiarities of the different SCF solutions is provided.

3.1) Receivable Purchase

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[1] According to the Financial Stability Board's definition, fintech refers to "business models, technology applications, operating processes, and innovative products that promote financial innovation through technical means and have a significant impact on financial markets, institutions, and financial services" (see FSB, 2017).

The background of this kind of SCF solution is the finalization of a commercial agreement between a firm selling goods and/or services (supplier) and a buyer company. The seller issues invoice with payment details. In particular, the sale contract (or service contract) establishes a payment obligation from the buyer to the seller, at a future pre-agreed date. Through receivables purchase the supplier transfers all or a part of its receivables to a finance provider. After the transfer, the ownership of these assets lies with the finance provider. In return, the supplier receives an early payment for the receivables, which may include a deduction denoting the quality of the receivables, and a charge based on the pricing arranged with the finance provider. The receivables involved in these transactions exist (i.e., they can be clearly identified and validated), are assignable and enforceable against its debtor in the debtor's jurisdiction.

Included in this category are:

- receivables discounting;
- factoring;
- forfaiting;
- payables finance.

Receivables discounting is usually offered by finance providers to larger enterprises selling to multiple buyers. It may be provided on a one-off, seasonal or continuous program basis. According to these contracts, the seller trades single or multiple receivables - represented by outstanding invoices - to a finance provider at a discount. The finance provider may discount up to 100% of the receivables up front or apply a security margin to cover for possible credit deterioration. Typically, the finance provider will limit such offering to a client base, whose receivables comply with certain criteria, such as a minimum credit rating. The payment of the underlying invoices is hinged on the buyer company, so the buyer coverage depends on the number of buyers for which the finance provider is willing to take credit risk. The finance provider may limit its own risk exposure, by insuring or sharing the credit risk with a third party (e.g., trade credit insurance). If the transfer of receivables is disclosed to the buyer, the collection of the receivables may be undertaken by either the seller (acting as agent for the finance provider) or by the finance provider. The buyer may be required to confirm that specific invoices are authentic and approve invoices for payment within a certain timeframe. Figure 4 shows the functioning of a receivable discounting agreement.
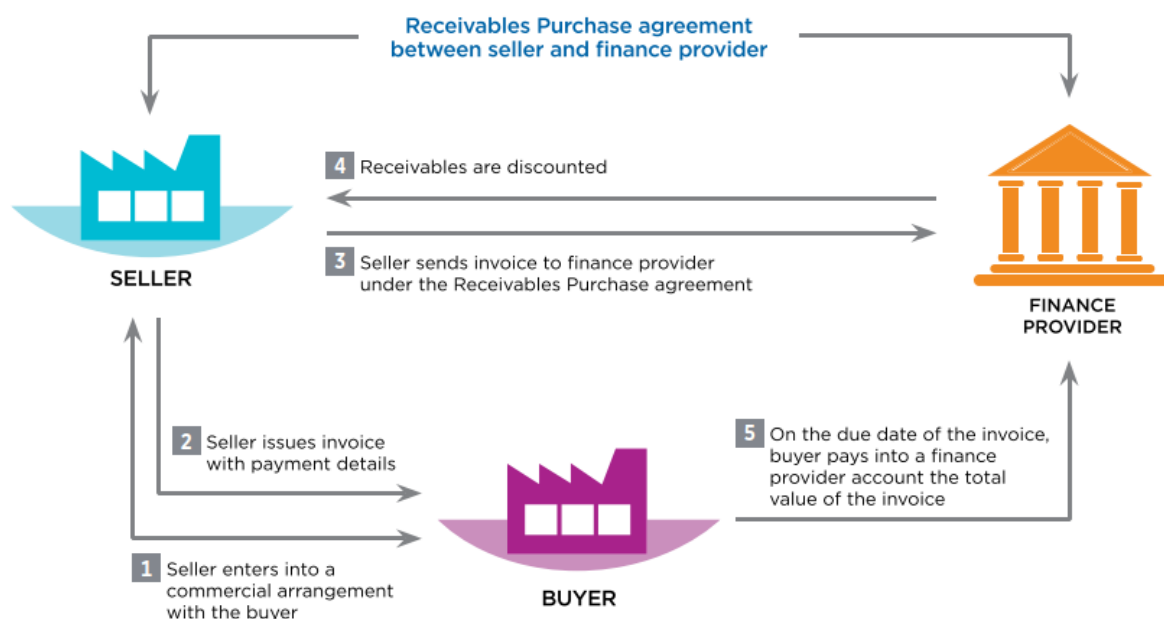
Figure 4. Illustration of the process for receivables discounting – Source: Global Supply Chain Forum (2016).

Factoring too implies the transfer of a seller's receivables at a discount to a finance provider (named 'factor'). The seller may assign all invoices or allowable invoices to the factor (whole turnover factoring), or it may select a range of invoices to be assigned (selective factoring). Usually, the factor is a specialized finance provider serving a variety of suppliers including most SMEs but also large value transactions.

The nature of factoring is the same as in receivables discounting, but the content of the contract may be slightly different, since the factor typically becomes responsible for managing the debtor portfolio and collecting the payment of the underlying receivables. Moreover, it often offers protection against the insolvency of the buyer. Within the factoring model, some variations exist. The buyer and the seller may be located in the same country (domestic factoring) or in two different countries (international factoring). In the second case, two factors are involved, one in each country.
In addition, the advanced payment to the seller can be provided with recourse (the factor has recourse to the seller in the case of buyer default) or without recourse, depending on aspects such as credit insurance, jurisdiction and market practice. Generally, the buyer is generally notified of the receivables assignment (disclosed factoring) and settles the invoice with the finance provider on the due date. In some cases, the invoice bears no notice of assignment, and the buyer is not aware of the factoring agreement between the seller and the finance provider (confidential factoring).
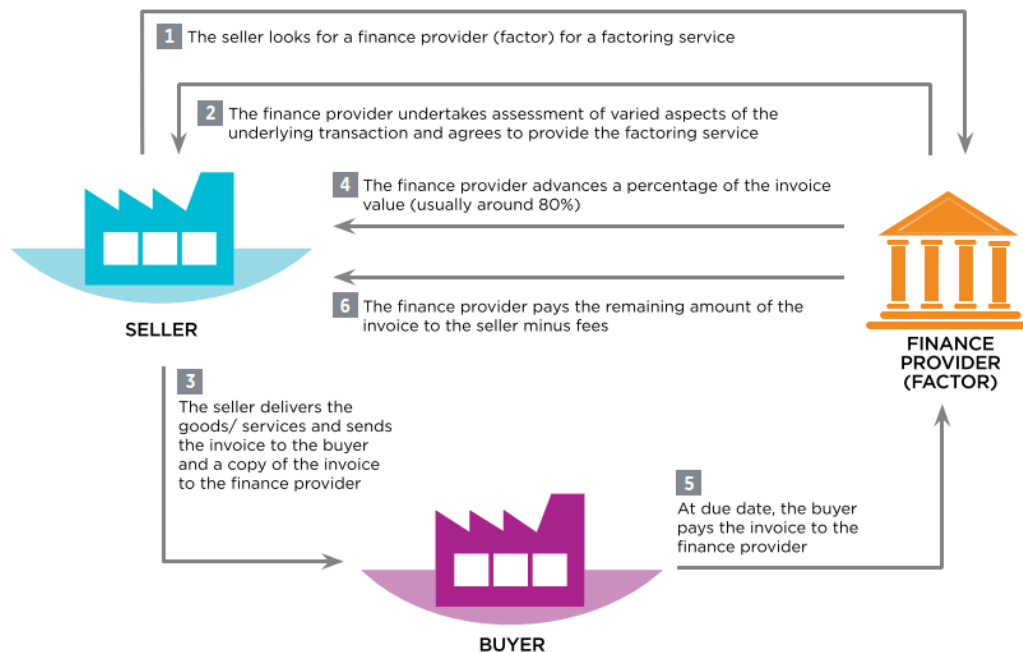
Figure 5. Illustration of a factoring scheme – Source: Global Supply Chain Forum (2016).

In forfaiting, the commercial contract between the buyer and seller is essentially the same as the one underlying receivable discounting and factoring. Differently form the two previous SCF solutions, forfaiting requires the existence of an underlying payment obligation that is usually embodied in a financial instrument or payment bond (normally in negotiable or transferable form), distinct from the commercial transaction (e.g., exports, imports) that originated it. Forfaiting consists in the without recourse sale of the payment obligations at a discount or at face value, in return for financing. These obligations may or may not be guaranteed by third parties (e.g., banks). Tenors can differ from one month to several years, unlike factoring and receivables purchase that are conducted on a short-term basis.

There is a primary and secondary forfaiting market. In the primary market, transactions are originated, and obligations can be purchased from sellers of goods/services or their buyers. Thus, there is a seller of goods/services or buyer that act as seller of the instrument or payment obligation to the initial finance provider (commonly the primary forfaiter). In this market, forfaiting is undertaken without recourse to the seller of goods and services. In the secondary market there are sellers and buyers (forfaiting), usually composed of finance providers and investors. In this case, forfaiting is undertaken without recourse to the seller of the forfaited asset.
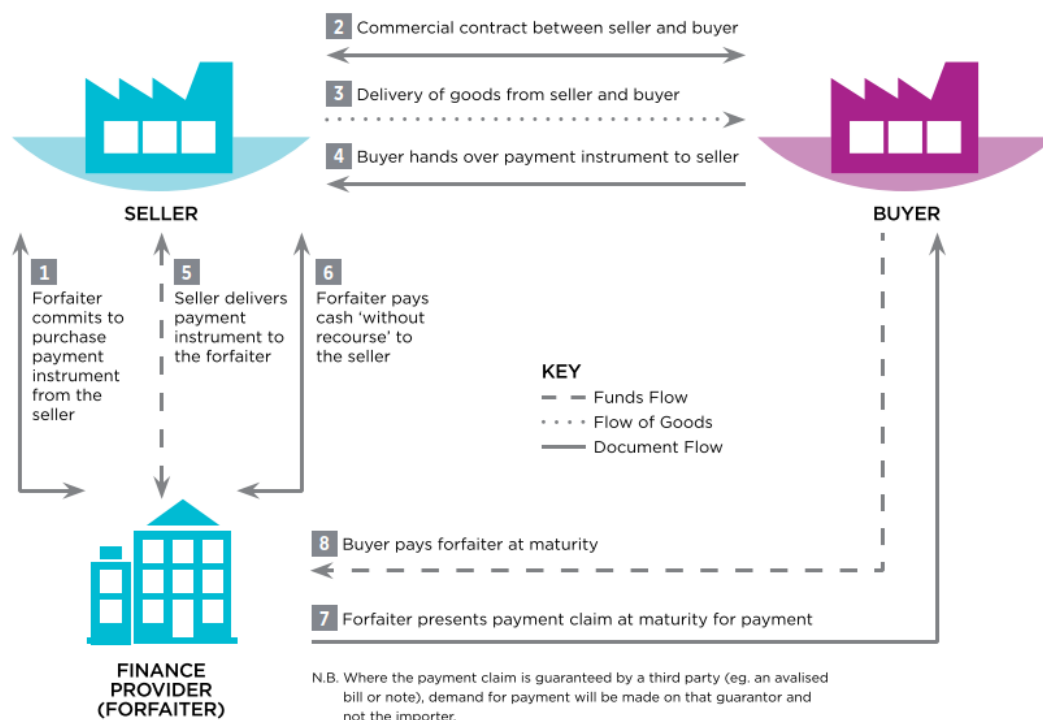
Figure 6. The phases of the process for forfaiting – Source: Global Supply Chain Forum (2016).

While the SCF techniques described so far are conducted based on a seller-led process, payables finance is provided through a buyer-led program in which sellers involved the buyer's supply chain are able to access finance by means of receivables purchase. Given its structure, this SCF solution is commonly known as 'reverse factoring'.

Reverse factoring is usually directed to large buyer companies that have a wide and fragmented portfolio of suppliers and are willing to rationalize their accounts payable cycle. The buyer acts as 'anchor party' or program arranger: it sets up a payables finance program with one or more finance providers, according to which each seller is entitled to receive early payment of receivables (represented by outstanding invoices), at discount and at a financing cost aligned with the buyer's creditworthiness. Usually, the loan is provided 'without recourse' to the seller. Following the stipulation of the reverse factoring agreement, the leading company (i.e., the buyer) provides the financial provider with the names of the strategic suppliers on which to operate. The financial provider admits the sellers that are characterized by an economic-financial situation demonstrating business continuity, albeit in the presence of temporary financial difficulties. At the invoice maturity date, the buyer makes the payment of the principal amount to the financial provider. Often, the buyer company is granted a deferment on the original payment terms, which allows it to benefit from an extension of the duration of its accounts payable cycle, without negatively influencing the cash flows of its supplier base.

Thanks to this SCF solution, the suppliers have the opportunity of accessing dedicated credit lines at preferential conditions, benefiting from the credit standing of the buyer company. Signing a reverse factoring agreement has many advantages also form the buyer's point of view. The main benefit for the anchor party consists in the optimization and planning of treasury and financial flows. A further significant advantage concerns the uniformity and simplification of the administrative procedures related to accounts payable. The fact of interacting with a single interlocutor for the transmission of the flow of information, acknowledgments and payments, leads to a reduction in operating costs. Furthermore, these operations strengthen the relationships with suppliers, with a potential improvement in the quality-of-service levels. Finally, as anticipated, the buyer has the option of requesting an extension of the payment terms.
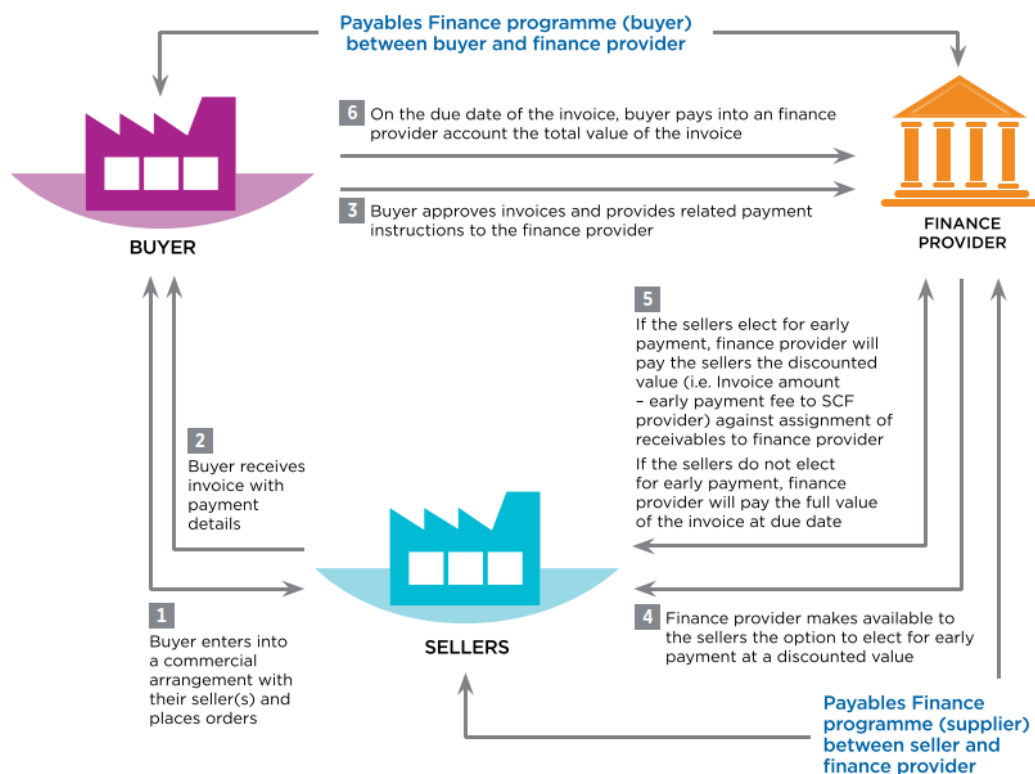
Figure 7. The structure of a payables finance program – Source: Global Supply Chain Forum (2016).

3.2) Advanced Payable

The second type of SCF techniques includes corporate payment undertaking and dynamic discounting.

Like payables finance, corporate payment undertaking is a buyer-led program which entitles sellers in the buyer's supply chain to receive in advance the payment of the discounted value of outstanding invoices (unconditionally approved by the buyer that will pay on the due date) from a finance provider, at a financing cost that is aligned to buyer's creditworthiness.

The peculiarity of corporate payment undertaking is that the early payment does not require receivables purchase but may ask the seller to confirm the finance provider's right to receive buyer payment and/or pass-through arrangements and/or acceptance as full payment of the approved invoice amount. Thus, in such a scheme:

- The buyer identifies the invoices to be involved on the SCF solution and after approval, it submits the details of the invoices to the finance provider, together with a corporate payment undertaking to make payment of the approved amounts to the finance provider at the confirmed invoice due dates.
- The seller has the option to request an early payment from the finance provider at a discount.
- The finance provider relies on the buyer's credit standing and generally provides early payments 'without recourse' to the seller.
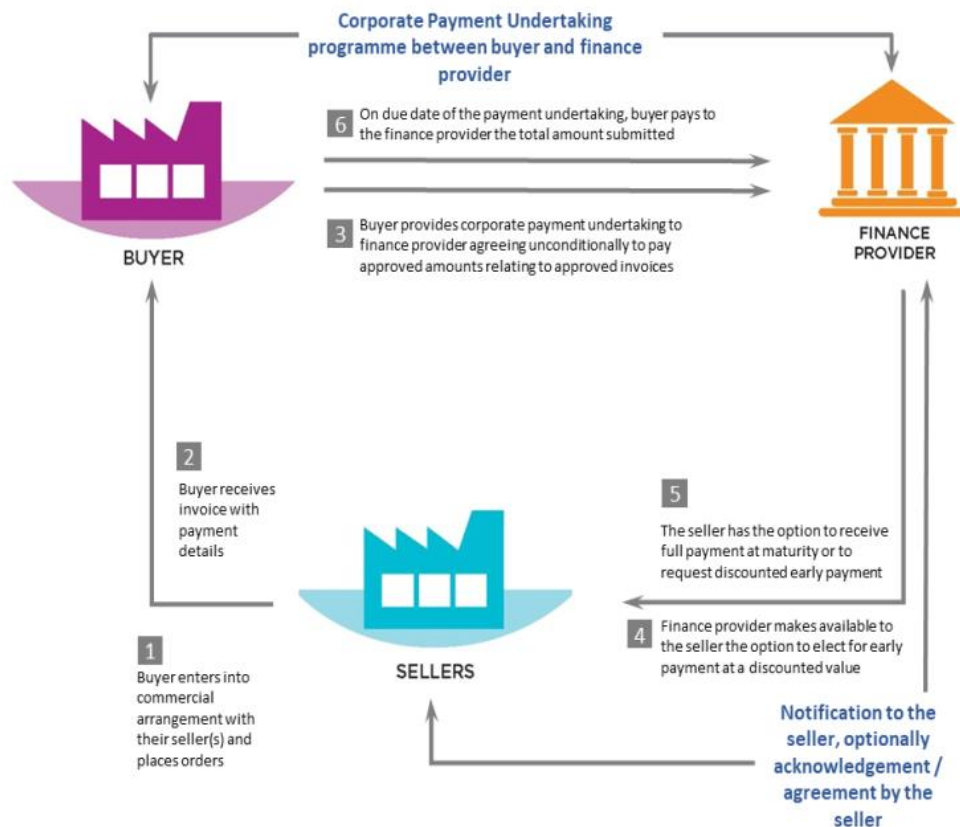
Figure 8. The scheme of a corporate payment undertaking – Source: Global Supply Chain Forum (2021b).

A dynamic discounting transaction is a peculiar form of SCF, since it consists in an advanced payment that is directly made by the buyer to its sellers. Differently from other SCF solutions, there is no financing from the finance provider. It is the buyer that finances its suppliers with early payment terms. Thus, from the seller company's perspective, dynamic discounting is a discounted payment. More precisely, buyers are allowed to choose the amount and timing of payment to their suppliers in exchange for a lower price or discount for the purchased goods and/or services. The discount is defined as 'dynamic', since it depends on the dates of payment to suppliers: the earlier the payment takes place, the greater the discount recognized to the buyer. Specific online platforms are used to initiate the collaboration between buyers and suppliers and to negotiate the discounts. The buyer is given flexibility in choosing how and when to pay the seller. It specifies how much of its own capital is available to generate additional purchasing discounts and it can also choose to approve individual invoices for early payment. From their side, the suppliers can ask advanced payment for specific invoices. Once the discounts are agreed between the parties, they are automatically initiated.
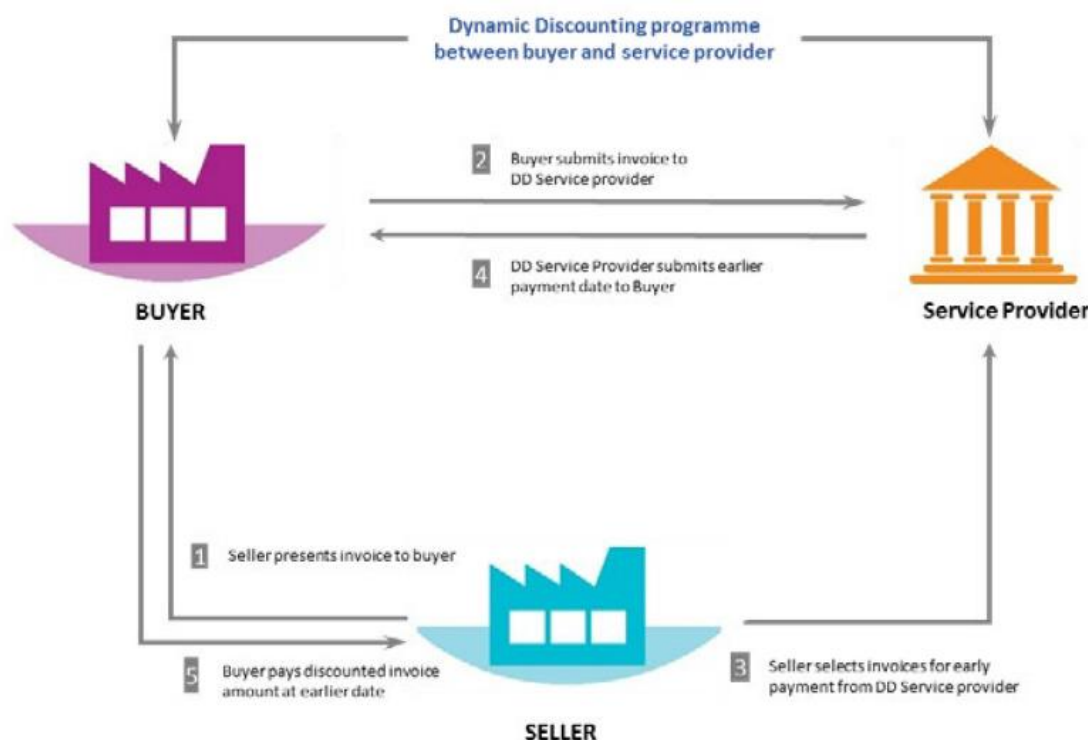
Figure 9. Illustration of dynamic discounting – Source: Global Supply Chain Forum (2021c).

3.3) SCF Loans

The third cluster of SCF solutions include loans and advances made against receivables, rather than by means of purchase, as seen before. In particular, the following transactions are provided:

- loans against trade receivables;
- loans against inventory;
- distributor finance;
- pre-shipment finance.

