RISK MANAGEMENT MAGAZINE

Vol. 20, Issue 3
September – December 2025

In collaboration with

Editor in chief: Maurizio Vallino – Managing Editor: Corrado Meglio
INDEPENDENT JOURNAL WITH NO PUBLIC FUNDING (Law 250/1990)
Poste Italiane - Spedizione in abbonamento postale – 70% aut. DCB / Genova nr. 569 anno 2005

IN THIS ISSUE

PAPERS SUBMITTED TO
DOUBLE-BLIND PEER REVIEW

Scientific journal recognized by ANVUR and AIDEA

Risk Management Magazine
Volume 20, Issue 3, Semptember - December 2025

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Registration number at Court of Genova (Italy) n. 1054 dated 12 February 2025

ISSN Print 2612-3665 – ISSN Online 2724-2153
DOI 10.47473/2016rrm

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Printing
Algraphy S.n.c. - Passo Ponte Carrega 62-62r 16141 Genova

The authors bear sole responsibility for the opinions expressed in the articles

MAILED TO AIFIRM SUBSCRIBERS WHO ARE RESIDENT IN ITALY AND DULY REGISTERED

Journal printed on 12th December 2025

Editorial

Recent scholarly contributions underscore the growing complexity of financial systems and the imperative for robust analytical and risk management frameworks. The valuation of hybrid instruments, such as convertible bonds, remains a critical area of inquiry. One study addresses this by implementing alternative stochastic binomial tree models—specifically the Haahtela, Jarrow-Rudd, and Tian schemes—while examining their convergence properties and reliability. Through an empirical application to the German market, the research fills a notable gap in the literature on pricing methodologies for these instruments.

Corporate finance dynamics in emerging economies also warrant attention. Evidence from South African firms between 2012 and 2023 reveals a significant inverse relationship between long-term debt ratios and profitability, measured by ROA and ROE. Conversely, liquidity indicators exhibit a positive and robust effect on both financial performance and market valuation. These findings provide actionable insights for optimizing capital structure and liquidity management in volatile environments.

Operational risk management has gained renewed prominence under Basel IV, which introduces a standardized approach for calculating operational risk capital. The framework's emphasis on risk sensitivity and heightened capital requirements compels financial institutions to reassess internal processes and resilience strategies. This regulatory evolution signals a paradigm shift toward more stringent governance and systemic stability.

Finally, the integration of risk management practices into public procurement processes emerges as a critical determinant of efficiency and accountability. Empirical evidence from Nigeria's Lower Benue River Basin Development