Loans against receivables refer to financing made available to a party involved in a supply chain on the expectation of repayment from funds generated from current or future trade receivables. The necessary background for these loans to be made is that the seller has or will acquire receivables, deriving from the sale of goods or services. If the seller has already the relevant receivables at its disposal at the time the loan is granted, the financing assumes the form of a secured loan collateralized such receivables. In some cases, the lender may consider the receivables as ideally satisfying financial covenants required to the seller. If these assets do not exist yet, the loan is disbursed on an expectation of such receivables arising at a future date.
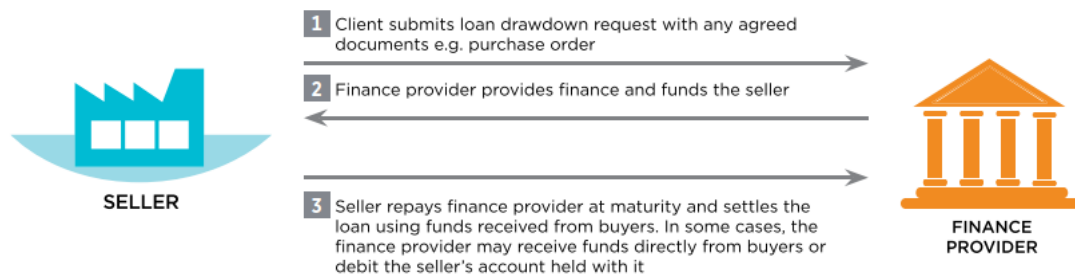
Figure 10. Illustration of loans against receivables – Source: Global Supply Chain Forum (2016).

Loans against inventory implies financing provided to a buyer or seller involved in a supply chain for the holding or warehousing of goods (pre-sold, un-sold, or hedged). This form of SCF technique may be used at any stage and by any party in a supply chain acting as seller and/or a buyer, and the tenor of transactions is short term.

A financing agreement is settled between the finance provider and the borrower (which could be a seller or buyer). In addition, a security agreement is established covering title to the underlying inventory and covering warehouse receipts (evidencing storage of the goods in the warehouse).

A third-party warehouse (certified or recognized by governmental or trade bodies) may also be involved, thus requiring ancillary agreements as well as third party collateral management or inspection agents.
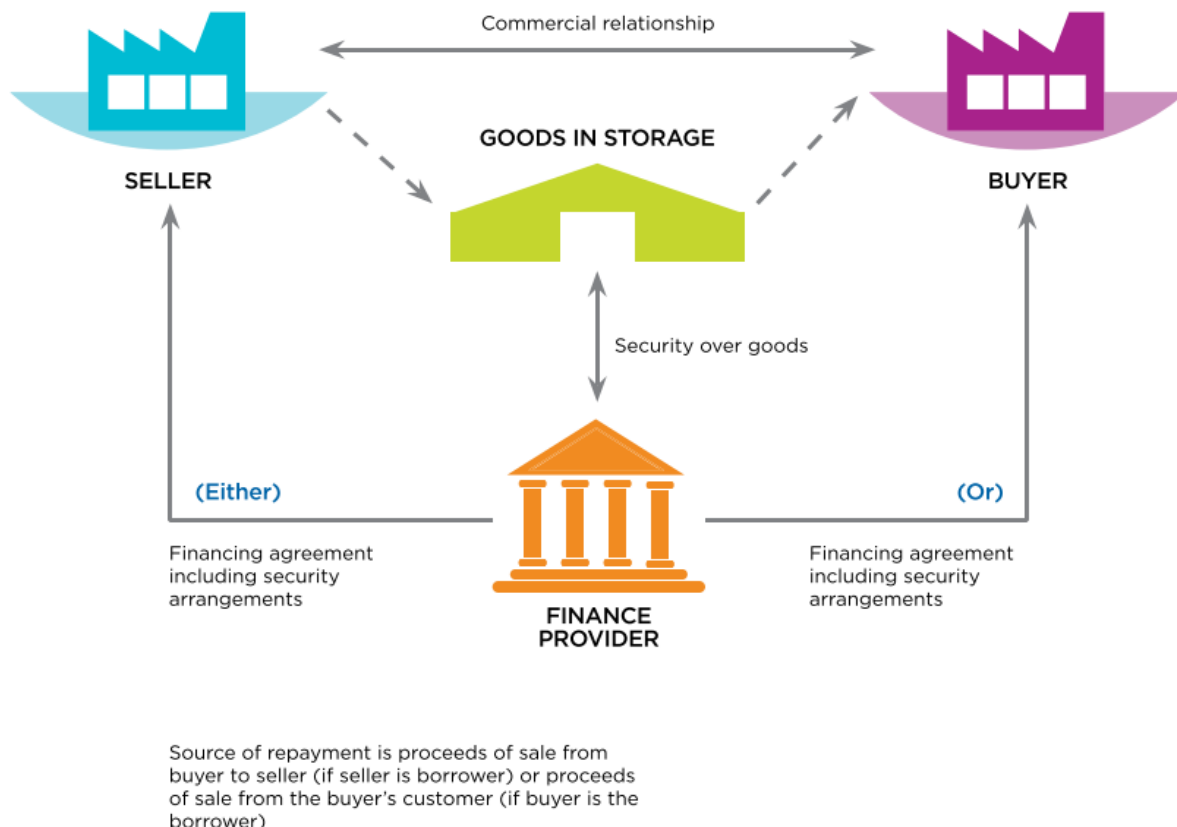
Figure 11. The parties involved in loans against inventory – Source: Global Supply Chain Forum (2016).

Distributor finance consists of a funding facility provided to the distributors of a large manufacturing firm/exporter acting as a seller (often called 'anchor party'). The financing is intended to cover the holding of goods for re-sale to bridge the liquidity gap until the collection of funds from receivables following the sale of goods to retailers or to final consumers.

This facility is typically used for funding inventory and receivables on a short-term basis and is subject to annual review. The distributors and the finance provider directly set up a financing agreement (or sign a facility letter). Moreover, the seller and the financing provider establish a master agreement including the terms of engagement for the finance provider to grant loans for multiple distributors in a variety of global territories, any agreed risk-sharing arrangements, and the operating model applicable to the three parties involved.
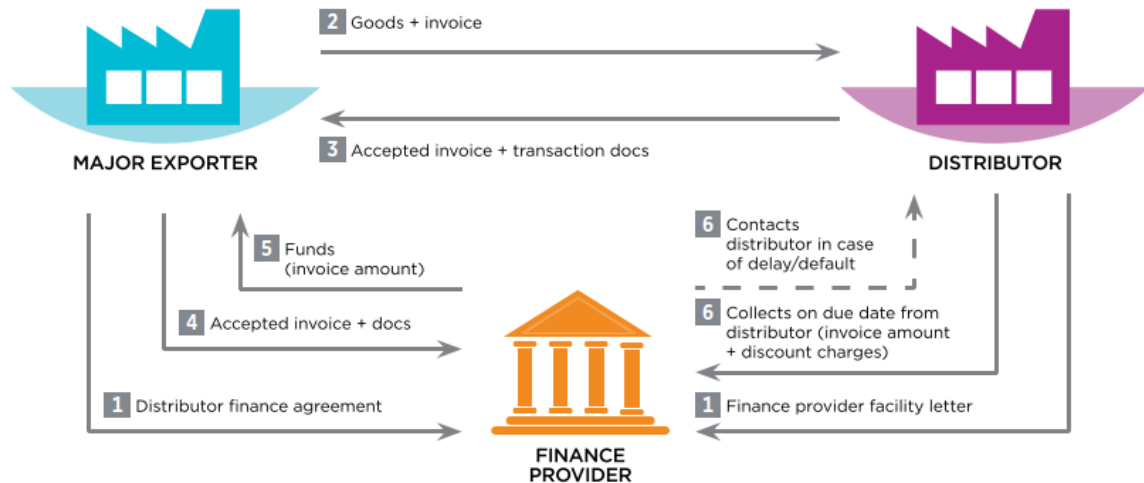
Figure 12. The scheme of distributor finance – Source: Global Supply Chain Forum (2016).

Pre-shipment finance (or purchase order finance) includes loans granted by a finance provider to a manufacturing or service company acting as a seller, to finance its operating cycle (purchase of raw materials and goods, transformation into semi-finished or finished products). Thus, the financing covers the seller's working-capital needs, following the issuance of a purchase order for goods and/or services by a client company (i.e., the buyer). The purchase order from high standing buyer in favor of the seller is the key element motivating the financing (in addition to the ability of the seller to fulfill delivery to its buyer). The seller and the finance provider sign a financing agreement detailing the terms of the loan structure. Sometimes a security agreement is also included, covering assignment of rights (transfer of title or a pledge) to the underlying work in progress and finished goods prior to shipment. The source of repayment usually comes from the flow of sales.
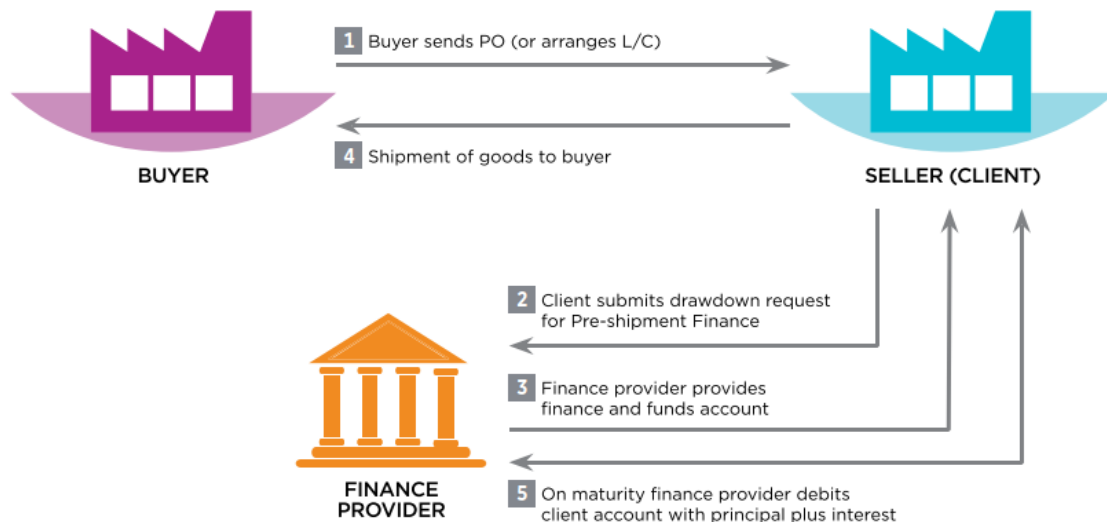
Figure 13. The architecture of pre-shipment finance – Source: Global Supply Chain Forum (2016)

4) The sources of risks in Supply Chain Finance

Risks in SCF can be distinguished in three main types: transaction flow risks, financial flow risks, and information flow risks.

Transaction flow risks involve the uncoordinated commodity flow and logistics flow of the parties involved in the supply chain. This type of risk affects:

- The procurement phase, in which supplier selection may be crucial. Choosing a single supplier often has a negative impact on the straight operations of the supply chain, as high concentration of supply can cause instability in procurement (Peck, 2005).

Relying on multiple suppliers gives firms more flexibility, allowing them to loosen the constraints on resource. At the same time this choice generates hidden costs. Levary (2007) points out that supplier selection requires a comprehensive consideration of supplier stability and reliability, national risks, and transport reliability. Besides supplier selection, risk factors to be considered in the procurement phase include lead time, supplied product monitoring, and control of supply capacity.

- The production phase in which supply chain operations may be negatively impacted by design risks for products and processes and the inability of firms to promptly update products or processes to answer to rapid market changes (Khan et al., 2008). These aspects increase the cost sustained by enterprises to position their products or processes in the market, exposing them to economic losses. Another important variable to consider in the production phase is production capacity, that is related to technology, skills, and quality capabilities (Korpela et al., 2002). Indeed, a shortage of resources and capabilities may prevent firms from carrying out normal production and operations. Moreover, the production and operations of an enterprise might not be carried out properly for either internal or external reasons (e.g., mismanagement, operational problems, economic downturn) that cause severe losses along the supply chain (Mizgier et al., 2015; Wagner et al., 2017).
- The distribution phase, one of the most important kinds of risk refer to excess and obsolete inventory resulting from fast changes in technologies or customer demand (Narayanan and Raman, 2004). In addition, the volatility or seasonality of demand is a key source of risk, that is likely to exacerbate uncertainty. The difficulties to determine the range and volumes of products or services to meet the needs of floating demand and markets, limit efficient and effective distribution (Wong and Hvolby, 2007).

Financial flow risks usually entail factors such as exchange rate risk, price risk and cost risk, and financial problems of supply chain partners. Exchange rate risk significantly affects firms that are involved in global supply chain operations, since exchange rate variations may directly influence after-tax profits, supplier selection, market development, and other related decisions. Exchange rate fluctuations may also cause increases in procurement costs of raw materials or input, as well as decrease in the price of firms' products (Papadakis, 2006; Christopher and Holweg, 2011).

Another important risk originates from the financial problems of partners in the supply chain, which may produce adverse consequences on upstream and downstream firms, which in turn may contaminate the entire supply chain (Tang and Nurmaya Musa, 2011).

Finally, risks in the information flow (Tang and Nurmaya Musa, 2011) mainly pertain to:
- The correctness of information. The efficient and effective functioning of the supply chain is based on the availability of several kinds of information that represent an important basis for decision-making, such as demand information, inventory information, order information, and production capacity information. If information is incorrect or not punctually available, the supply chain decision will be distorted. Information accuracy is linked to information accessibility, information efficiency, and data precision (Lee, 2002; Nishat Faisal et al., 2007). The failure of lead firms to build a smooth information sharing platform, the lack of transparency in each node of the supply chain, or the deliberate use of the supply chain by firms to disseminate false and poor-quality information in pursuit of their interests will damage the transmission and communication of information, resulting in a lack of reliability, integrity, and authenticity of information (Wu and Wu, 2022).
- The security and disruption of information systems that may arise from internal mismanagement, or hackers and natural disasters (Nishat Faisal et al., 2007), or application, firm-level, or cross-firm variables (Finch, 2004).
- Intellectual property rights. Intellectual property risks may result from the inability to protect the information shared due to dramatic increase in the amount of information in the supply chain, which also boosts the risk of data loss and confidential information leakage (Wu and Wu, 2022).
- Outsourcing data analysis and processing, which makes room to the opportunism of information service providers (Nishat Faisal et al., 2007).

Risks in SCF arise from two main sources: supply chain operations and moral hazard of the involved parties.

A physical supply chain involves production and distribution, and connects to wholesale, retail and end-users, relating to the whole value chain from raw material procurement to production, sales and end-users. Therefore, the first element that threatens the regular functioning of SCF is the disruption of supply chain operations, that negatively affects the possibility of completing the transactions and the future expected return of the whole supply chain. Thus, the transaction risk in the supply chain constitutes the financial risk in the supply chain (Sun, 2022).

The unstable states or uncertain resources that lead to supply chain disruptions refer to events that have a small probability of occurrence but may happen suddenly. Moreover, once these events happen, they produce a continuous harmful effect on the whole system (Tang and Nurmaya Musa, 2011).

The sources of risks affecting supply chain operations may be different: environmental factors, industry factors, and organizational factors (Rao and Goldsby, 2009). Environmental factors include macro-systemic variables (Miller, 1992; Wu and Wu, 2022) such as political stability, changes in government policies, macroeconomic uncertainty (e.g., fluctuations in bank interest rates or market prices due to an unstable economic environment), legal loopholes (due to inadequacy of laws and regulations), and natural uncertainty (e.g., natural disasters such as earthquakes, floods, and plagues). They are external events that affect the business scenario of each industry, or, rather, external factors that can negatively affect the industry (Ritchie and Marshall, 1993). Given their external nature, these risks are often more difficult for enterprises to control and manage. Differently from environmental variables, industry factors may not affect all industries of the economy, but specific sectors (Ritchie and Marshall, 1993).

These sources of risk regard either the uncertainty affecting the industry's production input factors, or unexpected variations in corporate or industrial demand (Miller, 1992), or competitive uncertainty. While input market uncertainty and product market uncertainty can arise both from changes in customer demand and the arrival of substitutes (Tang and Nurmaya Musa, 2011), competitive uncertainty is associated to market competition deriving from existing competitors and potential new entrants). Supply chain operations may be negatively influenced also by organizational features of enterprises (Ritchie and Marshall, 1993).

Despite their firm-specific nature, these variables may affect the entire supply chain. They include a wide array of uncertainties regarding:

a) corporate operations: events occurring to firms during the settlement of their supply chain activities e.g., changes in employee productivity, uncertainty in production input elements such as raw materials, components, and equipment failure (Rao & Goldsby, 2009). Uncertainty may also derive from the inadequate knowledge base of staff, as well as substandard business skills resulting in operational errors (Wu and Wu, 2022).

b) Outsourcing decisions: enterprises may outsource certain business processes to others, but outsourcing may increase the enterprises' vulnerability, making them lose control over those processes (Hallikas and Lintukangas, 2016; König and Spinler, 2016).

c) Liabilities: unexpected events related to the production or consumption of a firm's products (e.g., issues in product safety, or environmental hazards).

d) Inventory: market fluctuations (the longer the inventory is held, the more significant the impact of market price fluctuations it receives) and difficulty in preservation (due to the perishability of inventory and to the inability of the supervising company to match the various commodities with advanced preservation facilities for long-term storage) can affect the value of inventory.

e) Credits: the insolvency of a customer may cause liquidity pressures to an enterprise that, in turn, may delay payments to its suppliers. Business or financial problems of strategic customers or suppliers are likely to spread to the entire supply chain, since each node of the supply chain involves the exchange and operation of financial flows, and changes in the financial position of individual firms can pass on financial risk to other participants in the supply chain (Finch, 2004; Kleindorfer and Saad, 2005; Wu and Wu, 2022).

As mentioned before, SCF risks may also originate from opportunistic behaviors of the participants involved in a transaction or exchange, who violate the agreements in order to act in their own interest at the expense of others. Often, opportunism arises from ex ante and ex post information asymmetry.

Ex ante information asymmetry occurs before a transaction or a lending business and leads to adverse selection. As borrowers, SMEs may disclose positive information in their favor to lenders and hide negative information, to put themselves in a good light and obtain loans. In addition, ex ante information asymmetry may also refer to guarantees that are usually considered an effective means of risk mitigation. Financial institutions may fail to effectively reduce the risk of loss in case of the borrower's default due to the shortage of information about the true state or the quality of the guarantees. In practice, ex ante opportunistic behavior mainly includes fictitious trade, illegal contracts with multiple parties, and financing under false guarantees. Differently from other lending transactions, the lending decision in SCF is based on the transaction structure and trade relationship developed by upstream and downstream firms along the supply chain. Some participants may use fictitious trade to fraudulently obtain funds from financial institutions: they set up a false trade relationship with their related parties and forge a series of transaction contracts and documents, to recreate a substantial transaction flow (buying and selling) and logistics flow (provision of logistics services) among different participants or service providers. The aim of this set-up is either to allow the related parties to falsify operating income and profits, or to obtain low-cost financing. Another form of ex ante opportunism reflects in the borrower obtaining funds from multiple parties thanks to real transactions and assets that are exploited in a variety of occasions. Since there is almost no information sharing between different financial institutions and supply chain financial service providers, borrowers may use their transactions or logistics business or their certain assets to apply for lending from multiple entities to raise more funds. Another practice that is related to ex ante opportunistic behavior is financing under false guarantees. When guarantees and pledges are requested by lenders as a prerequisite for lending, firms may ask fictitious guarantors, usually relatives, friends, or closely related parties.

Ex post information asymmetry influences the ability of the lenders to monitor the borrowers throughout the loan life, which may lead to moral hazard. Borrowers may take advantage of the difficulty to fully oversee their behaviors, the true movement of funds as well as their actions, and they may misuse the funds received by the lenders. A common fraud phenomenon in SCF is the duplication of false warehouse receipts. Borrowing firms collude with warehousing enterprises or related personnel to deceitfully issue or repeatedly issue warehouse receipts for some goods to the borrowing firms who again and again pledge the warehouse receipts to different lenders, in order to achieve many loans.

5) Improving the management of supply chain risks through digitization

The full adoption of modern information and communication technologies allows to control the various factors underlying supply chain risks. First of all, digital technologies optimize operations across the entire supply chain by enabling connectivity, data management, insights, and smart automation. Second, digital technology can reduce risks stemming from both asymmetric information and supply chain disruptions.

Regarding risks stemming from incomplete and asymmetric information, the transaction information between the upstream and downstream of the supply chain can be incorporated into a unified information platform, thanks to the distributed ledger of blockchain. Thus, all the parties involved are able to access information and confirm its accuracy through consensus authentication. Enterprises with financing needs register their contracts, debts and other proofs on the chain, which prevent the tampering or copying of these elements after digitization. The automatic contract system allows to predefine the transaction procedure and complete the transaction process automatically after the transaction passes the consensus authentication, thereby improving the efficiency and security of supply chain management and reducing the moral hazard risk deriving from asymmetric information. Besides, should a dispute arise, it would be easy to rapidly solve it, since all information is open, transparent and traceable (Sun, 2022).

As for supply chain disruptions, digitalization can provide a contribution in several fields (McKinsey & Company, 2016), as illustrated by Figure 14:

- planning;
- physical flow;
- performance management;
- order management.

Big data, advanced analytics, and the automation of knowledge work play a significant role either in demand planning or, production planning, or inventory control. An accurate and granular demand plan can be achieved thanks to predictive analytics that analyze thousands of internal and external demand-influencing variables (e.g., weather, trends from social networks, sensor data) with Bayesian network and machine learning approaches. Thanks to these new technologies, the accuracy of demand forecast improves dramatically, as the forecasting error is reduced by 30 to 50%. Moreover, the use of closed loop manufacturing resource planning software enables plans to be continuously checked and adjusted. Exploiting an information feedback feature, the purchasing or materials procurement plans are synchronized with the master production schedule. In addition, depending on the stock levels, expected demand, and capability to replenish, prices can be dynamically adjusted to optimize the overall profit.

Warehousing operations can benefit from the use of modern technologies too. Lead times, transportation costs, product handling and environmental cost are the main areas than can be positively influenced by the adoption of autonomous and smart vehicles. For example, automated-guided vehicles are increasingly present as transportation solutions in industrial environments, as they ensure that automated storage and retrieval systems are agile and safe, while reducing load movement times. In particular, these vehicles these vehicles have a prefixed route that ensures the continuous, ensuring a uniform flow of goods. In addition, they allow 24-hour work cycles, since the automatization of the flow of goods allow operators to concentrate on higher value work. These vehicles also incorporate safety systems to optimally navigate the warehouse, avoiding interaction with operators an limiting the risk of accidents. Physical flows in the supply chain can also benefit from additive manufacturing. For example, 3-D printing has become much more relevant for a broad range of business applications, such as local production of slowly moving spare parts or tools (McKinsey & Company, 2016). The expanding range of printing materials, rapidly declining prices for the printers, and increased precision and quality boost the development of these tools.

As mentioned before, disruptions in supply chain operations are one of the main sources of SCF risks. Understanding the reasons for these events is therefore critically important. Root cause analysis is a process that allows the identifications of the causes for disruptions. A large number of data attributes go along with a given order e.g., the quantity of the item being ordered, its value, the plant that manufactures it, all events logged during the lifetime of the order, the supply levels for the item, the agreed delivery date, the order route, and many further factors. Causal discovery algorithms help mapping all cause-and-effect relationships among these variables, that is how these variables are interrelated and ultimately how they relate to the order being delayed or not. Once identified the root causes of an exception (e.g., on-hand inventory level falls below a minimum), the performance management system automatically triggers countermeasures, such as activating a replenishment order or changing parameter settings in the planning systems (e.g., safety stocks).

As for order management, it may be improved by no-touch order processing and real-time re-planning, which lead to lower costs through automation of efforts, higher reliability due to granular feedback, and superior customer experience through immediate and reliable responses (McKinsey & Company, 2016). No-touch order processing is the logical next step after implementing a reliable available-to-promise process (that provides a response to customer order inquiries, based on resource availability). No-touch order processing can be used to completely automate the ordering process, requiring no manual intervention between order intake and order confirmation. In addition to simplifying the ordering and sales process, touchless order processing integrates the business with the customer's needs, ensuring on-time delivery. In particular, this solution allows a drastic reduction of time to process orders, the elimination of human errors, a greater visibility of the entire process by the company, customers and suppliers, an increase in customer satisfaction, and a growth in productivity. The second technology-driven element that can positively impact on order management is real-time re-planning. The production process requires a re-planning to handle unexpected events occurring in the day-to-day operations of a production plant (i.e., disruptions to schedules, such as last-minute orders, out-of-stock raw materials, equipment failures) that can cause production or capacity problems along the supply chain. Internet of things (IoT) made real-time data available on equipment performance and analyze it in a few minutes to predict when a part is going to fail before it does (predictive maintenance).
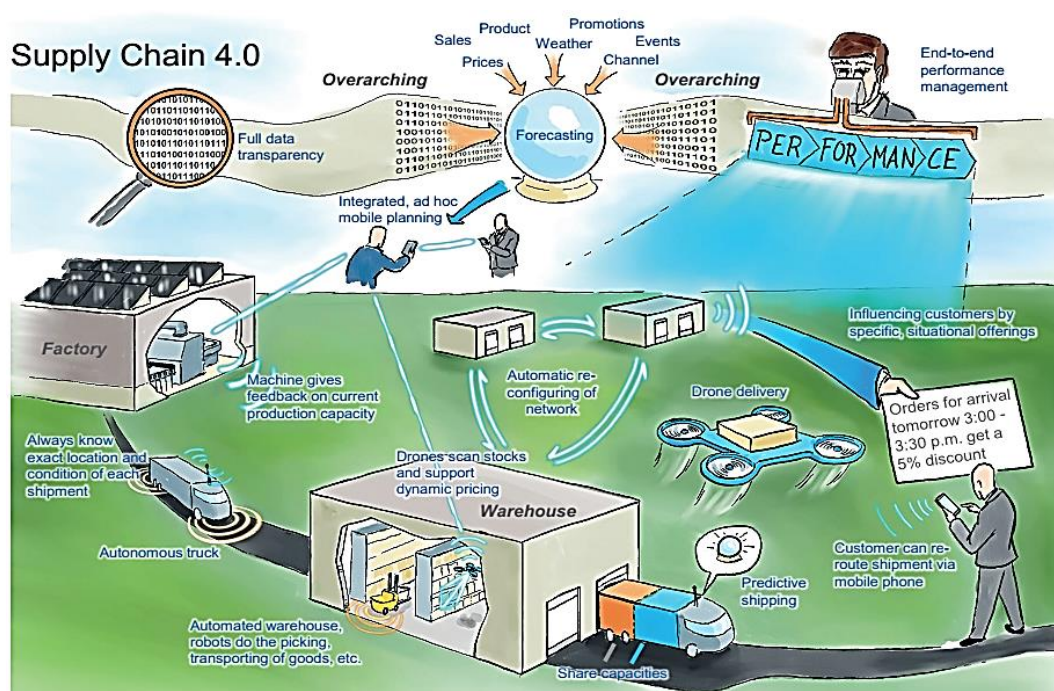

Figure 14. The components of Digital Supply Chain – Source: McKinsey & company (2016).

According to a recent survey published by Capgemini (2023), global supply chain bottlenecks and trade/ logistical disruptions are perceived to be the biggest risk to growth in the next 12–18 months. Coherently, 43% of executives across sectors including life sciences, retail, industrial manufacturing, and consumer products, say that the principal focus areas for investment by their organizations will be in supply chain technologies (enabling agility/transparency/visibility of supply chains) and diversification (of supplier base, production, and transportation partners). Nevertheless, the acquisition of information and digital technologies is not sufficient to improve supply chain risk management. Underlying processes and working methods of people need to be reviewed or redesigned, making use of the support of competent partners. Another fundamental element to carry out a successful digitization project is the ability to find and integrate within the organization the vertical skills relating to the specific technologies that are required to guide the transformation process.

Conclusions

Each SCF techniques shows its own peculiarities, depending on the financial needs of the parties involved and the stage of the supply chain in which they are settled. However, they are all exposed to several risk sources. While traditional forms of financing expose lenders to risks deriving above all from the borrower's features, besides factors related to macro-economic conditions, the functioning of SCF solutions can be negatively affected by supply chain disruptions and exceptions. Risk sources include a wide variety of uncertainties regarding corporate operations, outsourcing decisions, liabilities, inventory, and credits. In addition, SCF risks may originate from opportunistic behaviors of the participants involved in a transaction, who infringe the agreements in order to act in their own interest to the detriment of others. All these factors lead to serious troubles regarding the commodity and logistics flow along the supply chain, as well as the correctness/availability/security of information and the solvency of supply chain partners. The use of digital technologies can help managing the various factors underlying supply chain risks. As far as supply chain disruptions are concerned, the contribution of supply chain digitization is manifold as it can improve demand, inventory and production planning, physical flow, performance management, and order management. With regard to risks arising from incomplete and asymmetric information, the distributed ledger of blockchain can ease information access by all the parties involved in SCF, through the incorporation of transaction information between the upstream and downstream of the supply chain into a unified information platform.

Several organizational and strategic challenges remain to overcome to foster the success of the digital supply chain implementation. On the one hand, there must be a deep knowledge of the available technologies, which are fundamental for innovation. On the other hand, it is crucial to identify a cultural and organizational transformation strategy, establishing new processes and working methods in enterprises.

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Implementation of variance reduction techniques applied to the pricing of investment certificates

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Abstract

Certificates are structured financial instruments that aim to provide investors with investment solutions tailored to their needs. Certificates can be modeled using a bond component and a derivative component, typically an options strategy. The pricing of certificates is typically performed using the Monte Carlo numerical methodology. Such method allows for projections of the underlying using series of random numbers. The results obtained display an error (standard deviation) that depends on the number of simulations used and on the specific characteristics of the structured product. This work has the objective of minimizing the experimental error, and, consequently, of accelerating the speed of convergence using statistical techniques known in the literature as variance reduction methods. The most popular stochastic dynamics have been analyzed, like the classical Black and Scholes model, the Local Volatility model and the Heston model. Three certificates are analyzed in the paper and they are characterized by different payoffs. The variance reduction techniques, implemented in different programming languages (Python, Matlab and R), are: Latin Hypercube, Stratified Sampling, Antithetic Variables, Importance Sampling, Moment Matching and Control Variates.

1) Introduction

This introductory section aims to highlight the innovative nature of the application of variance reduction techniques to instruments of a hybrid nature, such as investment certificates, through a literature review. The implementation of this category of statistical techniques especially connected with the Monte Carlo method is certainly not new in quantitative finance. Indeed, these approaches are considered important as they are able to improve the performance of a numerical scheme that allows the integration of a stochastic differential equation aimed at solving problems connected with the determination of the fair value and the estimation of the related sensitivity measures (typically the Greeks).

Most studies concentrate on the quantitative analysis of the most common path-dependent exotic options: Asian, barrier and lookback (Avramidis and L'Ecuyer, 2006). In particular, focusing on options whose pay-off depends on the average of the underlying values assumed during the life of the derivative, i.e. Asian-type options, several studies apply the variance reduction techniques to different types of stochastic processes always aimed at valuating the derivative.

By way of example, Zhao, Liu and Gu (2013) implement the Importance Sampling technique for an Asian call option based on the arithmetic mean by adopting a traditional Geometric Brownian Motion to simulate the forward projections of the spot level.

As highlighted by Moretto, Pasquali and Trivellato (2017), the importance of extending these methodologies also to contexts where stochastic dynamics can improve the modeling of the phenomenon of fat tails is extremely important.

Indeed, the Importance Sampling technique has been used for pricing Asian options whose pay-off was grounded both on the geometric and arithmetic mean of the underlying, together with a more complex stochastic dynamics used for modeling the prospective values of the underlying based on a stochastic process dedicated to describing the dynamics assumed by variance (Fouque and Han, 2004). Dingeç, Sak and Hörmann (2014) instead proposed a Control Variates approach applied to a conditional Monte Carlo and showed its efficiency with a huge number of different stochastic processes implemented for the projection. However, not only first generation Asian options have been considered in the previous literature; for example Giribone and Ligato (2013) used a combination of variance reduction methods (Importance Sampling, Control Variates and Latin Hypercube) able to significantly reduce the simulation error committed on a 2-asset Asian spread put.

Barrier options have been studied in detail by Hieber and Scherer (2010), who suggested to use Antithetical Variables and Control Variates in Monte Carlo simulations for pricing barrier options in a Markov-switching model, while Zhang (2020) and Ameur, L'Ecuyer and Lemieux (1999) provide interesting insights on the application of variance reduction techniques to the pricing of lookback options. Another widespread application of variance reduction techniques is related to the pricing of American options, i.e. derivatives with an early exercise feature. For example, Boire, Reesor and Stentoft (2021a-b) have experimented an extremely efficient usage of Antithetical Variables, Control Variates and Importance Sampling to this end. Moreover, Lemieux and La (2005) showed how a Randomized Quasi-Monte Carlo can be successfully integrated with the algorithm of Longstaff and Schwartz (2001), which is the most widespread regression-based technique for estimating early-exercise in an engine based on numerical integration of stochastic differential equations.

Finally, Areal, Rodrigues and Armada (2008) have extended previous studies by combining different regression methods and different variance reduction techniques, aiming at further improving the numerical results of the previous Randomized Quasi-Monte Carlo implementation.

Within the different types of regressive methods that can be combined with the Longstaff and Schwartz approach, machine learning was also taken into consideration by Goudenège, Molent and Zanette (2021) that show how Control Variates can also be applied in this statistical context.

The literature also includes interesting examples of application of variance reduction techniques for the estimation of the main risk measures of financial portfolios, like Value-at-Risk (Glasserman, Heidelberger and Shahabuddin, 2000), Expected Shortfall (Xing, Sit and Wong, 2022) and fat tails (Hsieh *et al.*, 2019).

To the best of our knowledge, scientific studies that apply variance reduction techniques to certificates are practically absent so that this work can be considered original from this perspective.

Moreover, this study can be of great practical value for the Italian financial industry. To better understand its practical importance, in Section 2 we analyze the Italian market of certificates on the basis of the statistics compiled on a quarterly basis by ACEPI, the Italian Association of Certificates and Investment Products and we review the most widespread investment certificates, the different product features and trading venues.

In the following sections we discuss the application of variance reduction techniques to the quantitative analysis of such financial instruments. A "bottom-up" discussion is proposed where we start from discussing the problem that need to be solved (description of the product to be analyzed) and then we deal with the pricing model which is usually adopted by financial market participants (i.e. the as-is solution for the problem).

We present the best practice for estimating the fair value of the observed financial instrument and we proceed to implement the statistical methods which allow an increase in the convergence performance in terms of dispersion respect to the expected value (i.e. improvement of the current solution). This methodological approach is adopted for the analysis of three different structured products that display increasing levels of difficulty in their pricing.

Section 3 provides a comprehensive quantitative analysis for "Banca Ponti Certificate Protection Cap Euro Stoxx 50 25/12/2026" (ISIN: IT0005468142). The pricing of this structured product is relatively simple, since the optional component can be modeled as a long position in a bull call spread at maturity, thus the theoretical quotation was tackled with a traditional Black-Scholes pricing framework.

The choice of this dynamic is motivated by the fact that the pay-off can be replicated with a short and a long position of a call option written on the underlying index and an expiration date equal to the maturity date of the certificate. Therefore an analyst can trade the two vanilla options separately on the financial markets, typically making use of a closed-formula valuation model (for example, a standard pricing module such as Bloomberg® OVME).

The possibility of replicating the Monte Carlo method with a closed formula derived by the analytical resolution of the fundamental Black-Scholes-Merton PDE, allows to check the correct theoretical convergence to the fair-value of the implemented techniques. This validation improves the confidence of correctly pricing more complex certificates that cannot be priced using an analytical closed-form approach.

Section 4 deals with the "Banca Ponti Certificate Protection Cap on FTSE MIB 08/03/2027" (ISIN: IT0005481442). Unlike the previous product, in this case, the option component consists in a strip of conditional amounts with the so-called memory effect, while the invested capital is fully protected. One of the reference pricing techniques for this category of certificates is the Local Volatility model and the pricing problem cannot be analytically handled in closed form due to the memory effect. We therefore proceed to verify which variance reduction techniques are best suited to improve the performance of this Monte Carlo simulator.

The last section considers another feature which is quite common in certificates, namely the "autocallability". The analyzed product is the "Phoenix reverse convertible linked to the FTSEMIB Index" issued by JP Morgan (ISIN: XS2168930340).

In this case the reference market model for the valuation is the Heston model. As in the previous cases, the pricing of the instrument is presented using a standard Monte Carlo method (known in jargon as "Crude Monte Carlo"); then the analysis shows that the performance can be optimized thanks to the code of a Randomized Monte Carlo.

In order not to burden the discussion, the working principles of the variance reduction techniques implemented in the paper are briefly reported in the appendix: Antithetic Variables (A.1), Stratified Sampling (A.2), Latin Hypercube (A.3), Control Variates (A.4), Moment Matching (A.5), Importance Sampling (A.6) and Randomized Quasi-Monte Carlo (A.7).

2) An overview of investment certificates and the Italian market

Investment certificates are financial instruments with different risk/return profiles that allow for the identification of financial solutions aligned with the needs of the investor (ACEPI, 2022).

Certificates are securitized derivatives, essentially a combination of financial contracts incorporated in a security, negotiable in the same way as a share.

They are usually issued by bank issuers in the form of notes (i.e., short-term bonds) and they are subject to issuer risk, a risk associated with the default of the obligations of the issuer in relation to the product, consistently with the contents of the prospectus.Certificates have a significant derivative component; therefore, their value is connected to the performance of an underlying, which can be a share, an index or a basket of them (Oliveri, 2018).

The types of certificates differ according to:

- Directionality of the underlying (there may be an exposure to the downside or upside of the asset).

- Distribution of payment flows during the life of the financial instrument (additional amounts).

- Possibility of early repayment with payment of a premium to the investor (Autocallable).

- Capital protection which can be total, partial or conditional.

- Repayment at maturity.

By accessing this type of investment, the investor renounces the dividends distributed by the underlying in order to benefit from the conditions defined in the derivative contract.

Specifically, there are three macro types of certificates: Investments Certificates (for investment purposes), Leverage Certificates (for trading or risk hedging purposes) and finally Credit Linked Notes.

There are also three sub-categories of Investment Certificates which are: Protected Capital, Conditionally Protected Capital and Unprotected Capital (ACEPI, 2022).

The protected capital certificates can be divided into:

- Equity protection: they can be long (i.e. the holder makes a profit if the underlying goes up) or short (i.e. the holder makes a profit if the underlying goes down) to be able to participate in the increases or decreases in the price of the underlying. However they protect the issue price or a percentage of it, at maturity.

- Digital: they allow for additional earnings if, on the observation dates, the underlying quotation is higher or lower than the initial quotation.

Conditionally protected capital certificates can be classified into:

- Bonus: they allow to participate in increases and decreases in the price of the underlying, also providing an additional income when certain conditions occur.

- Express: they can be subject to early maturity /reimbursement at issue price plus a premium, if the price of the underlying is greater than or equal to an initially pre-set level on the observation dates (this type is also called Autocallable).

- Twin win: there is a barrier level beyond which the capital is no longer protected. These instruments offer the possibility of a positive return both in the event of a rise and a slight fall in the underlying.

- Credit linked: they allow to obtain recurring bonuses conditional on the "non-default" status of one or more reference entities.

- Accelerator: they allow to participate in the increase or decrease of the performance of the underlying in a more than proportional way. In case of loss there is usually an "Airbag" factor. The Airbag is a factor (ratio between the final and initial value of the underlying) which is multiplied by the price at maturity (in the event that the underlying is below the floor barrier at maturity).

- Premium Certificate: they pay certain and fixed premiums during the life of the instrument regardless of the performance of the underlying and a premium at maturity conditional to the performance of the underlying. However, there is a barrier level beyond which the capital is no longer protected.

- Cash Collect: they have the peculiarity of distributing fixed premiums and conditional additional premiums, according to the performance of the underlying.

Certificates with unprotected capital, on the other hand, do not have sub-categories but have specific characteristics, they replicate the performance of the benchmark and can have more or less proportional fluctuations.
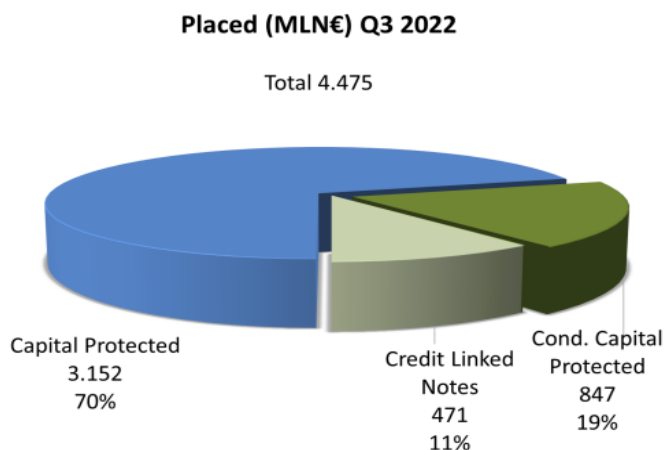
Placed (MLN€) Q3 2022

Total 4.475

Figure 1: Pie chart of certificates placed in the third quarter of 2022 - Source: Acepi (ITForum 16 June 2022)

Placed (MLN€) Q1 2019 - Q3 2022

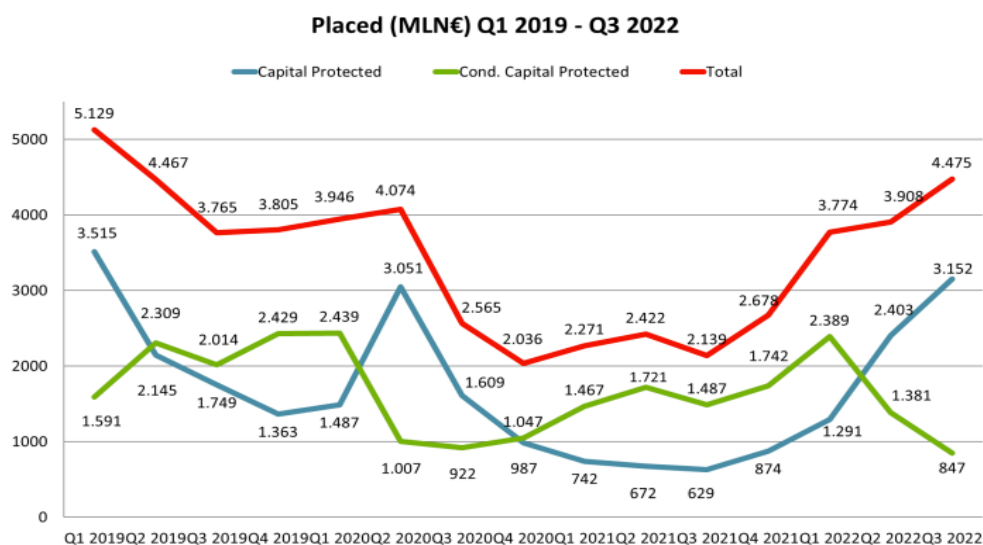Legend: Capital Protected — Cond. Capital Protected — Total

Figure 2: Development of certificates in years 2019-2022. Source: Acepi (ITForum 16 June 2022)

The graphs in Figures 1 and 2 are shown in order to explain that investors have constantly been looking for new investment solutions in recent years, which certificates are able to satisfy, as can be seen from the most recent placements (only a slight decrease can be seen in correspondence of 2020 due to the COVID-19 pandemic). Certificates can be traded on regulated markets, such as SeDex (Securities Derivative Exchange or Electronic Securitized Derivatives Market) or EuroTLX. This allows the instrument to be reasonably liquid and consequently to have systematic purchase and sale offers entered by an authorized operator.

The SeDex market is the market managed by Borsa Italiana on which Covered Warrants and Certificates are traded. As reported on the official Borsa Italiana website (borsaitaliana.it) this market is segmented as follows:

- "plain vanilla covered warrant segment" (covered warrants consisting of a call or a put option).

- "structured/exotic covered warrant segment" (covered warrants which are combinations of call and/or put options or which incorporate exotic options).

- "leverage certificates segment" (certificates that replicate the performance of the underlying asset, with a leverage effect).

- "investment certificates segment" (certificates which replicate the performance of the underlying asset without leverage effect, or which replicate the performance of the underlying asset and also include one or more ancillary structured or exotic options).

This market was created in 2004 as a segment dedicated to the negotiation of Certificates and Covered Warrants which are overall defined as securitized derivatives. The first covered warrant was listed on the stock market in 1998. The success of these instruments soon led to the listing of several thousand instruments and the creation of the Covered Warrant Market (MCW). Since the early years, alongside the covered warrants, the first certificates have also started to be traded. Over the years, the number of certificates and the related traded volumes have progressively been growing, as the variety of types listed.

For this reason, in 2004 Borsa Italiana replaced the MCW market with SeDeX. The opening hours of this market go from 9:05 am to 5:30 pm, Monday to Friday.

Investment Certificates traded on the SeDeX market segment of Borsa Italiana belong to two classes:

- Class A: they linearly replicate the performance of the underlying (so-called benchmark certificates).

- Class B: they replicate the performance of the underlying and include one or more structured options to provide further benefits (for example, total or partial capital protection with Protection Certificates).

For the purposes of admission to the SeDex market, the issuer must provide the following reports as indicated in the official regulation provided by Borsa Italiana:

- a prospectus approved by the competent authority in accordance with the Prospectus legislation.

- a base prospectus approved by the competent authority in accordance with the Prospectus legislation, supplemented by the final terms.

- a registration document accompanied by the securities note on the financial instruments approved by the competent authority in compliance with the Prospectus legislation or, alternatively, accompanied by the KID required by Regulation 2014/1286.

- an admission document containing the information referred to attachments 6 or 7 and 14 or 15 of the EU Delegated Regulation 2019/980. In any case, this information must be sufficient to allow investors to make an informed investment decision on the financial instruments and it must contain any other information that Borsa Italiana may request to facilitate the orderly conduct of trading.

The focus on this type of financial instrument is due to the fact that, in a context of very low interest rates and low returns, investors are looking for a solution that can provide a suitable return and a protection from strong market fluctuations.

In particular, we will analyze certificates that provide investors with total protection or partial, but still high, protection.

The first certificate, for example, has a protection of 90% (section 3) of the amount initially invested while the second has total protection i.e., 100% (section 4).

In addition to the level of protection, investors can obtain additional amounts, conditional or not, depending on the specific characteristics of the product (section 5). We believe one of the main contributions of this study is the application of variance reduction techniques to the pricing of investment certificates as these techniques are typically only applied to individual options.

3) Analysis of the certificate - Banca Ponti Certificate Protection Cap Euro Stoxx 50 28/12/2026

The 90% protected certificate with a 115% cap is a partially protected capital "Investment Certificate" (ISIN: IT0005468142). This type of certificate allows the investor to protect his capital up to 90% therefore in the worst case he will obtain 90% of the invested capital at maturity.

Conversely, if the underlying outperforms, the customer will receive up to a maximum of 115% of the initial investment (this is due to the presence of the cap).

In addition to the capital at maturity, conditional to the performance of the underlying, the certificate distributes an unconditional additional payment of 1.5% of the nominal value on pre-established dates.

The valuation of the certificate is done on 31/03/2022, the issue date was 20/12/2021 while the expiry date is 28/12/2026. Table 1 shows the characteristics of the financial instrument obtained from the KID (Key Information Document). This financial product (as well as the subsequent case studies), can only be offered to a retail customer which has the following characteristics:

- Experience and knowledge of financial markets.

- Ability to bear capital losses.

- A high risk tolerance (equal to or higher than the risk indicator).

- Time horizon of the investment, at least medium term.

The payment dates of the unconditional amount are fixed in advance and indicated in Table 1. The minimum amount for buying this certificate is 1,000 euros and such minimum amount represents one unit.

The underlying index on which the performance of the certificate is conditioned is the EURO STOXX 50 (which represents the stock index including the 50 major Eurozone companies in the main industrial sectors). The strike price of the index, i.e., the 100% threshold, is set on 17/12/2021.

The costs incurred, useful for pricing the instrument, are: 2.50% for placement commissions and 1.5% for structuring costs. The following section highlights the pricing method (Monte Carlo technique) and subsequently the payoff, the Greeks and the variance reduction techniques will be analyzed.

Certificates Characteristics	
Underlying	EURO STOXX 50 index
Initial Valuation Date	17/12/2021
Initial Reference Value of the underlying	Closing Price of the underlying at the Initial Valuation Date
Issuing Date	20/12/2021
Maturity Date	28/12/2026
Nominal Value	1,000 EUR
Protection	90% of Nominal Value
Cap Level	115% of Nominal Value
Unconditional Amount	1.5% of Nominal Value
Payment Dates of the Unconditional Amount	21/12/2022, 21/12/2023, 23/12/2024, 22/12/2025, 28/12/2026
Final Valuation Date	21/12/2026
Final Reference Value of the underlying	Closing Price of the underlying at the Final Valuation Date

Table 1: Financial characteristics of the certificate retrieved from the KID - Source: Banca Cesare Ponti – Gruppo BPER

3.1) The Monte Carlo methodology and the Black-Scholes-Merton pricing framework

The value of a derivative, being closely linked to the performance of the underlying financial asset $S(t)$, depends on the possible trajectories that it will take in the future (Hunt & Kennedy, 2001). One of the most widespread stochastic processes for this purpose is the Geometric Brownian Motion, which, consistently with the Black-Scholes-Merton pricing framework, is represented by the following Stochastic Differential Equation (SDE):

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \quad (1)$$

Where:

μ = annualized expected return earned by an investor over the time period dt. In the pricing of options on equity or stock indexes it is usually set as: $r - q$, where r is the risk-free rate and q is the continuous dividend yield of the underlying.

σ = the annualized volatility of the asset. When vanilla option prices are available, the implicit volatility is usually used, otherwise a historical estimate is made.

dW_t = Wiener stochastic process.

The SDE can be numerically integrated through the Euler-Maruyama scheme and therefore implemented using a numerical processing software (in this study three languages are mostly used: Python, Matlab and R):

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \rightarrow \Delta S \simeq \mu S \Delta t + \sigma S \Delta W \ (2)$$

$$S_t \simeq S_{t-1} + \mu S_{t-1}\Delta t + \sigma S_{t-1}\epsilon\sqrt{\Delta t} \ (3)$$

where ϵ represents an extraction from a standard normal distribution and Δt the discretization interval (Kloeden & Platen, 1992). A Wiener process is a particular type of stochastic Markov process, used to model the Brownian motion of various random phenomena. Formally, $W(t)$ follows a Wiener process if it satisfies the following properties:

- $W(0) = 0$,

- $W(t)$ is (almost surely) continuous,

- random variable $W(t_2) - W(t_1)$, with $0 \le t_1 \le t_2$ is normally distributed with zero mean and standard deviation $t_2 - t_1$ and

- random variables $W(t_2) - W(t_1)$ and $W(t_4) - W(t_3)$ with $t_1 < t_2 < t_3 < t_4$ are independent random variables.

In our paper, the Monte Carlo engine is used with simulation brackets that correspond to a letter: A indicates a thousand simulations, B represents ten thousand simulations, C stands for a hundred thousand simulations and finally D stands for a million.

For each number of simulations indicated, 200 replications are made in order to extrapolate the great mean, which corresponds to the fair value of the structured product, the standard deviation (used as a measure of precision and for comparison purposes related to the improvement hopefully introduced by the variance reduction techniques) and the computing time (useful for understanding the "cost" in computational terms of increasing the number of simulations or the methodology).

For pricing purposes, the certificate can be broken down into two parts: the former concerns the flows of unconditional additional amounts and the latter regards the options strategy at maturity (bull call spread).

The calculation of the unconditional amounts was implemented discounting the future additional amounts, while the call-spread value at maturity was estimated simulating the value of the underlying using the Monte Carlo engine (Boyle, 1977).

Assuming that the notional is equal to 1,000 (minimum amount that can be purchased), the three different cases at maturity are analyzed:

- If the simulated underlying is less than 90% of the strike price at maturity, then the certificate will pay 900. The exercise price is the initial reference value of the index, on 17/12/2021, equal to EUR 4,161.35.

- If the simulated underlying is between 90% and 115% of the strike, then it will pay the corresponding value between EUR 900 and EUR 1,150.

- Finally, if the underlying is greater than 115% of the strike price, then the final redemption will be EUR 1,150.

Once the two components have been obtained, they are added together. The prices obtained on the valuation date 31/03/2022 using Bloomberg® market data are shown in Table 2:

	Mean	SD	Time
A4	923.901	2.89307618	0.11
B4	923.6242	0.96398002	0.41
C4	923.6088	0.28511702	4.22
D4	923.6304	0.08170546	52.11

Table 2: Pricing of certificate IT0005468142 with Crude Monte Carlo technique

The values obtained are in line with the market quotation observed on the Sedex on 31/03/2022 (923.75 – Source: Borsaitaliana.it) and also with the fair value of the hedge estimated using the traditional Black-Scholes-Merton closed formula for the long position in the call with the lower strike price and for the short position in the call with the higher strike price (Black & Scholes, 1973).

In order to complete the analysis, the price sensitivity measures were calculated with respect to the main inputs of the valuation model (Tables 3, 4 and 5):

Delta: $\Delta = \frac{\partial price}{\partial S}$, Vega: $\nu = \frac{\partial price}{\partial \sigma}$, Rho: $\rho = \frac{\partial price}{\partial r}$, Theta: $\theta = \frac{\partial price}{\partial T}$, Gamma: $\gamma = \frac{\partial^2 price}{\partial S^2} = \frac{\partial \Delta}{\partial S}$

	Mean	SD	Time
A4	0.004707454	3.14E-04	0.11
B4	0.004685893	8.70E-05	0.58
C4	0.004683118	3.01E-05	5.72
D4	0.00468321	9.00E-06	110.45

	Mean	SD	Time
A4	6.364351	0.62309148	0.1
B4	6.356778	0.1973578	0.88
C4	6.350574	0.06623772	9.06
D4	6.345232	0.01756761	115.8

Table 3: Numerical estimation of Delta (left) and Vega (right) with the 2-sided Finite Difference Method for certificate IT0005468142 with Crude Monte Carlo technique

	Mean	SD	Time
A4	443.3628	37.99339	0.09
B4	442.0137	11.707837	0.9
C4	441.593	3.987575	9.28
D4	441.3124	1.052667	104.52

	Mean	SD	Time
A4	0.09256348	0.007277801	0.11
B4	0.09184981	0.002338103	0.96
C4	0.09187552	0.000724441	9.39
D4	0.09193785	0.00023892	98.17

Table 4: Numerical estimation of Rho (left) and Theta (right) using the Finite Difference Method, respectively 2-sided and 1-sided, for certificate IT0005468142 with Crude Monte Carlo technique

	Mean	SD	Time
A4	2.24E-07	1.92E-06	0.17
B4	4.29E-07	6.25E-07	1.35
C4	4.19E-07	2.01E-07	13.11
D4	4.38E-07	6.07E-08	156.95

Table 5: Numerical estimation of Gamma with the Finite Difference Method for certificate IT0005468142 with Crude Monte Carlo technique

The estimated experimental error for a Monte Carlo simulation is given by the standard deviation of the results: a small value implies a more accurate convergence to the expected value. The stochastic process of the variables underlying a derivative is replicated implementing a huge number of simulations in order to estimate the value of the financial instrument with a certain level of accuracy. The greater the number of simulations, the greater the accuracy accompanied by a significant increase in computing time. In this context, the variance reduction techniques can be used to improve the accuracy of the Monte Carlo engine results and to also save on computing time (Giribone & Ligato, 2013). The most popular techniques in the literature are: Antithetic Variables, Stratified Sampling, Latin Hypercube, Control Variates, Moment Matching and Importance Sampling (Botev & Ridder, 2017). A short summary of their working principles can be found in the Appendix.

3.2) Implementation of variance reduction techniques in the first certificate

This section shows the results of the variance reduction techniques applied to the first certificate (ISIN:IT0005468142). In addition to the tables displaying the results, the graphs are also shown in order to illustrate that the Monte Carlo engine reaches convergence at the theoretical price of the Black and Scholes closed formula in a different way between the Crude Monte Carlo and the one improved by the various reduction techniques implemented.

Tables: First Certificate, Variance Reduction Techniques

	Mean	SD	Time
LHS_A4	923.6255	4.68E-03	0.15
LHS_B4	923.6246	1.37E-04	1.28
LHS_C4	923.6246	4.50E-06	13.26
LHS_D4	923.6246	1.42E-07	89.4

Table 6: Latin Hypercube

	Mean	SD	Time
AV_A4	923.5843	2.03215726	0.05
AV_B4	923.679	0.61434496	0.44
AV_C4	923.6512	0.21246318	4.17
AV_D4	923.6261	0.06487431	46.37

Table 7: Antithetic Variables

	Mean	SD	Time
SS_A4	923.6742	0.41395448	0.08
SS_B4	923.6265	0.13254349	0.52
SS_C4	923.627	0.0447108	4.94
SS_D4	923.6259	0.01270018	48.74

Table 8: Stratified Sampling

	Mean	SD	Time
MM_A4	923.752	1.35238255	0.27
MM_B4	923.6802	0.51091336	0.51
MM_C4	923.6519	0.151696	5.05
MM_D4	923.6224	0.04911662	59.57

Table 9: Moment Matching

All tables showing reduction techniques results display a reduction in standard deviation. In particular, for the first certificate, the technique that largely outperforms is the Latin Hypercube Sampling.

It is also clear from the tables that there is an inverse relation between the computational time and the reduction: the designer has to find the optimal trade-off between these two quantities.

In our study, the increase in the implementation times using the variance reduction techniques is more than compensated by the significant reduction of the standard deviation.

The graphs for the variance reduction techniques are shown below (Figure 3) and refer to the long position of the hedging call: the prices of the Crude Monte Carlo are shown in green, while those of the reduction techniques are shown in blue.

The horizontal red line indicates the theoretical price of the Call obtained with the Black and Scholes closed formula and equal to 529.70.

The y-axis indicates the prices of the Long Call while the x-axis indicates the number of simulations performed:

from 0 to 200, prices are based on 10^3 simulations (Case A),

from 200 to 400, on 10^4 simulations (Case B),

from 400 to 600, on 10^5 simulations (Case C),

from 600 to 800, on 10^6 simulations (Case D).
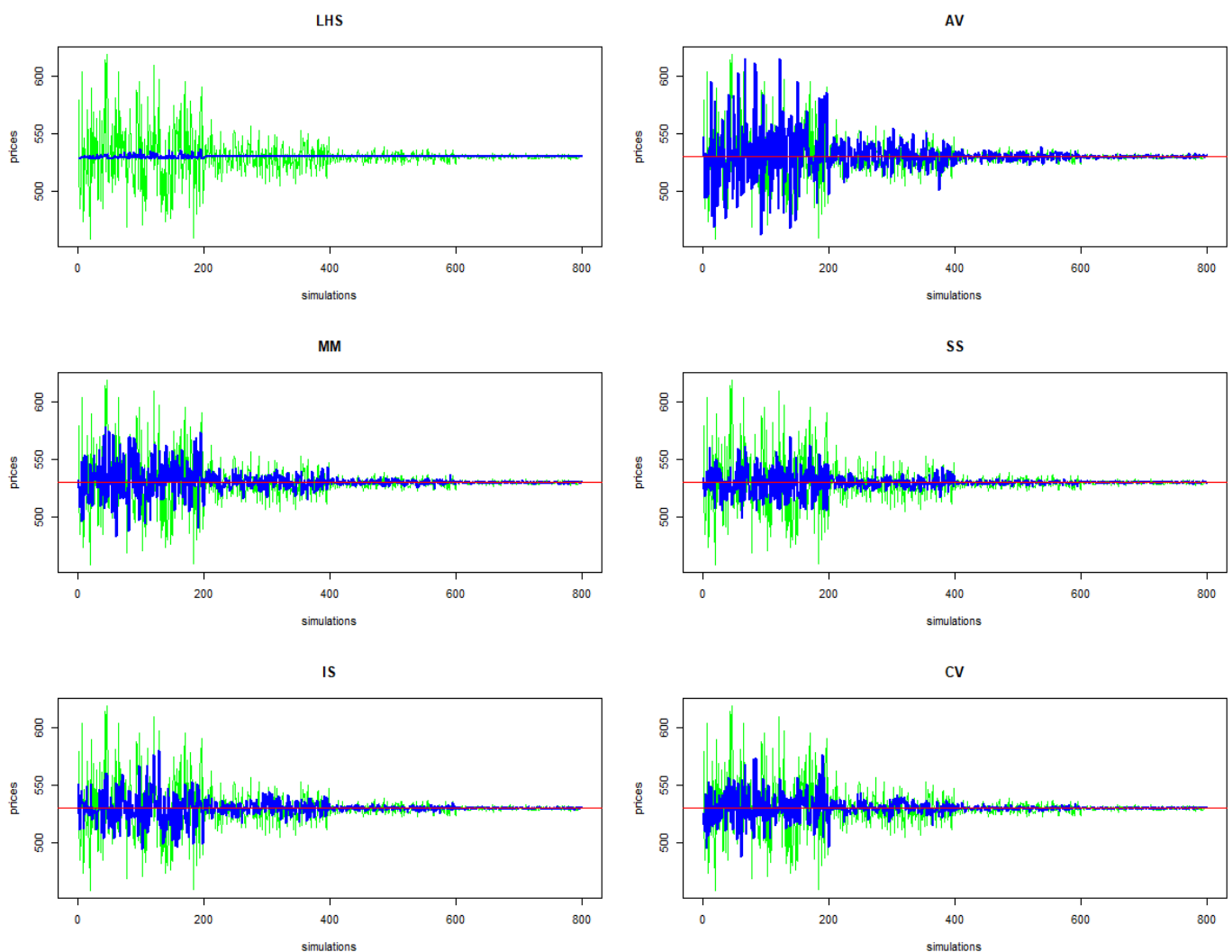
Figure 3: Latin hypercube (LHS), Antithetic Variables (AV), Moment Matching (MM), Stratified Sampling (SS), Importance Sampling (IS) and Control Variates (CV)

The graphs relating to the pricing of the entire certificate are shown below (Figure 4). In this case, the red line indicates the theoretical price of the financial instrument, equal to 923.62.

The Latin Hypercube, Moment Matching, Antithetic Variables and Stratified Sampling are the applicable reduction techniques.

The implemented techniques have all performed well but the Latin Hypercube has decisively and significantly reduced the standard deviation.

In Figure 4 representing the pricing of the certificate, the trend is not clearly visible due to the very low standard deviation (the price converges already from the very first simulations).

In conclusion, the reduction technique that showed the best performance was the Latin Hypercube Sampling that drastically reduced the standard deviation.

Figure 5 shows how the prices are distributed in the Monte Carlo Crude, where the blue line is the distribution of the crude and the red line represents the distribution of the Latin Hypercube. In the enlargement in Figure 6, the variance reduction technique performs excellently, and is characterized by an infinitesimal standard deviation.

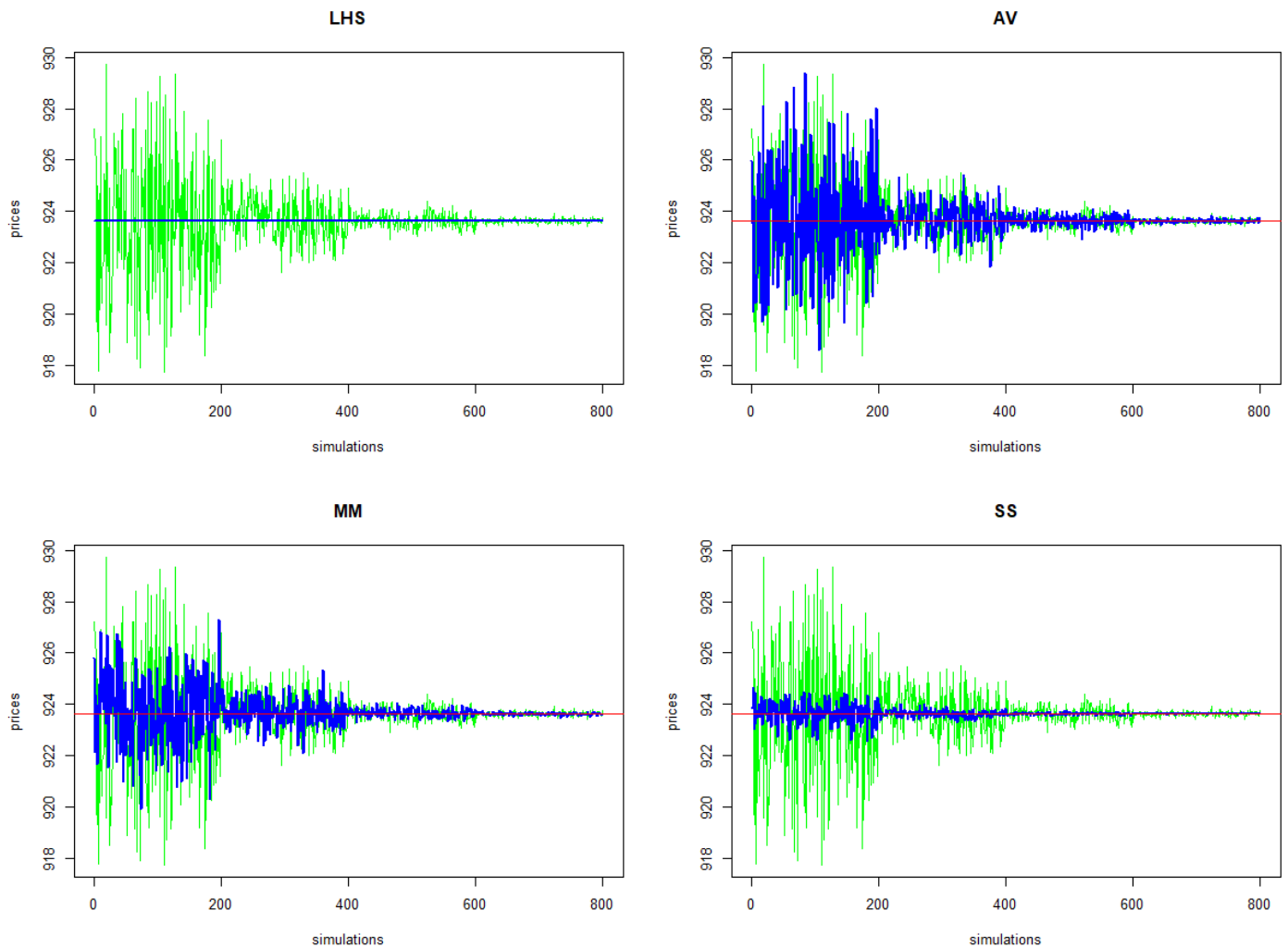*Figure 4: Convergences of Latin hypercube (LHS), Antithetic Variables (AV), Moment Matching (MM) and Stratified Sampling (SS) to the fair value of the certificate*
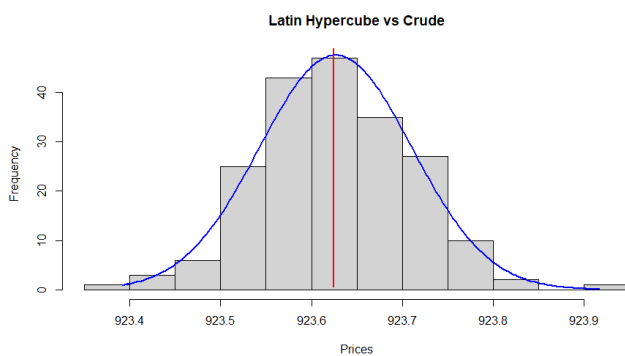

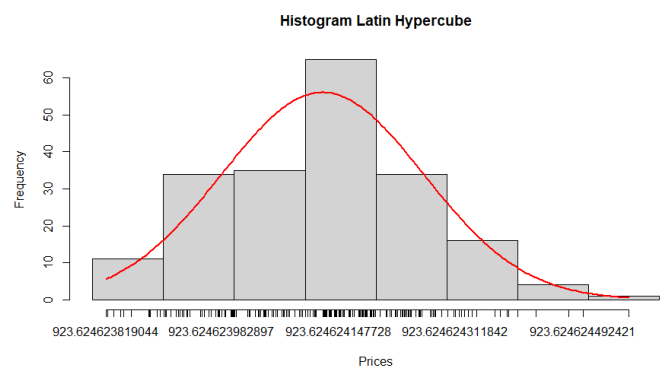*Figure 5: Crude price distribution and Latin Hypercube*

Figure 6: Latin Hypercube distribution enlarged

4) Analysis of the certificate - Banca Ponti Certificate Protection Cap on FTSE MIB 08/03/2027

The 100% protection certificate is an "Investment Certificate" with fully protected capital; therefore, the investor will receive the entire invested amount at maturity. The certificate provides for the payment of a conditional additional amount, on pre-established dates. If on the valuation date for the t-th conditional additional amount, the underlying is at a level equal to or higher than the initial reference value, then the investor receives an amount equal to 3% of the invested capital.

The initial reference of the underlying is defined as the closing price of the underlying on the initial reference date. The peculiarity of this additional amount is that it benefits from the memory effect.

With respect to structured products with a barrier mechanism, the so-called Memory coupon is an interest payment that is carried over to the next observation dates if the note, at a given observation date, fails to meet the coupon payment requirements as defined in the structure. However, if payment requirements are met at a certain observation date, all coupons that have not previously been paid will fall due for payment at such date.

We can express the rule for the payment of the t-th conditional additional amount in accordance with the following formula:

$$Nominal\ Value \times [t - th\ Additional\ Amount[\%] + Memory\ Additional\ Amount[\%] \times (t - k - 1)] \ (4)$$

Where k can assume values between 0 and 4 and indicates the value of t corresponding to the last Conditional Additional Amount Event that occurred. In particular, if t = 1 then k = 0, while if a Conditional Additional Amount Event has never occurred, k = 0.

The underlying of this certificate is the FTSE MIB index which measures the performance of 40 Italian stocks and aims at duplicating the broader sector of the Italian equity market.

Pricing is in any case carried out as of 31/03/2022 using market data from the Bloomberg® info-provider as in the first certificate. Table 10 shows the main characteristics of the certificate retrieved from the KID.

Certificates Characteristics	
Underlying	FTSEMIB Index
Initial Valuation Date	04/03/2022
Initial Reference Value of the underlying	Closing Price of the underlying at the Initial Valuation Date
Issuing Date	07/03/2022
Maturity Date	08/03/2027
Nominal Value	1,000 EUR
Protection	100% of Nominal Value
Cap Level	100% of Nominal Value
Conditional Additional Amount	3%
Valuation Date of the Conditional Additional Amount	28/02/2023, 28/02/2024, 28/02/2025, 02/03/2026, 01/03/2027
Payment Date of the Conditional Additional Amount	07/03/2023, 07/03/2024, 07/03/2025, 09/03/2026, 08/03/2027
Memory effect	The memory effect allows to receive, at any Payment Date of the Conditional Additional Amount, any Conditional Additional Amounts which were not paid because the Condition was not satisfied
Condition for the payment of Conditional Additional Amount	Conditional Additional Amount is payed if, on the Valuation Date of the Conditional Additional Amount, the Underlying value is equal to or higher than the Initial Reference Value of the Underlying
Final Valuation Date	01/03/2027
Final Reference Value of the underlying	Closing Price of the underlying at the Final Valuation Date

Table 10: Protected Capital Certificate on FTSEMIB (from the KID) – Source: Banca Cesare Ponti (Gruppo BPER)

The valuation model used is the local volatility model, proposed in 1994 by Dupire for the continuous case (Dupire, 1994). Initially, it is necessary to define the concept of implied volatility, which corresponds to the expectation of the market for the volatility of the underlying stock in the future. Implied volatility is one of six inputs used in the Black and Scholes option pricing model, but it cannot be directly observed in the market (Haug, 2007). The standard method to determine it, knowing the option price and the other five parameters (Spot, Strike, Dividend Yield, Time to Maturity and interest rate), is to solve the equation of the Black and Scholes model for the unknown parameter (i.e., the implied volatility). The important observation is that the "volatility smile" should never occur based on the Black and Scholes theory, as it theoretically assumes a completely flat volatility curve. The first case that drew attention to the volatility smile was the stock market crash that occurred in 1987. The reason for the need to move to a more sophisticated model lies precisely in that historical event, in fact Derman describe this model change as follows: "After the crash,

and ever since, equity index option markets have displayed a volatility smile, an anomaly in blatant disagreement with the Black-Scholes-Merton model. Since then, quants around the world have labored to extend the model to accommodate this anomaly" (Itkin, 2020).

Essentially, the authors showed that new models have been developed from that anomaly, and that quants aim to move from the Black–Scholes-Merton model to more complex models that better describe the trend and reality of the market.

The Local Volatility model implements a different Geometric Brownian Motion, since in this case volatility will depend on time and on the performance of the underlying in a time-varying way.

In order to take into consideration the Local volatility model, starting from the standard Geometric Brownian Motion, see Eq (1), we have to set the volatility in function of the S_t and t. Consequently the new dynamics can be represented by the following Stochastic Differential Equation:

$$dS_t = (r - q)S_t dt + \sigma(S_t, t)S_t dW_t \quad (5)$$

Basically, the Black and Scholes model uses the same level of volatility for the entire life of the option, while the Local Volatility model estimates its own level of volatility for each individual option in order to reflect a more accurate theoretical value of the option (Dupire, 1998).

The calculation of the volatility through the Local Volatility is implemented starting from the implied volatilities deriving from the traded prices.A series of Call and/or Put price vectors for different strikes and different maturities by means of interpolations allows to attain a Local Volatility surface. The connection relates the different volatility structure for different strikes and for different maturities. The formula for calculating the local volatility area is given below (Di Franco, Polimeni & Proietti, 2002):

$$\sigma(K,T) = \sqrt{\frac{\sigma_{IMP}^2 + 2T\sigma_{IMP}\frac{\partial \sigma_{IMP}}{\partial T} + 2rKT\sigma_{IMP}\frac{\partial \sigma_{IMP}}{\partial K}}{\left[1 + Kd_1\sqrt{T}\frac{\partial \sigma_{IMP}}{\partial K}\right]^2 + K^2 T\sigma_{IMP}\left(\frac{\partial^2 \sigma_{IMP}}{\partial K^2} - d_1\left(\frac{\partial \sigma_{IMP}}{\partial K}\right)^2\sqrt{T}\right)}} \quad (6)$$

With:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma_{imp}^2}{2}\right)T}{\sigma_{IMP}\sqrt{T}} \quad (7)$$

Where σ_{IMP} is the function of the implied volatility estimated from the numerical inversion of the Black-Scholes-Merton pricing formula starting from the market quoted prices.
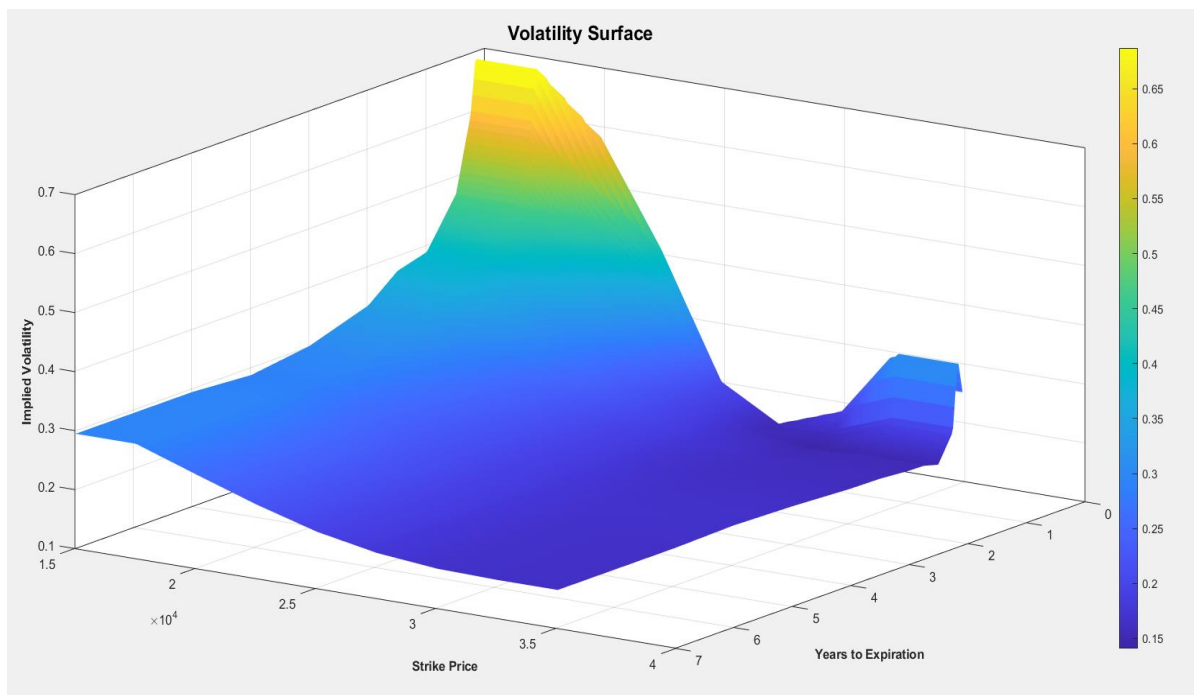

Figure 7: Local Volatility of FTSEMIB as of 31/03/2022

The implied volatility surface used to calculate the Local Volatility of this certificate is that of the FTSEMIB index (Source: Bloomberg®).

Contrary to the previous case, due to the path dependency given by the memory effect of the coupon strip, closed formulas cannot be used for the pricing of digital options (Giribone & Revetria, 2021).

Given that no closed-formulas exist for pricing this optionality, the numerical methodology of the Monte Carlo engine with local volatility can be implemented for the pricing, also applying the variance reduction techniques.

Specifically, the techniques used are: Antithetic Variables (AV), Latin Hypercube Sampling (LHS), Moment Matching (MM) and finally Stratified Sampling (SS). The table below displays values on the customer side:

	CRUDE		AV		SS		LHS		MM	
	MEAN	SD	MEAN	SD	MEAN	SD	MEAN	SD	MEAN	SD
A4	95.2255	0.204304	95.2014	0.11439	95.2383	0.103877	95.2163	0.103877	95.2075	0.193223
B4	95.2088	0.0666348	95.2109	0.0334144	95.2283	0.032937	95.2089	0.032937	95.2125	0.0559303
C4	95.2106	0.0202179	95.2124	0.0112894	95.2283	0.0155754	95.2123	0.0112894	95.2106	0.0180509
D4	95.2128	0.006367	95.2121	0.00479706	95.2287	0.006602981	95.2128	0.00489924	95.2117	0.0080949

Table 11: Crude, Antithetic Variables (AV), Stratified Sampling (SS), Latin Hypercube (LHS) and Moment Matching (MM)

In this case, the approach that did not impact so much is the Moment Matching, which failed to meet the expectations of a significative improvement measured in terms of standard deviation reduction (see Table 11, last columns).

The graphs of the pricing convergence of the financial instrument are also shown, using the same graphic conventions used in the previous certificate.
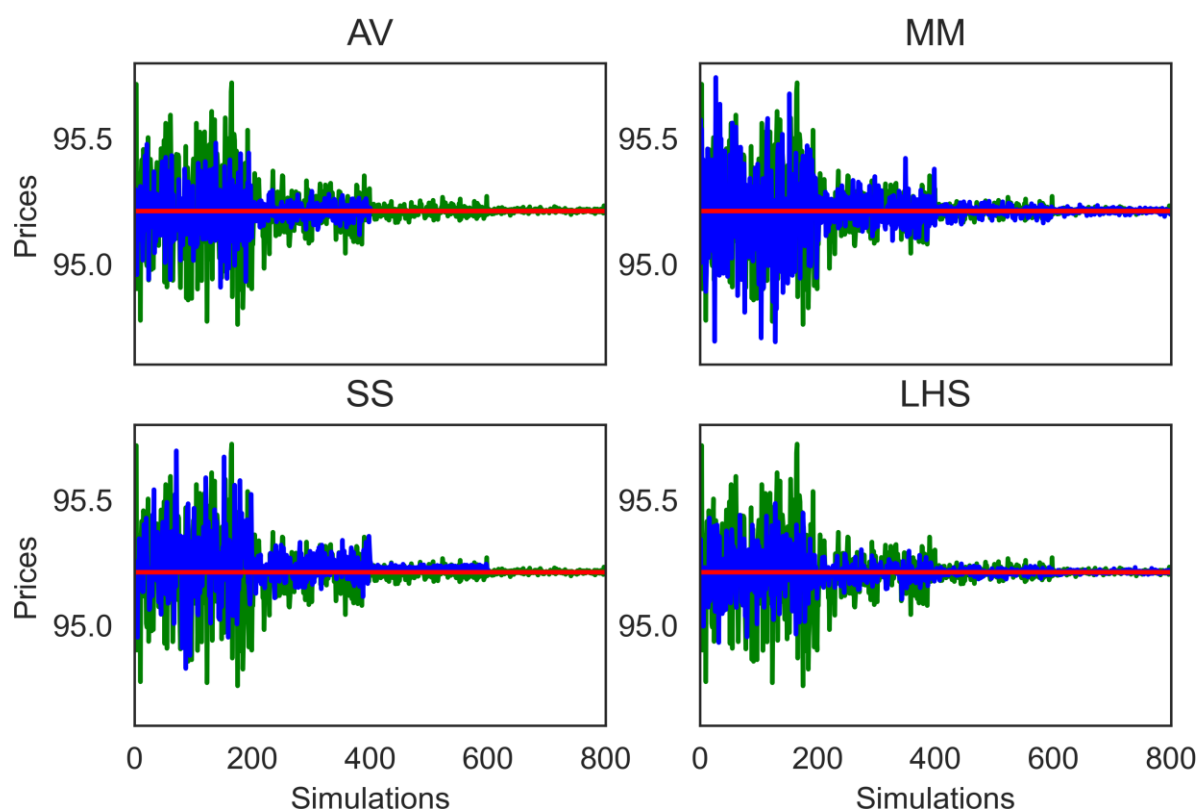

Figure 8: Antithetic Variables (AV), Moment Matching (MM), Stratified Sampling (SS), Latin Hypercube (LHS)

In the second certificate, where the model used is the Local Volatility, the variance reduction technique that performs best is the Antithetic Variables.
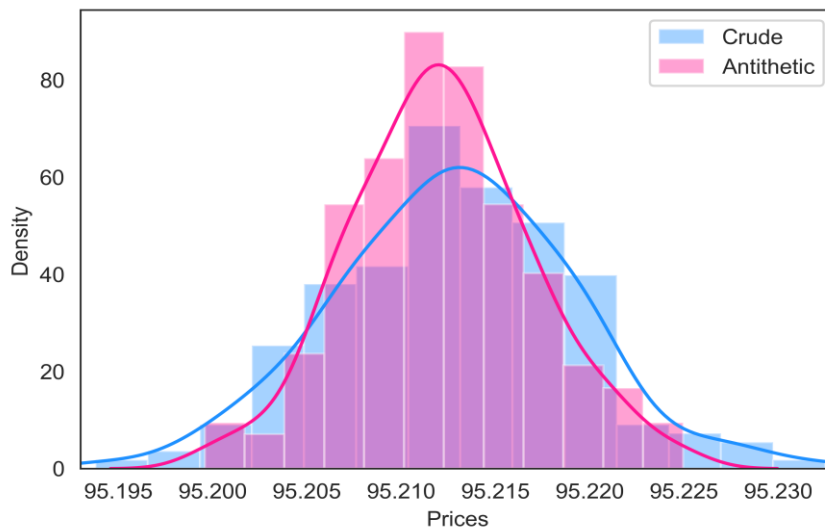
Figure 9: Crude and Antithetic Variables price distributions

5) Analysis of the certificate – Phoenix Reverse Convertible Linked to the FTSEMIB Index

The Phoenix Reverse Convertible linked to the FTSEMIB certificate is a structured certificate from JP Morgan. The certificate is characterized by the payment of conditional additional amounts and also by the payment of an amount in the event of early termination. The timing and amount of such payments will depend on the performance of the underlying.

The financial product has in any case a fixed maturity and will terminate on the maturity date. The certificate has a barrier option at 70%. This implies that if the price of the underlying falls below the barrier level at maturity, the product will pay an amount lower than the nominal amount, which could theoretically, in the most extreme case, be equal to zero.

On the coupon payment dates the investor will receive the coupon rate of 3.55% to be multiplied by the notional, if the reference level of the underlying is equal to or higher than the coupon barrier level (equal to 70% of the initial reference).

If this condition is not met, the investor will not receive any Coupon Payment on that Payment Date. The reference dates for the coupon payments are shown in Table 12:

Coupon Payment Table		
t	Coupon observation dates	Coupon payment dates
1	20 September 2022	4 October 2022
2	20 September 2023	4 October 2023
3	20 September 2024	4 October 2024
4	22 September 2025	6 October 2025
5	Final Valuation Date	Final Date

Table 12: Coupon payment dates. Source: JP Morgan

On the maturity date, the investor will receive:

- in the event that the final reference level is equal to or higher than the barrier level, a final payment equal to the nominal value.

- in case the price level is below the barrier level, the amount of the final payment will be directly linked to the performance of the underlying. The amount of such final payment will be equal to the notional amount multiplied by the final reference level divided by the exercise level.

The certificate also has a peculiar feature, i.e. the possibility of being redeemed before maturity. This feature is "mentioned" in the name of the certificate, since the "Phoenix" type indicates the autocallability by the issuer, as well as the fact that the coupons can be cumulated.

In the event that the underlying, on the coupon observation dates indicated above, is higher than the initial reference value, observed at the issue of the certificate, then the early termination is triggered and the investor will receive a final payment of 100% of the nominal value, plus the corresponding coupon.

This peculiarity implies that a different model has to be used for calculating the volatility: the Heston Volatility Model, which will be described in the next sub-section.

The characteristics of the structured product described previously are shown in Table 13:

Certificates Characteristics			
Underlying	FTSEMIB Index	Barrier Level	70% of the initial Reference Level
Underlying Market	Stock Market	Reference Level	Closing Level of the underlying provided by the Reference source
Product Nominal Amount	1,000 EUR	Reference source	FTSE international limited
Issue price	1,000 EUR	Final Reference Level	Reference Level at the final valuation Date
Product Currency	Euro (EUR)	Initial Valuation Date	20 September 2021
Underlying Currency	EUR	Final Valuation Date	21 September 2026
Subscription period	From 16 August 2021 to 17 September 2021	Final Date	5 October 2026
Issue Date	4 October 2021	Coupon Barrier Level	70% of the initial Reference Level
Initial Reference Level	Value of the Underlying at the Initial Valuation Date	Exercise Level	100% of the initial Reference Level

Table 13: Characteristics of certificate (ISIN: XS2168930340). Source: JP Morgan

5.1) Heston stochastic volatility model

Volatility is a difficult parameter to measure, as it is constantly changing and it becomes natural to consider the use of a stochastic model intended to model this particular parameter (Rouah, 2013). The most popular method for modeling stochastic volatility certificates characterized by a highly non-linear pay-off is the Heston Volatility Model (Heston, 1993). The proposed model is the following:

$$dS_t = (r - q)S_t dt + \sqrt{V_t} S_t dW_t^1 \quad (8)$$

$$dV_t = k(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^2 \quad (9)$$

$$dW_t^2 \, dW_t^1 = \rho dt \quad (10)$$

Where S_t and V_t are the price and its variance respectively, while W_t^1 W_t^2 are correlated Wiener processes with correlation parameter ρ. Moreover, θ is the long-term mean of variance, k is the rate of adjustment of the long-term mean, σ is the volatility of volatility and ρ is the correlation parameter.

The parameter ρ can be considered as the correlation between the log-returns and the volatility of the asset and it impacts the asymmetric width of the tails.

Intuitively if ρ is greater than zero then volatility will increase, as will the price of the underlying asset. In this case, the right tail will increase while the left tail will decrease, creating a distribution with a "fat" right tail.

Conversely, if the correlation parameter is less than zero then volatility will increase, while the value of the asset decreases, which produces an effect on the distribution leading to a "fat" left tail.

The parameter σ impacts the kurtosis (which indicates the "fatness" of the tails) of the returns; when it is equal to zero, the variance V is deterministic, and the log-returns are distributed according to a normal.

Increasing the sigma will result in an increase in kurtosis, which will lead to greater heaviness in the tails on both sides, consequently the volatility surface will have a more accentuated smile (meaning that the market has higher probability of having extreme movements).

This insertion made in Heston's model allows the distribution to be adapted to the one observed on the market, approximating not only the first two moments (mean and variance) but also the third and the fourth (skewness and kurtosis).

These adjustment factors allow this model to be better than others, in terms of predictive capabilities and consequently able to provide a better estimation of the instrument sensitivities in relation to a structured product characterized by the autocallability feature, like the Phoenix Reverse Convertible (Hansson, 2012).

The parameters are calculated starting from the implied volatility surface of the underlying index (in this case, the FTSEMIB index).

5.2) Estimation of the parameters of the Heston model

The Heston model implies that the price trend and volatility of a security follow certain laws according to 5 parameters: $V_t, \theta, k, \sigma, \rho$. These parameters cannot be directly observed on the market therefore they must be calibrated in order to enter them into the Monte Carlo pricing engine. For the calibration, we start from the implied volatility surface of the FTSEMIB and take the volatilities of the options traded on the market, then all the parameters are estimated together using a least squares minimization (Mrázek & Pospíšil, 2017). It is also possible to assign weights, for example, giving importance to the volatilities deriving from the most traded options on the market (the most liquid ones). Such implied surface has strikes ranging from 80% to 120% in terms of moneyness while the maturities range from 1 month to 7 years. The calibration is presented as a five-dimension minimization problem where we try to minimize the least squares of the differences between the volatilities obtained from the model and those observed on the market. Therefore, defining the implied volatility of an option as $I(V_i)$, the problem is as follows:

$$\min \sum_{i=1}^n (I(V_i^{model}(S, t_i, K_i, \bar{\phi})) - I(V_i^{market}(S, t_i, K_i))^2 \quad (11)$$

With $\overline{\phi} = (V_t, \theta, k, \sigma, \rho)$ under the following conditions: $V_t \geq 0, \ \theta \geq 0, \ k \geq 0, \ \sigma \geq 0 \ \text{e} -1 \leq \rho \leq +1$

The closed formula for pricing a European Call Option that pays a continuous dividend within the Heston model pricing framework is (Heston, 1993):

$$C(S_t V_t, t, T) = S_t P_1 - K e^{-r(T-t)} P_2 \quad (12)$$

Where:

$$P_j(x, V_t, T, K) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re}\left(\frac{e^{-i\phi \ln(K)} f_j(x, V_t, T, \phi)}{i\phi} \right) d\phi \quad (13)$$

$$x = \ln(S_t) \quad (35)$$

$$f_j(x, V_t, T, \phi) = \exp\{C(T - t, \phi) + D(T - t, \phi)V_t + i\phi x\} \quad (14)$$

$$C(T - t, \phi) = r\phi i(r - q)\frac{a}{\sigma^2}\left[(b_j - \rho\sigma\phi i + d)\tau - 2\ln\left(\frac{1 - ge^{d\tau}}{1 - g}\right)\right] \quad (15)$$

$$D(T - t, \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left(\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}\right) \quad (16)$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d} \quad (17)$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)} \quad (18)$$

For $j = 1, 2$ where: $u_1 = \frac{1}{2}, \ u_2 = -\frac{1}{2}, \ a = k\theta, \ b_1 = k - \rho\sigma, \ b_2 = k$

In the formulas, i represents the imaginary unit.

Having found the parameters that regulate the Heston dynamics, we then move on to the description of the numerical integration schemes implemented.

5.3) Integration schemes for the Heston model

The most classic integration scheme is the Euler discretization. For each step of the Monte Carlo, the time horizon $[0, T]$ is partitioned into M periods with a constant duration of Δt.

$$[0 = t_0 < t_1 < \cdots t_M = T], t_i = \frac{iT}{M}, i = 0, 1, \ldots, M \quad (19)$$

The discretization for the price process is (Kloeden, Platen & Schurz, 1997):

$$S_{t_i} = S_{t_{i-1}} + \mu S_{t_{i-1}}\Delta t + \sqrt{v_{t_{i-1}}}\sigma\Delta W_{t_i}^1 \quad (20)$$

And here is the volatility process:

$$v_{t_i} = V_{t_{i-1}} + k(\theta - V_{t_{i-1}})\Delta t + \sqrt{v_{t_{i-1}}}\sigma\Delta W_{t_i}^2 \quad (21)$$

With $\Delta W_{t_i}^1 = W_{t_i}^1 - W_{t_{i-1}}^1$ and $\Delta W_{t_i}^2 = W_{t_i}^2 - W_{t_{i-1}}^2$

The k represents the speed of adjustment for variance to return towards the long-term average (θ).

The correlation constraint must be respected: $\rho dt = (W^1, W^2)_t$ in accordance with the numerical implementation of the Cholesky decomposition.

Euler's discretization method presents a major problem, namely that volatility can reach a negative value with non-zero probability, contrary to the definition of variance. To ensure that this does not happen, the well-known Feller condition has to be checked on the variance process: $2k\theta \geq \sigma^2$ (Rouah, 2013).

If this inequality holds, then the drift is sufficiently large for the variance process to be guaranteed positive and not reach zero. Given that the calibration of the five parameters that characterized the Heston dynamics strictly depend on the raw market data used for this aim (i.e. European call/put market prices or their implied log-normal volatilities), we are not sure that this condition can definitely be met. For this reason, we prefer to implement a more robust and reliable discretization scheme.

The Broadie-Kaya scheme for calculating the Monte Carlo steps is based on the possibility of having exact values of the probability distributions of the underlying price and its volatility at each discrete time increment (Broadie & Kaya, 2006). For each time interval we have that:

$$S_t = S_u \exp\left[(r - q)\Delta t - \frac{1}{2}\int_u^t v_s ds + \rho \int_u^t \sqrt{v_s} dW^1\right] \quad (22)$$

$$v_t = v_u + k\theta\Delta t - k\int_u^t v_s ds + \sigma \int_u^t \sqrt{v_s} dW^2 \quad (23)$$

The steps to be followed are:

Step 1: Generate a sample from the distribution v_t given v_u.

Step 2: Generate a sample from the distribution of $\int_u^t v_s ds$ given v_t and v_u.

Step 3: Recover $\int_u^t \sqrt{v_s} dW^1$ from v_t and v_u and $\int_u^t v_s ds$.

Step 4: Generate a sample from a distribution of S_t given $\int_u^t \sqrt{v_s} dW^1$ and $\int_u^t v_s ds$.

The two suggested methods offer two different solutions, Euler offers a lower level of approximation with the same time step Δt compared to the exact solution from Broadie-Kaya. The implementation of the Broadie-Kaya scheme corresponds to the method with the highest precision thus it remains important for verifying the validity of the model to be used.
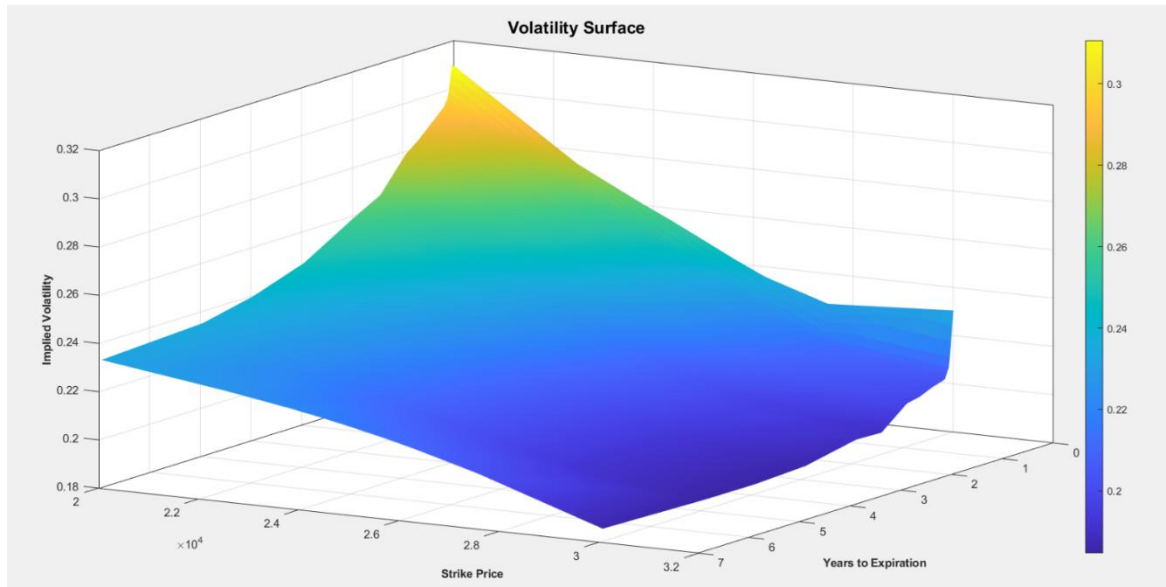

Figure 10: Surface of implied volatilities obtained by inverting the pricing formula for a Call in the Heston model

The Heston model is therefore one of the stochastic volatility models that best represents the pace of market prices. However, we emphasize the high sensitivity of the five parameters calibration of the dynamics to market data and the absence of closed formulas for non-European contracts.

A "Low-Discrepancy Sequence", also called Quasi-random Sequence, is a sequence of a representative sample from a probability distribution (Jäckel, 2002). The feature of these sequences is that they have a standard error proportional to $1/M$ instead of $1/\sqrt{M}$, where M is the sample size. Quasi-random Sampling is very similar to Stratified Sampling: the goal is to sample representative values for the underlying variables. In Stratified Sampling we assume we know in advance how many samples will be considered, while in low discrepancy sequences, the procedure is more flexible.

In this case the samples are taken in such a way that the gaps between the created sample and the existing one are filled. With each simulation, the sampled points are approximately uniformly arranged within the probability space. The resulting (deterministic) sampling is entered into the Monte Carlo engine replacing the typical random sampling (Giribone & Ligato, 2014).

The advantage of using this methodology is to obtain the same result as the stochastic methodology with a lower number of simulations. For a brief summary of the working principle, see appendix A.7.

In this certificate pricing context, we decided to introduce a normal random outcome by means of a Low Discrepancy Sequence (LDS) for the generation of dW_t^1, and a normal random output by means of a traditional uniform distribution for dW_t^2 since this combination of random number generation has proven to be empirically more efficient compared to a standard variance reduction methodology.

Consequently, the output of the SDE system is not deterministic, but based on LDS, and it is associated with the Randomized QMC methods (L'Ecuyer & Lemieux, 2002).

5.4) Certificate valuation and performance comparison

The reduction techniques applied to this Certificate are: Antithetic Variables, Latin Hypercube Sampling, Randomized Halton and Randomized Sobol.

The result of the Crude Monte Carlo was obtained using the Euler-Maruyama integration scheme while the result obtained using the Broadie-Kaya scheme is also reported, which indicates an improvement in the standard deviation, but at the cost of a higher computing time.

	CRUDE		AV		SOBOL		LHS		HALTON		KAYA	
	MEAN	SD	MEAN	SD	MEAN	SD	MEAN	SD	MEAN	SD	MEAN	SD
A4	90.001	0.91522	90.181	0.87588	89.893	0.43746	90.152	0.67218	90.098	0.39637	90.711	0.66602
B4	90.167	0.29263	90.155	0.24733	90.124	0.13369	90.137	0.21777	90.119	0.13619	90.302	0.20321
C4	90.122	0.098405	90.134	0.077158	90.137	0.046531	90.14	0.072935	90.137	0.045183	-	-
D4	90.142	0.030451	90.144	0.025676	90.142	0.014625	90.145	0.022101	90.143	0.013439	-	-

Table 14: Crude, Antithetic Variables (AV), Sobol, Latin Hypercube (LHS), Randomized Halton and Kaya

The two techniques that performed best for this certificate, in terms of standard deviation and in terms of computing time were Halton and Sobol. On the other hand, the Monte Carlo engine implemented with the Broadie-Kaya scheme has brought a huge increase in the computing time despite a good reduction of the standard deviation and an expected value slightly higher compared to the other methodologies. For the Broadie-Kaya method, only cases with 1,000 and 10,000 simulations have been reported due to the high computing time and the negligible improvement in standard deviation compared to the other techniques.
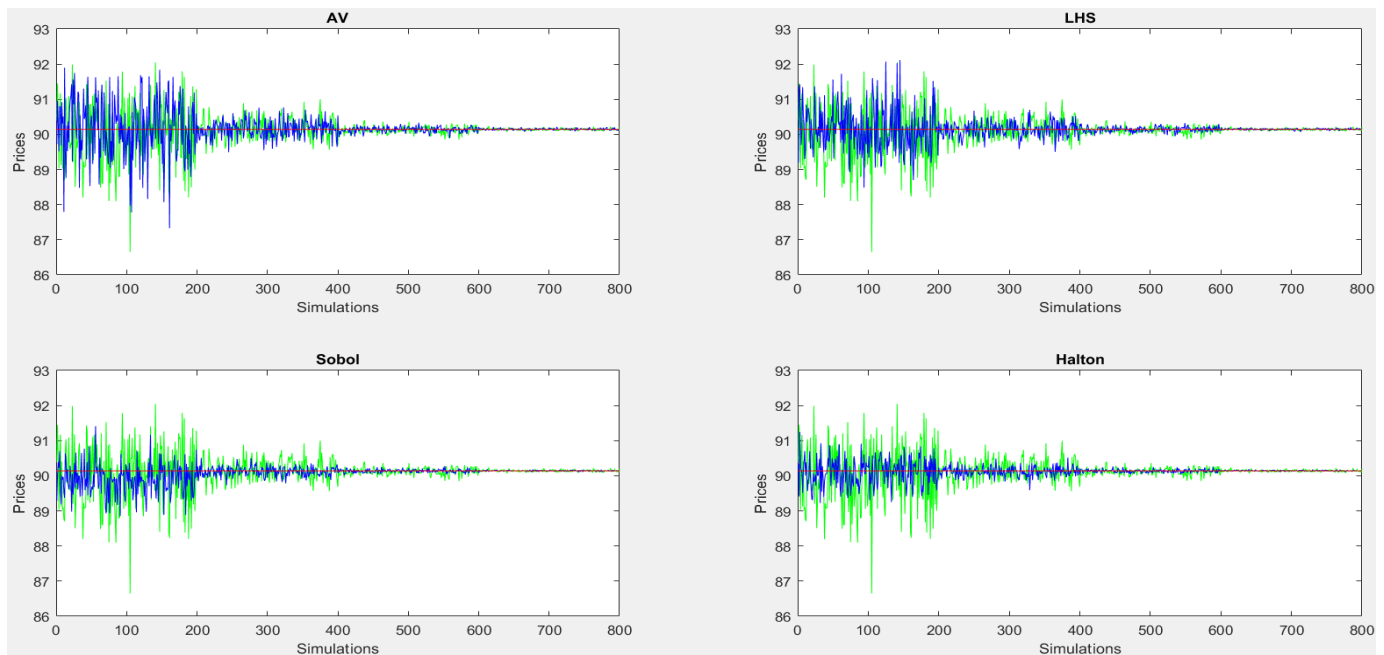

Figure 11: Antithetic Variables (AV), Latin Hypercube (LHS), Randomized Sobol and Halton

For the last certificate, the Randomized Sobol performed better than the other reduction techniques, i.e. the Randomized Halton and the Broadie-Kaya. In the last case we analyzed, we also achieved a reduction in the standard deviation.
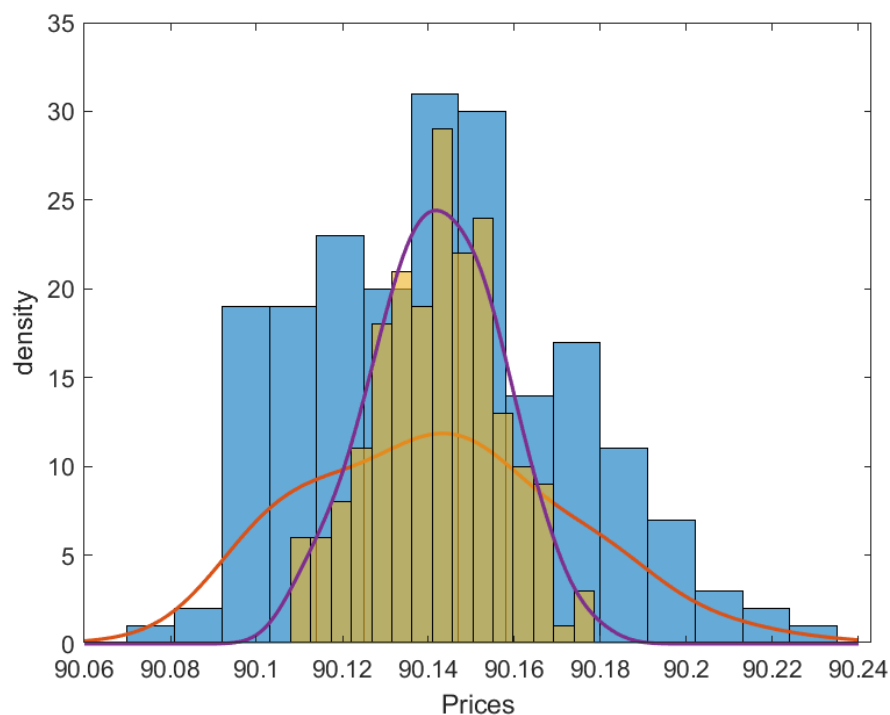

Figure 12: Crude and Randomized Sobol price distributions

6) Conclusions

The purpose of this paper is to study and implement variance reduction techniques for structured and complex products such as investment certificates. These techniques are usually applied to single options and their application to certificates represents an innovative approach that has never been reported in the previous literature. Indeed, the market for these financial instruments has grown considerably in recent years, as highlighted in the first part of the paper, so that our analysis takes on particular relevance. Three pricing models have been used: the first is the traditional Black and Scholes model, the second is the Local Volatility model and lastly the Heston model. Each model has been associated with a specific certificate, each of them being characterized by a different payoff. The pricing and the volatility model analysis have been performed with three different programming languages: Python, Matlab and R. The market data have been provided by the info-provider Bloomberg®, one of the most widely used info-providers in the global financial sector. The certificates payoffs have been replicated analyzing their Termsheets and verifying that the estimated theoretical price reflects the market price or it was calculated independently using the Bloomberg® calculation modules (DLIB and OVME). The variance reduction techniques applied are: Latin Hypercube, Stratified Sampling, Antithetic Variables, Importance Sampling, Moment Matching and Control Variates.

Overall results suggest that our research objective have been reached for all three certificates we have examined. Ultimately, we can state that the formulations of the initial hypotheses have been demonstrated with highly satisfactory results.

This study might be extended by combining different variance reduction techniques in order to study if further standard deviation reductions can be achieved; furthermore, the efficiency of the techniques could be verified with other types of structured products, since the certificate industry includes extremely heterogeneous kinds of products.

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Appendix

A.1) Antithetic Variables

In the Antithetic Variables technique, the simulation by the Monte Carlo engine is made through the estimation of two values of the derivative to be evaluated (He & Chen, 2011). The first value f_1 is calculated normally while the second value f_2 is calculated by changing the sign to the whole random sample deriving from the standard normal distribution. The sample of final values of a simulation will be the mean of the values f_1 and f_2. This method works well when one value is above the exact value while the other tends to be below and vice versa. The mean of the two values is calculated:

$$\bar{f} = \frac{f_1 + f_2}{2} \text{ (A1)}$$

The final estimate of the value of the derivative is the mean of the obtained values $\bar{f}$. Considering $\bar{\omega}$ as the standard deviation of the $\bar{f}$ values and N as the number of simulations then the estimation error will be $\frac{\bar{\omega}}{\sqrt{N}}$. The final result should have a lower error compared to using $2N$ random simulations (Hull, 2015).

A.2) Stratified Sampling

Stratified Sampling consists in dividing the sample domain into smaller areas, and for each of them, a representative value of the function is selected (Hull, 2015). The fundamental concept is to limit the extractions in specific subsets of the random number generation domain without introducing biases. Let us presume we need to estimate $E[X]$, with X real-valued random variable and let us consider $A_1, \dots, A_\mathcal{S}$ the disjunct sets for which the probability $P(X \in \cup A_i)$ is one:

$$\sum_{i=1}^{\mathcal{S}} P(X \in A_i) E[X|X \in A_i] = \frac{1}{n} \sum_{i=1}^{\mathcal{S}} p_i E[X|X \in A_i] \text{ (A2)}$$

With $p_i = P(X \in A_i)$

In a random sampling, $X_1, \dots, X_\mathcal{S}$ are independently generated and they have the same distribution as X. The fraction of such extractions that fall into partition A_i generally does not equal p_i. In Stratified Sampling, on the other hand, the portion of the sampling that should be obtained from each stratum A_i is decided in advance. The simplest and most common application of this methodology consists in partitioning the random variables distributed according to layers:

$$A_1 = \left(0, \frac{1}{n}\right], A_2 = \left(\frac{1}{n}, \frac{2}{n}\right], \dots, A_i = \left(\frac{i-1}{n}, \frac{i}{n}\right], A_n = \left(\frac{n-1}{n}, 1\right] \text{ (A3)}$$

Each one of these ranges has a probability of being selected. Instead, if operating with the SS logic, a random variable selected within the designated interval will be generated. By sampling the random variable conditional to belonging to one of the designed proportional partitions, iteration after iteration, the contribution due to the sampling variability is eliminated.

A.3) Latin Hypercube Sampling

Latin Hypercube Sampling is obtained by dividing a cumulative density function into n equal parts and then selecting a random point in each division (Iman, Davenport & Zeigler, 1980). This technique allows to extend Stratified Sampling in a multi-dimensional context. To generate a Latin Hypercube of dimension $\mathcal{S}$ in d dimensions, let us consider $U_i^{(j)}$, the random variables generated by U(0,1) for i = 1,…,d , j = 1,…., $\mathcal{S}$.

Let us consider $\pi_1, \dots, \pi_d$ random permutations in {1,…, $\mathcal{S}$} and let us define:

$$V_i^{(j)} = \frac{\pi_i(j) - 1 + U_i^{(j)}}{\mathcal{S}} \text{ (A4)}$$

With i = 1,…,d and j = 1,…., $\mathcal{S}$

The sample consists in selecting the $\mathcal{S}$ points: $\{V_i^{(j)}, \dots, V_d^{(j)}\}$.

A.4) Control Variates

The method is based on the idea of obtaining information about the errors in the estimates of known quantities in order to reduce the error in estimating unknown quantities (Lemieux, 2017).

In order to describe the mathematical procedure, we consider a random variable X and another random variable Y correlated with X. We suppose that Y is the control variable for X.

Assuming that the statistical average of Y is known, we define a new variable X^* as follows:

$$X^* = X + \alpha[Y - E(Y)] \text{ (A5)}$$

Note that the statistical average of X^* is equal to the mean of X: $m_X = m_{X^*}$, consequently we can estimate the average of X rather than m_{X^*}.

Thus, the goal is to choose a parameter α that minimizes the variance of X^*.

$$Var\,(X^*) = E[(X^* - m_{X^*})^2] = E[(X + \alpha[Y - E(Y)] - m_{X^*})^2] = Var(X) + \alpha^2 Var(Y) + 2\,\alpha\,Cov(X,Y) \text{ (A6)}$$

Where $Cov(X,Y) = E[(X - m_X)(Y - m_Y)]$

The optimal choice is to set the first derivative equal to zero:

$$\frac{dVar(X^*)}{d\alpha} = 0 \rightarrow 2\alpha^* Var(Y) + 2\,Cov(X,Y) = 0 \rightarrow \alpha^* = -\frac{Cov(X,Y)}{Var(Y)} \text{ (A7)}$$

$$Var(X^*) = Var(X) - \frac{[Cov(X,Y)]^2}{Var(Y)} \text{ (A8)}$$

Being $\frac{[Cov(X,Y)]^2}{Var(Y)} \geq 0$, it follows that $Var(X^*) \leq Var(X)$.

The same logical steps can be generalized for a k- dimensional space.

In this context, we generate Y_k control variables for X. The new variables X^* can be expressed in the form:

$$X^* = X + \alpha_1[Y_1 - E(Y_1)] + \cdots + \alpha_k[Y_k - E(Y_k)] \text{ (A9)}$$

Or, in a vectorized notation we can express the same relationship in a more concise way:

$$X^* = X + \vec{\alpha}^T(\vec{Y} - \vec{m}_{\vec{Y}}) \text{ (A10)}$$

The variance of X^* becomes:

$$Var(X^*) = E[(X^* - m_{X^*})^2] = E\left[\left(X - m_X + \vec{\alpha}^T(\vec{Y} - \vec{m}_{\vec{Y}})\right)\left(X - m_X + (\vec{Y} - \vec{m}_{\vec{Y}})^T\vec{\alpha}\right)\right] = Var(X) + \vec{\alpha}^T\Lambda_{\vec{Y}}\vec{\alpha} + 2\vec{\alpha}^T\vec{P} \text{ (A11)}$$

Where Λ is the variance-covariance matrix of the array $(X, \vec{Y})$ and $\vec{P}$ is the covariance array:

$$\Lambda = E\left[(X - m_X, \vec{Y} - \vec{m}_{\vec{Y}})^T(X - m_X, \vec{Y} - \vec{m}_{\vec{Y}})\right] \text{ (A12)}$$

$$\vec{P} = E[(\vec{Y} - \vec{m}_{\vec{Y}})(X - m_X)] = [Cov(X,Y_1), \dots, Cov(X,Y_k)]^T \text{ (A13)}$$

Assuming that the covariance matrix is positively defined, the optimal choice for $\vec{\alpha}$ is:

$$\frac{d\,Var(X^*)}{d\vec{\alpha}} = \left(\frac{d\,Var(X^*)}{d\alpha_1}, \dots, \frac{d\,Var(X^*)}{d\alpha_k}\right)^T = 0 \rightarrow \frac{d\,Var(X^*)}{d\vec{\alpha}} = 2\vec{P} + 2\Lambda_{\vec{Y}}\vec{\alpha} = 0 \rightarrow \vec{\alpha}^* = -\Lambda_{\vec{Y}}^{-1}\vec{P} \text{ (A14)}$$

With this optimal choice, the variance of X^* is:

$$Var(X^*) = Var(X) - \vec{P}^T\Lambda_{\vec{Y}}^{-1}\vec{P} \text{ (A15)}$$

Given that $\Lambda_{\vec{Y}}$ is positively defined, $\Lambda_{\vec{Y}}^{-1} > 0$. Being the quadratic form, $\vec{P}^T\Lambda_{\vec{Y}}^{-1}\vec{P}$ positive, it follows that $Var(X^*) \leq Var(X)$.

A.5) Moment Matching

This technique consists in transforming the paths in order to match the moments of the distribution (Glasserman, 2003). This can be done for example in the simulation of an underlying asset (S_t) simulated under the assumption of risk neutrality and with a constant interest rate. If the underlying pays no dividends, then we have:

$$E[S(t)] = e^{rt}S(0) \quad \text{(A16)}$$

We assume to simulate n independent copies of the process $S_1, \dots, S_n$ and we define the mean of the sample:

$$\bar{S}(t) = \frac{1}{n}\sum_{i=1}^{n} S_t(t) \quad \text{(A17)}$$

For a finite number of n, the mean of the sample will not coincide with $E[S(t)]$, therefore it would occur that the current value of the simulations would not coincide with the real current value of the underlying:

$$S(0) \neq e^{-rt}\bar{S}(t) \quad \text{(A18)}$$

The left side of the equation is the current underlying while the right side represents the estimated and discounted simulation. A possible remedy that Moment Matching attempts to bring is to transform the simulated paths:

$$\widetilde{S}_i(t) = S_i(t)\frac{E[S(t)]}{\bar{S}(t)}, \quad i = 1, \dots, n \quad \text{(A19)}$$

$$\widetilde{S}_i(t) = S_i(t) + E[S(t)] - \bar{S}(t) \quad i = 1, \dots, n \quad \text{(A20)}$$

At this stage, we use $\widetilde{S}_i(t)$ instead of S_i for the price of the derivative.

A.6) Importance Sampling

The Importance Sampling tries to accelerate the convergence rate of the Monte Carlo engine by changing the probability measure from which the paths are generated (Glasserman, 2003). This methodology focuses on those regions that contribute to the mean of the Monte Carlo integration procedure. An example to understand this technique consists in calculating the price of a European "deep-out-of-the-money" Call option with strike price K and maturity T.

In this case, the sample of values at time T will lead to a zero payoff most times. This is a computational waste of time because the paths that lead to zero contribute very little to the value of the option. The operation to be implemented in this case is to only select the important paths, i.e., those where the price of the underlying is higher than the strike price at maturity (for the Call option; vice versa for a Put option).

To further clarify the reasoning described above, we proceed with the mathematical steps. Let us suppose we have to estimate:

$$\lambda = E[h(X)] = \int h(x)f(x)\,dx \quad \text{(A21)}$$

Where X is a random variable in $\Re^d$ with probability density f and h is a function $\Re^d \to \Re$. The estimator in an ordinary Monte Carlo is:

$$\hat{\lambda} = \hat{\lambda}(n) = \frac{1}{n}\sum_{i=1}^{n} h(X_i) \quad \text{(A22)}$$

With $X_1, \dots, X_n$ being independent variables extracted from f. Let g be any other probability density in $\Re^d$ that satisfies the condition: $f(x) > 0 \to g(x) > 0, \forall x \in \Re^d$

In this case we can also represent λ as:

$$\lambda = \int h(x)\frac{f(x)}{g(x)}g(x)dx \quad \text{(A23)}$$

This integral can be interpreted as the expected value with respect to density g:

$$\lambda = \tilde{E}\left[h(X)\frac{f(X)}{g(X)}\right] \quad \text{(A24)}$$

If $X_1, \dots, X_n$ are now random variables extracted from g, the estimator of the Importance Sampling associated with g is:

$$\hat{\lambda}_g = \hat{\lambda}_g(n) = \frac{1}{n}\sum_{i=1}^{n} h(X_i)\frac{f(X_i)}{g(X_i)} \quad \text{(A25)}$$

Where the ratio $\frac{f(X_i)}{g(X_i)}$ is the stochastic derivative of Radon-Nikodym calculated in X_i. Since the following is true:

$$\lambda = \tilde{E}\left[h(X)\frac{f(X)}{g(X)}\right] \quad \text{(A26)}$$

it follows that $\tilde{E}[\hat{\lambda}_g]$ is an unbiased estimator of λ in order to compare the variances with and without the technique in question and it is sufficient to analyze the second moments.

With the Importance Sampling, we have:

$$\tilde{E}\left[\left(h(X)\frac{f(X)}{g(X)}\right)^2\right] = E\left[h(X)^2\frac{f(X)}{g(X)}\right] \quad \text{(A27)}$$

Such figure can be greater or smaller than the second moment of the ordinary Monte Carlo, $E[h(X)^2]$ depending on how the density function g is designed, which is strongly related to the type of option we want to valuate. The difficulty in defining a suitable density function makes it complicated to generalize the method and therefore to integrate it into automated pricing systems.

A.7) Randomized Quasi-Monte Carlo

The Quasi-Monte Carlo (QMC) method approximates the integral using:

$$\int_{[0,1)^d} f(x)\, dx \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i) \quad \text{(A28)}$$

For points $x_1, x_2, \dots x_n$ suitably extracted in a deterministic and sequential way from a numerical series within the unitary hypercube $[0,1)^d$. The issue with Quasi-Monte Carlo techniques is that they are related to the dimensionality of the problem, and it is actually essential to know d in order to correctly produce the sequences of points. Starting from the Van der Corput series, the basis of most of the QMC simulations, the Halton and Sobol sequences will then be obtained, which are the ones used in this certificate. Given a collection X of subsets measurable according to Lebesgue of A=$[0,1)^d$, the discrepancy D of the set of points $\{x_1, \dots x_n\}$ relative to X is defined as follows:

$$D(x_1, \dots x_n; X) = \sup_{A \in X}\left|\frac{\#\{x_i \in A\}}{n} - vol(A)\right| \quad \text{(A29)}$$

Where:

$\#\{x_i \in A\}$ indicates the number of x_i in A,
vol(A) is the measure of A in the reference space.

The LDS numerical sequences manage to minimize the quantity D and to solve the integration problem (Huynh, Lai & Soumare, 2008). To obtain the Van der Corput sequences, the following steps must be followed:

- Choosing a prime number b

- Writing n in base b. This operation allows to find the unknown $a_j(n)$ which satisfies the equation:

$n = \sum_{j=0}^{m} a_j(n)b^j$ where m is the smallest integer so that $a_j(n) = 0, \forall j > m$

- Converting n to decimal base to find the n-th element, indicated as:

$$b_n = \Phi_b(n) = \sum_{j=0}^{m} \frac{a_j(n)}{b^{j+1}} \epsilon [0,1) \quad \text{(A30)}$$

After introducing the Van der Corput sequence, we can introduce the Halton sequence which constitutes a multi-dimensional extension of the Van der Corput sequence. The implementation of this sequence is very simple, and it is sufficient to select a different base b for each dimension, to build it. The Sobol sequence is built starting from the Van der Corput sequence, alternating its elements for each dimension of the problem: we have the same fundamental elements, but the order is permuted (Sobol, 1967). The values are calculated according to the following logic: let us suppose we want to generate the term n+1 of sequence j, indicated as x_{n+1}^j. We start with the sequence $x_1 = 0.5$ in base ten or 0.1 in base two and we write n in binary notation:

$$n = n_\omega 2^{\omega-1} + n_{\omega-1}2^{\omega-2} + \cdots + n_2 2^1 + n_1 \quad \text{(A31)}$$

We define k as the subscript i of the first coefficient $n_i \neq 0$. Finally, element x_{n+1} is calculated through the operation: $x_{n+1} = x_n \oplus v_k$, where v_k is the direction k for the considered dimension j. The $\oplus$ operator represents the XOR binary operator. The deterministic numbers v_k are generated by iteratively applying the recurrence:

$$v_i = a_1 v_{i-1} \oplus a_2 v_{i-2} \oplus \dots \oplus a_{q-1}v_{i-q+1} \oplus v_{i-q} \oplus \frac{v_{i-q}}{2^q} \quad \text{(A32)}$$

Where i > q and a are the coefficients (0/1) of the primitive polynomial of degree q of the following form:

$$P(X) = x^q + a_1 x^{q-1} + a_2 x^{q-2} + \cdots + a_{q-1} x^1 + 1 \quad \text{(A33)}$$

To achieve greater tractability, the theory of numbers suggests performing the transformation of a variable: $M_i = 2^i v_i$ (Huynh, Lai & Soumare, 2008). These values satisfy the recurrence:

$$M_i = 2a_1 M_{i-1} \oplus 2^2 a_2 M_{i-2} \oplus .. \oplus 2^{q-1} a_{q-1} M_{i-q+1} \oplus 2^q M_{i-q} \oplus M_{i-q} \quad \text{(A34)}$$

Given the properties of v_i , then M_i is an odd integer between 0 and 2^i. Instead of generating the values of x_i, in order to simplify the algorithm, we will rescale the values to generate the variables y_i, which will be multipliers of x_i. The theoretical procedure can be efficiently implemented in a programming environment:

- Select a primitive polynomial for dimension j.

- Select a starting value for the first figure M_i with Eq. (A34)

- Determine k in the base 2 decomposition of n Eq. (A31)

- Calculate a new value for M_i .

- Calculate $y_{n+1} = \begin{cases} y_n \oplus 2^{m_{n+1}-k} M_k \ if \ m_n = m_{n+1} \\ 2y_n \oplus 2^{m_{n+1}-k} M_k \ if \ m_n \neq m_{n+1} \end{cases}$ $x_{n+1}: x_{n+1} = y_{n+1}/2^{m_{n+1}}$ (A35)

In the implemented Randomized Quasi-Monte Carlo we have used this algorithm for generating the Wiener process associated with the spot level projection in the Heston model.

Approaching IRRBB and CSRBB: a case study in line with the EBA approach

Michail Michoulas (Wolters Kluwer) and Ioannis Akkizidis (Wolters Kluwer)

Introduction

EBA guidelines on Interest Rate Risk in the Banking Book (IRRBB) are designed to help EU banks effectively manage their interest rate risk and maintain a stable earnings stream. EBA also requires the credit spread risk from the banking book (CSRBB). Banks can effectively manage their exposure to interest rates and spread risks by implementing a comprehensive IRRBB and CSRBB management framework that includes: regular stress testing, sensitivity analysis, effective hedging strategies, and appropriate governance and risk management structures. Under those frameworks, banks must regularly monitor and report interest rates and credit spread risk metrics.

EBA's updated guidelines for IRBB and CSRBB

The EBA released updated guidelines for IRRBB and CSRBB in October 2022. The refreshed guidelines underlined the increased regulatory focus on both risks. There was continuity with the 2018 versions, but also included new aspects including identifying non-satisfactory internal models for IRRBB management[1]. With market volatility as it is, the need to be stressing books has been rising and this directive from the regulator cements that.

Specifically, the EBA guidelines require banks to:

1. Establish a comprehensive IRRBB management framework that covers all interest rate-sensitive measures applied to portfolios and accounts of loans, deposits, trading activities, and derivatives placed on off-balance-sheet portfolios.

2. Establish an appropriate governance structure, including an independent risk management function, a risk committee, and a board of directors that oversees IRRBB activities.

3. Establish and maintain an appropriate risk appetite for interest rate risk and to set limits on exposure to this risk.

EBA also expects financial institutions to consider the credit spread risk in the banking book (CSRBB). The CSRBB considers the risk arising from changes in non-trading book instruments' credit spread, such as loans and deposits.

The are two main elements in capturing CSRBB:

1. The changes in the market price of credit risk are driven by the idiosyncratic credit spread and defining the credit risk premium required by market participants and are applied to the financial instruments' level.
2. The changes in the "market liquidity spread" are driven by the willingness of the counterparties to fulfil their credit obligations. Thus, it defines the liquidity premium expressed by the market participants.

CSRBB can significantly impact a bank's earnings and capital adequacy, particularly in the event of a sharp and unexpected widening of credit spreads. Banks must therefore measure and manage CSRBB as part of their overall interest rate risk management framework.

IRRBB and CSRBB measure the Economic Value (EV) measures and Net interest income (NII) measures plus market value changes, applied in the context of the sensitivity analysis. The IRRBB refers to changes in interest rates, and the CSRBB to changes in market credit/liquidity spreads. Banks may also perform and report the Economic value of equity (EVE) and the measure of EV where the cash flows reference to the equity are excluded.

Banks must reconsider their existing IRRBB and CSRBB management framework and ensure they have a comprehensive basis that includes policies, procedures, risk limits and covers all sources of interest rate risk.

EBA Stress test guidelines

The EBA guidelines require banks to perform regular stress tests to assess the impact of different scenarios on their balance sheet, including both parallel and non-parallel interest rate shocks. The stress tests should be based on a range of economic scenarios and include an assessment of the impact of changes in customer behaviour, such as prepayments or withdrawals.

The analysis must be performed in both Run-off and Dynamic balance. Thus, financial contracts in both on- and off-balance-sheet accounts are incorporated into future business strategies and consistently adjusted for the relevant scenario.
The EBA guidelines also require banks to have appropriate hedging strategies in place to manage their interest rate risk. These strategies should be regularly reviewed and updated to ensure that they remain effective in mitigating the bank's exposure to interest rate risk.

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[1] Guidelines on IRRBB and CSRBB | European Banking Authority (europa.eu)

Supervisors expect the banks to identify all underlying risk factors impacting interest rates and to conduct stress testing to assess their resilience to adverse interest rate scenarios. In this context, banks must develop a comprehensive IRRBB management framework that includes policies, procedures, and risk limits which covers all sources of interest rate risk in the banking book.

Cases in line with the EBA approach

Taking all the guidelines into account, let's speak through some cases which have been assumed for a typical bank's requirements.

Let's assume a financial credit institution has a significant portion of its balance sheet invested in:

- long-term, fixed and variable-rate loans
- short-term fixed and floating-rate deposit loans

The asset classes are exposed and thus sensitive to both interest rates and credit spreads. The deposits are exposed to interest rates and behaviour risk driven by the interest rates. The main scenarios are the interest rates rising, where the bank experiences a significant impact on net interest income and economic value in the non-trading book activities.

Performing stress and sensitivity analysis

Case 1: the bank performs deterministic shocks on IR and CS-sensitive products to perform Interest rate risk (IRRBB), and Credit spread risk (CSRBB) analysis.
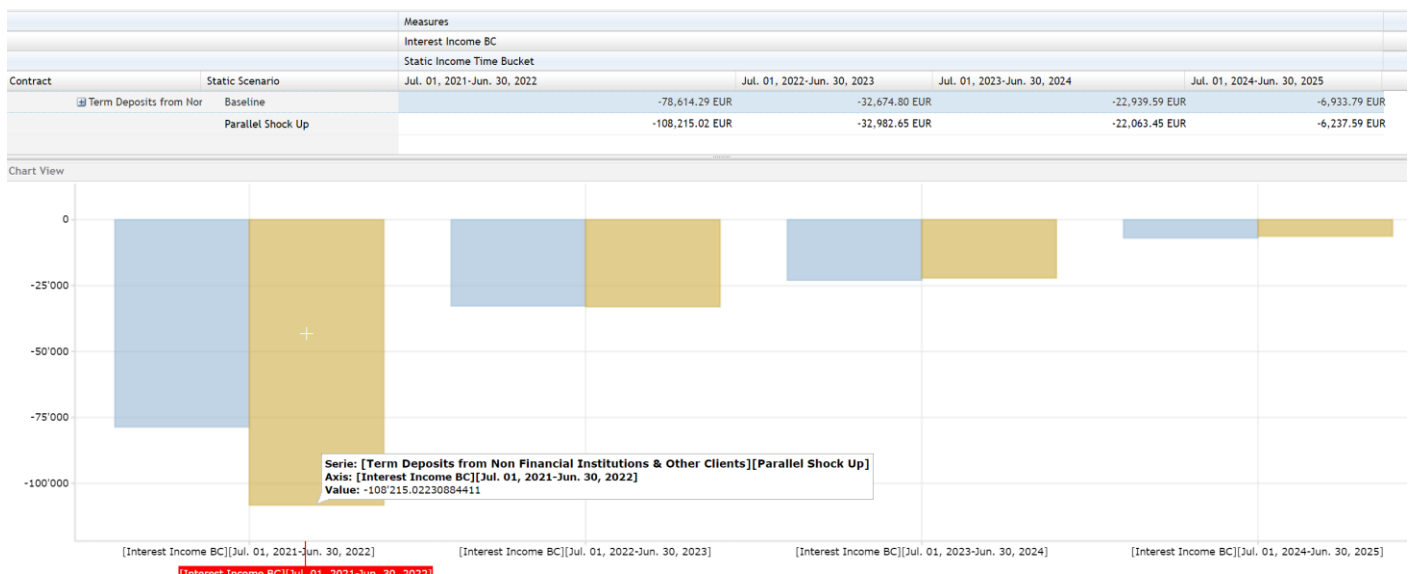
Figure 1

	Static Scenario			
	Base Case	Interest and Credit Spread shock	Credit Spread Shock	Interest Rate Shock
	Measures	Measures	Measures	Measures
Contract	NPV BC	NPV BC	NPV BC	NPV BC
⊟ Floating Mortgages	1,033,100.63 EUR	1,018,434.20 EUR	1,026,531.79 EUR	1,024,881.42 EUR
🔍 Mortgage 2 (Credit) Unsecured Lending to Corporate	515,982.78 EUR	505,435.63 EUR	509,413.94 EUR	511,882.85 EUR
🔍 Mortgage 1 Unsecured Lending to Corporate	517,117.85 EUR	512,998.57 EUR	517,117.85 EUR	512,998.57 EUR

The output of this analysis is illustrated in figure 1 above, where the shocks on the sample contracts mortgage portfolio are broken down into the interest rate and the credit spread impact.

Case 2: Based on a static run-off strategy, the bank has to perform transactional deposit and accounts sensitivity analysis to behaviour risk that is sensitive to the IR and CS risk factors.

Figure 2

	Measures				
	Interest Income BC				
	Static Income Time Bucket				
Contract	Static Scenario	Jul. 01, 2021-Jun. 30, 2022	Jul. 01, 2022-Jun. 30, 2023	Jul. 01, 2023-Jun. 30, 2024	Jul. 01, 2024-Jun. 30, 2025
⊞ Term Deposits from Nor	Baseline	-78,614.29 EUR	-32,674.80 EUR	-22,939.59 EUR	-6,933.79 EUR
	Parallel Shock Up	-108,215.02 EUR	-32,982.65 EUR	-22,063.45 EUR	-6,237.59 EUR

Serie: [Term Deposits from Non Financial Institutions & Other Clients][Parallel Shock Up]
Axis: [Interest Income BC][Jul. 01, 2021-Jun. 30, 2022]
Value: -108'215.02230884411

As can be seen in figure 2, under the parallel shock-up scenario, the increase in the interest rates triggers an increase in interest expenses in the short run. However, the depositors are running off to other asset classes with higher yields, paying back the outstanding loans and returning the interest expenses to balance.

Case 3: Performing and reporting net interest income measures within a given time horizon resulting from the movements of IR (IRRBB) and credit spreads (CSRBB).

<u>Figure 3</u>

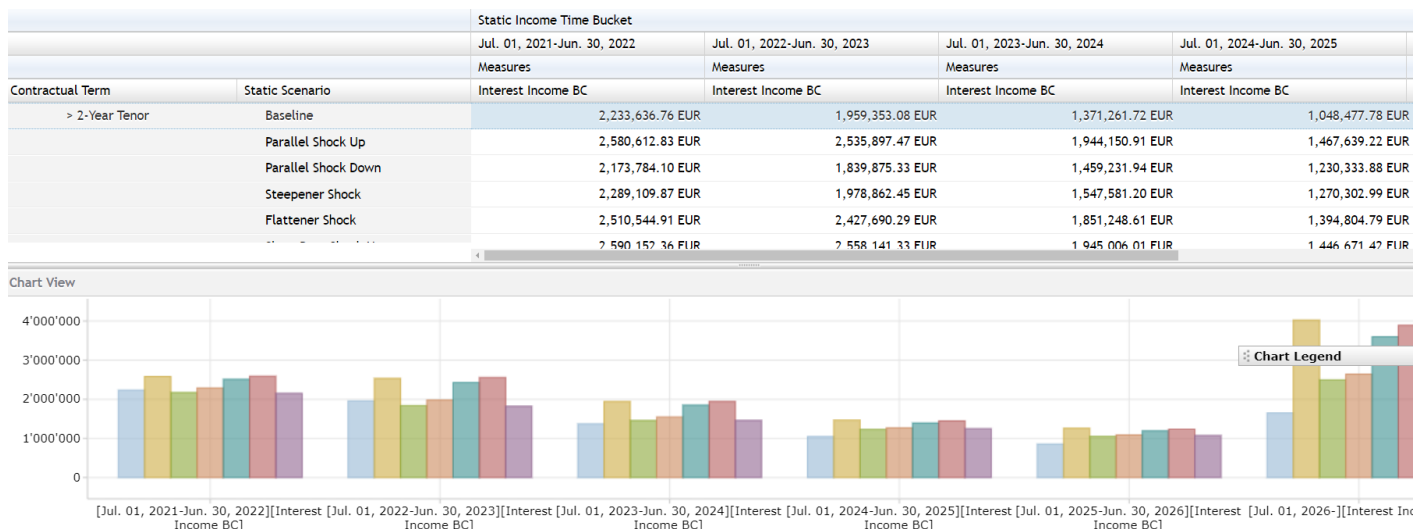| | | Static Income Time Bucket | | | |
		Jul. 01, 2021-Jun. 30, 2022	Jul. 01, 2022-Jun. 30, 2023	Jul. 01, 2023-Jun. 30, 2024	Jul. 01, 2024-Jun. 30, 2025
		Measures	Measures	Measures	Measures
Contractual Term	Static Scenario	Interest Income BC	Interest Income BC	Interest Income BC	Interest Income BC
> 2-Year Tenor	Baseline	2,233,636.76 EUR	1,959,353.08 EUR	1,371,261.72 EUR	1,048,477.78 EUR
	Parallel Shock Up	2,580,612.83 EUR	2,535,897.47 EUR	1,944,150.91 EUR	1,467,639.22 EUR
	Parallel Shock Down	2,173,784.10 EUR	1,839,875.33 EUR	1,459,231.94 EUR	1,230,333.88 EUR
	Steepener Shock	2,289,109.87 EUR	1,978,862.45 EUR	1,547,581.20 EUR	1,270,302.99 EUR
	Flattener Shock	2,510,544.91 EUR	2,427,690.29 EUR	1,851,248.61 EUR	1,394,804.79 EUR
		2,590,152.36 EUR	2,558,141.33 EUR	1,945,006.01 EUR	1,446,671.42 EUR

Chart View

Figure 3 illustrates the evolution of the impact of the interest rate shocks per scenario on the medium-term contracts in the banking book. The bank has to report, over calculation time steps, the effects of shock scenarios on the interest income.

Case 4: Bank is performing and reporting sensitivity GAPs under different terms and the underlying interest rate risk factors, as illustrated in figures 4 and 5.
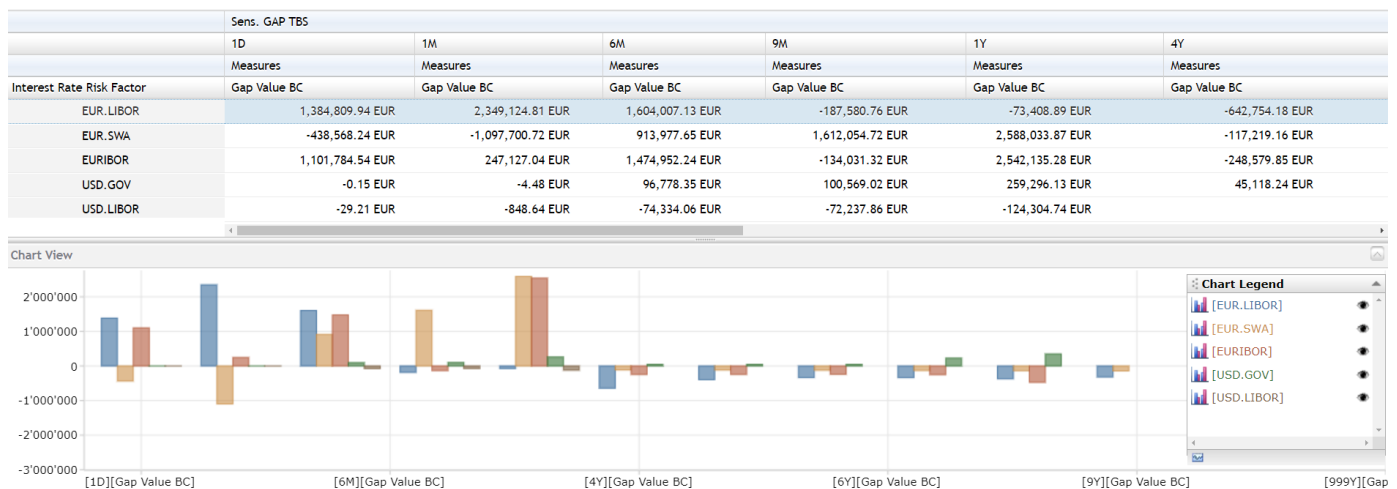
<u>Figure 4</u>

	Sens. GAP TBS					
	1D	1M	6M	9M	1Y	4Y
	Measures	Measures	Measures	Measures	Measures	Measures
Interest Rate Risk Factor	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC
EUR.LIBOR	1,384,809.94 EUR	2,349,124.81 EUR	1,604,007.13 EUR	-187,580.76 EUR	-73,408.89 EUR	-642,754.18 EUR
EUR.SWA	-438,568.24 EUR	-1,097,700.72 EUR	913,977.65 EUR	1,612,054.72 EUR	2,588,033.87 EUR	-117,219.16 EUR
EURIBOR	1,101,784.54 EUR	247,127.04 EUR	1,474,952.24 EUR	-134,031.32 EUR	2,542,135.28 EUR	-248,579.85 EUR
USD.GOV	-0.15 EUR	-4.48 EUR	96,778.35 EUR	100,569.02 EUR	259,296.13 EUR	45,118.24 EUR
USD.LIBOR	-29.21 EUR	-848.64 EUR	-74,334.06 EUR	-72,237.86 EUR	-124,304.74 EUR	

Chart View

Figure 4 illustrates the sensitivity GAP reports driven by the Parallel shock scenario.

<u>Figure 5</u>

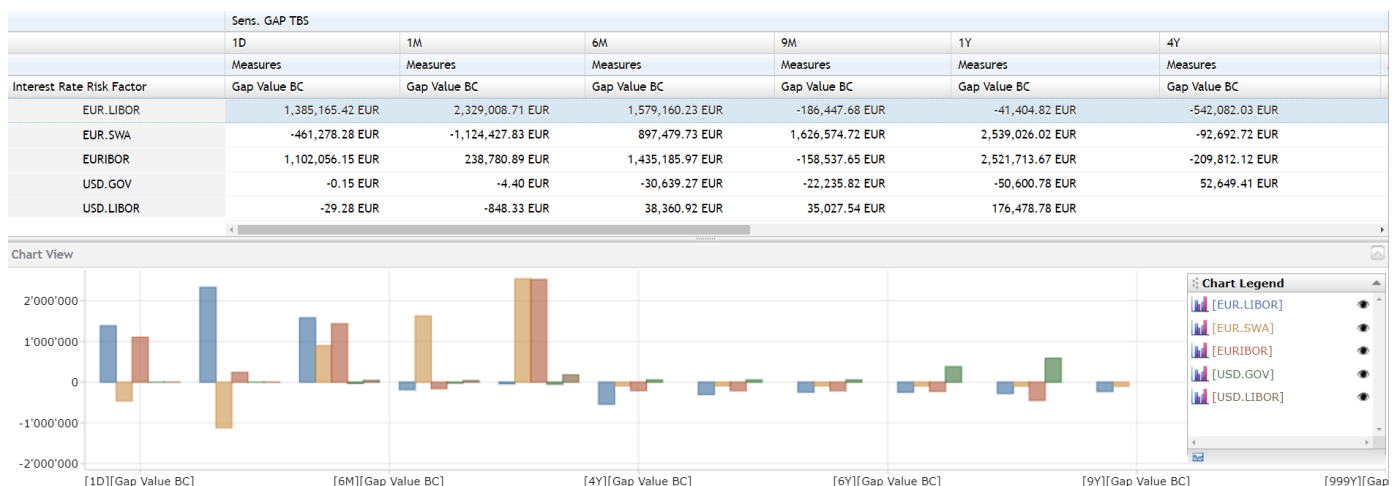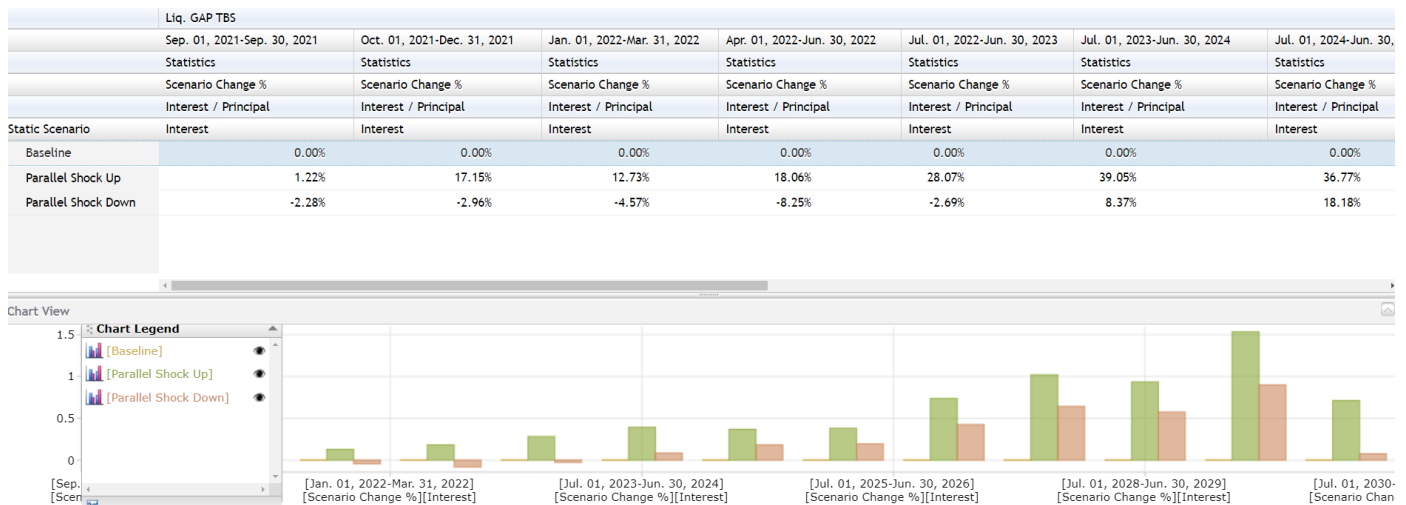| | Sens. GAP TBS | | | | | |
	1D	1M	6M	9M	1Y	4Y
	Measures	Measures	Measures	Measures	Measures	Measures
Interest Rate Risk Factor	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC	Gap Value BC
EUR.LIBOR	1,385,165.42 EUR	2,329,008.71 EUR	1,579,160.23 EUR	-186,447.68 EUR	-41,404.82 EUR	-542,082.03 EUR
EUR.SWA	-461,278.28 EUR	-1,124,427.83 EUR	897,479.73 EUR	1,626,574.72 EUR	2,539,026.02 EUR	-92,692.72 EUR
EURIBOR	1,102,056.15 EUR	238,780.89 EUR	1,435,185.97 EUR	-158,537.65 EUR	2,521,713.67 EUR	-209,812.12 EUR
USD.GOV	-0.15 EUR	-4.40 EUR	-30,639.27 EUR	-22,235.82 EUR	-50,600.78 EUR	52,649.41 EUR
USD.LIBOR	-29.28 EUR	-848.33 EUR	38,360.92 EUR	35,027.54 EUR	176,478.78 EUR	

Chart View

Figure 5 illustrates the sensitivity GAP reports driven by the Parallel shock-down scenario.

Case 5: In this case, the bank has to evaluate the cash flows that are conditional and unconditional to interest rate (IR) movements. Figure 6 illustrates the contribution of parallel interest rate up vs parallel interest rate down. As expected, the shock up over time contributes much more to the shock down.
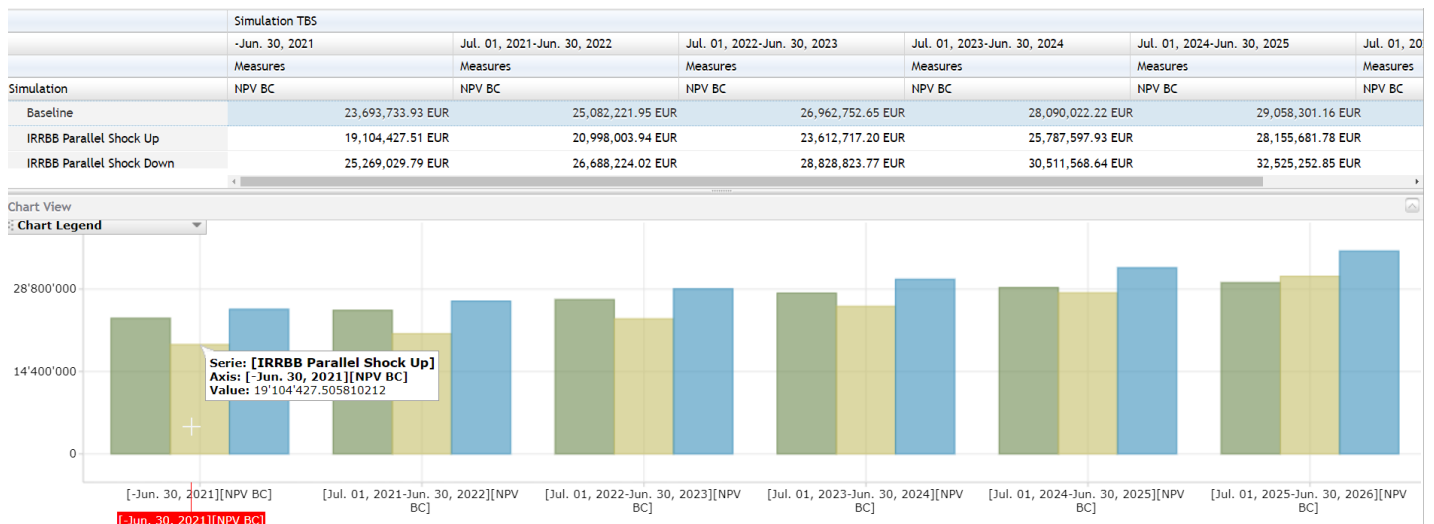
Figure 6

Liq. GAP TBS							
	Sep. 01, 2021-Sep. 30, 2021	Oct. 01, 2021-Dec. 31, 2021	Jan. 01, 2022-Mar. 31, 2022	Apr. 01, 2022-Jun. 30, 2022	Jul. 01, 2022-Jun. 30, 2023	Jul. 01, 2023-Jun. 30, 2024	Jul. 01, 2024-Jun. 30,
	Statistics	Statistics	Statistics	Statistics	Statistics	Statistics	Statistics
	Scenario Change %	Scenario Change %	Scenario Change %	Scenario Change %	Scenario Change %	Scenario Change %	Scenario Change %
	Interest / Principal	Interest / Principal	Interest / Principal	Interest / Principal	Interest / Principal	Interest / Principal	Interest / Principal
Static Scenario	Interest	Interest	Interest	Interest	Interest	Interest	Interest
Baseline	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Parallel Shock Up	1.22%	17.15%	12.73%	18.06%	28.07%	39.05%	36.77%
Parallel Shock Down	-2.28%	-2.96%	-4.57%	-8.25%	-2.69%	8.37%	18.18%

Case 6: In the dynamic balance sheet, the bank applies strategies for future business expectations and performs Economic value (EV) and Economic value of equity (EVE) measures.
In the analysis and the report illustrated in figure 7, the first time bucket, one can see the impact of the scenarios in the banking book on balancing accounts. The evolution of the economic value (i.e. dynamic balance sheet) is shown through all simulation time buckets under all three selected scenarios - the baseline, the parallel shock up and the parallel shock down.

Figure 7

Simulation TBS					
	-Jun. 30, 2021	Jul. 01, 2021-Jun. 30, 2022	Jul. 01, 2022-Jun. 30, 2023	Jul. 01, 2023-Jun. 30, 2024	Jul. 01, 2024-Jun. 30, 2025	Jul. 01, 20
	Measures	Measures	Measures	Measures	Measures	Measures
Simulation	NPV BC	NPV BC	NPV BC	NPV BC	NPV BC	NPV BC
Baseline	23,693,733.93 EUR	25,082,221.95 EUR	26,962,752.65 EUR	28,090,022.22 EUR	29,058,301.16 EUR	
IRRBB Parallel Shock Up	19,104,427.51 EUR	20,998,003.94 EUR	23,612,717.20 EUR	25,787,597.93 EUR	28,155,681.78 EUR	
IRRBB Parallel Shock Down	25,269,029.79 EUR	26,688,224.02 EUR	28,828,823.77 EUR	30,511,568.64 EUR	32,525,252.85 EUR	

As we can see in the above case, interest and spreads significantly impact the Interest income under sensitivity and stress analysis. Thus, banks must perform scenarios and strategies per the EBA expectations. The reports illustrate some of the typical results that the Bank should be able to report.

Conclusion

Even though risk managers have an ever-growing catalogue of priorities, the requirements from the EBA and BCBS on IRRBB and CSRBB should be high on that list. By setting up robust frameworks, a bank can remain compliant, optimized and organized. And of course, interest rates do not only affect IRRBB, but all risk types, including credit and behaviour risks, impacting value liquidity and P&L. A bank should be looking at risks holistically in order to drive a unified risk management strategy across the portfolio, while stress testing and scenario analysis shouldn't be static. The need to look ahead is paramount, where IRRBB and CSRBB should be measured and managed over the evolution of the markets and future portfolios.
Therefore, regulatory obligations aside, the analysis gained from thorough stressing and scenario building, can be priceless in understanding exactly how interest rates will affect banks' bottom lines.

New AIFIRM website section dedicated to free E-books publication

Editorial

AIFIRM is pleased to announce the launch of a new section of the Association's website dedicated to the publication of Free E-Books on Risk Management subject matters.

The direct link to access it is shown hereafter: https://www.aifirm.it/documenti/free-ebook/.

AIFIRM's commitment, as a publisher, is solely aimed at promoting the dissemination of culture and at fueling the debate on financial risk topics, while the author maintains sole responsibility and rights to the contents of the work.

The books – contrary to what happens for the AIFIRM POSITION PAPERS – are not subject to scientific assessment by the Technical Scientific Committee and the Council, but the authors' competence is still certified by AIFIRM through the verification that they have publications on risk management in scientific journals accredited by ANVUR or SCOPUS.

The first published contribution belongs to the series "AIFIRM Edizioni - Educational Series" and it is "Notes on Quantitative Financial Analysis" by Pier Giuseppe Giribone.

Notes on Quantitative Financial Analysis

This volume aggregates a set of notions which introduce the fundamentals of quantitative financial analysis in a clear and concise way, providing a very practical approach, as demonstrated by the discussion of numerous case studies. The work has an unconventional structure, presenting a reasoned collection of slides, which can be used by the reader immediately and easily, even for arranging a presentation for educational purposes. All material can be freely used, quoting the source. The codes, if not protected by copyright, are available upon request.

A specific background is not strictly required for the reader, although basic notions of economics and statistics would be recommended. The book is divided into seven sections and each of them has a chapter structure. Below is a brief summary of the covered topics:

Part I: Fixed Income Instruments

The first chapter is a summary of the main concepts of financial mathematics underlying quantitative analysis, up to the modeling of the interest rates term structure.

The second chapter shows the different types of bonds present in the financial markets, together with the assessment of the risks that an analyst must manage.

The third part explores the heart of quantitative analysis, introducing the best practices for estimating the fair value of a bond, together with its risk measures (duration, modified duration and convexity).

Part II: Futures and Forwards

After the description of the basic concepts for understanding this category of derivatives, the second chapter introduces the specific quantitative analysis of these instruments, with a particular focus on pricing and hedging.

Part III: Options

Given the inherent variety of topics connected to options, this section has been thoroughly covered. In addition to the description of the standard pay-off, the first chapter deals with the foundations of this derivative and introduces the mathematical properties, including the put-call parity.

The second chapter concerns the pricing of plain-vanilla options. The well-known Black-Scholes-Merton pricing framework has been introduced, showing how it can be applied to options written on different underlyings (equity, index, rates, futures and currencies). In addition to the fair value, the sensitivities (Greeks) are also estimated.

The third chapter deals with option strategies: combinations of plain-vanilla options with underlying and with other options, in order to create specific hedging and trading strategies. Among the strategies, covered call, protective put, bull/bear spread, butterfly spread, straddle, strip, strap and strangle are covered.

The fourth chapter reviews the main non-standard (i.e. exotic) options, characterized by special pay-offs. The lognormal pricing framework is extended to these types of options; among them: forward start, cliquet, digital, chooser, compound and path-dependent (barrier, Asian and lookback) options.

Not all options can be adequately priced using a closed formula. For those characterized by particularly non-linear pay-offs or by early exercise features, a numerical methodology has to be implemented.

Chapter 5 is therefore dedicated to binomial stochastic trees, particularly useful for dealing with derivatives characterized by the possibility of being exercised in advance, while chapter 6 is dedicated to the Monte Carlo technique, which is considered suitable for representing any type of pay-off, thanks to its flexibility. The working principle, the internal consistency, the pricing estimation, and the computation of the most important risk measures are illustrated for both algorithms. Once the reader has become confident on the correct approach for the quantitative analysis of the derivative, it is time to focus on the inputs of the model.

Finally, Chapter 7 centers on determining the inputs for the previously exposed techniques. A particular focus has been given to the estimation of volatility (both historical and implicit) and to the correlation.

Part IV: Swaps

Similarly to the previous scheme, the section dedicated to swaps is divided into two parts: the first chapter describes the fundamentals of the different types of swaps, while the second deals with the quantitative analysis of the instrument. Particular attention is paid to Interest Rate Swaps (IRS) and Currency Swaps. Two distinct valuation approaches are provided, i.e. considering the derivative as a portfolio of forward contracts, or as two positions (one long and one short) in two bonds. The second chapter concludes with the derivation of long-term spot rates from Interest Rate Swaps, a process known as swap curve stripping.

Part V: Credit Derivatives

This section consists of only one chapter in which Credit Default Swaps (CDS) are presented. It describes how premiums can be used to compute risk-adjusted discount factors in a fixed income instrument pricing context. The chapter ends with an introduction of the most popular models among analysts for the pricing of these derivatives.

Part VI: Aggregate Risk Measures

The risk measures discussed so far have addressed the single instrument and can hardly be extended to a portfolio, characterized by instruments of a different financial nature. Considering this need, the most common approaches to estimating Value-at-Risk have been introduced: parametric, full-evaluation, Monte Carlo backward and forward looking. The Expected Shortfall and the importance of conducting stress tests and back tests are briefly presented as well.

Part VII: Credit Risk

The first chapter focuses on the definition and on the mathematical models for estimating counterparty risk, which can be interpreted by its nature as a hybrid between financial risk and credit risk.

In particular, it has been shown that the probability of default can be inferred from Credit Default Swap (CDS) premiums, listed bond spreads or stock prices using the KMV (Kealhofer, Merton and Vasicek) model. The last part of the chapter highlights the structural limits of counterparty risk, validating the need to provide a more complete definition of credit risk. The latter is based on three pillars: the probability of default (PD), the Loss Given Default (LGD) and the Exposure at Default (EAD). An effectual discussion is dedicated to each of these three important components.

The second chapter deals with the statistical approaches that allow the estimation of PD starting from historical data (not necessarily market data), among which, the Altman's Z-Score, the Logit-Probit and the CreditGrades models are covered.

The third chapter introduces the regression models suitable for estimating and forecasting the Loss Given Default.

The fourth chapter deals with the estimation and the predictive models for EAD. In this context, a Monte Carlo model is introduced for the determination of the Credit Valuation Adjustment (CVA) with particular attention to the modeling of the Expected Exposure to the various future time buckets. Once the reader has acquired the required knowledge for a correct credit measurement, we move on to the concept of rating systems.

The fifth chapter introduces Rating Agencies and provides the basic notions for creating transition matrices. The Cohort approach and the Hazard approach are adequately discussed with the relative methods of calculating confidence intervals.

The sixth chapter deals with credit risk managed not on a single position, but at portfolio level. In this phase asset correlation has to be presented and, to this end, the Moment matching and the Maximum Likelihood approaches are explained. An example of estimating a Monte Carlo VaR and a C-VaR is also provided in the credit context.

The part dealing with credit concludes with the main methods for validating credit models. Among those, the Cumulative Accuracy Profile (CAP), the Receiver Operating Characteristics (ROC), the binomial test and the Brier Score are covered.

At the end of each chapter, further food for thought is provided through a bibliography of reference papers or books, which allow useful insights into each topic covered.

About the author

Pier Giuseppe Giribone is a Financial Engineer at BPER Banca, where he has been working since 2009, in particular handling the valuation of structured financial products and the calculation of their risk measures. Pier is also an Adjunct Professor at UNIGE in the Master's Degree «Economics and Data Science».
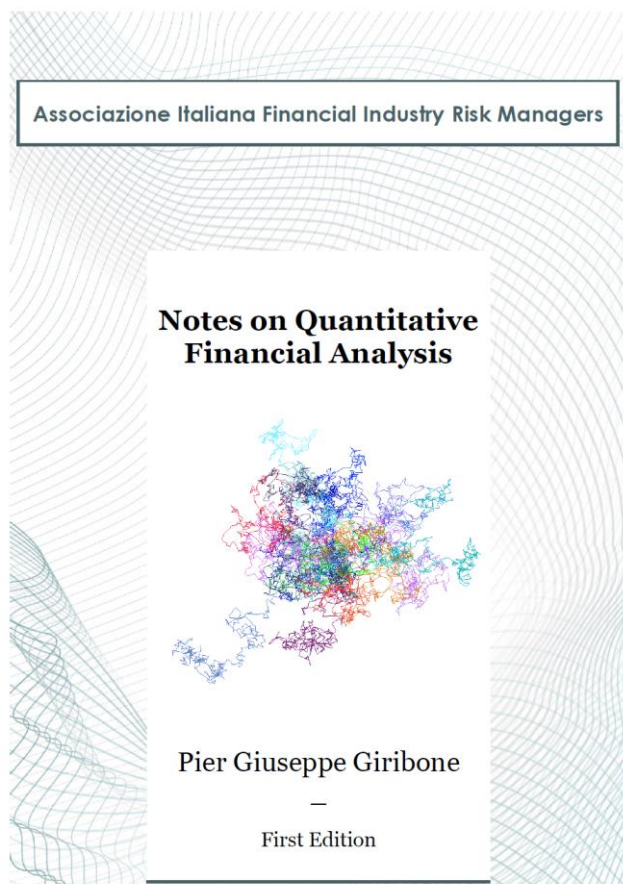
He has earned two PhDs: the first one in 2013 in Mathematical Engineering and the second one in 2021 in Economics, analyzing research topics in the scope SECS-S/06: Mathematical Methods in Economics and SECS-P/11: Economics for Financial Intermediaries.

In July 2022, he completed an Executive MBA, discussing his thesis «Essay on Banking Credit Management and Measurement».

Besides, while working for the bank, he has completed further master courses, among which: Banking and Financial Diploma, CIIA, CESGA and CIWM. Pier cooperates as a lecturer with "Il Sole 24 Ore" Business School for the master courses "Credit Risk Management" & "Data science, Big Data and Artificial Intelligence in Finance".

He has written more than 50 papers which were published in scientific magazines.

Finally, Pier has also been a reviewer for the American Mathematical Society (AMS) since 2020, as well as a referee for several international scientific journals, among which Springer Verlag and Frontiers in Artificial Intelligence.

Associazione Italiana Financial Industry Risk Managers

Notes on Quantitative Financial Analysis

Pier Giuseppe Giribone

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First Edition

SUMMARY

"Notes on Financial Quantitative Analysis" – E-book cover and summary

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Soddisfare i requisiti normativi, gestire le operazioni finanziarie, affrontare ogni singolo rischio ed elaborare una strategia integrata per rischio e finanza, sono tutti aspetti su cui lavoriamo insieme a piu' di 800 banche.

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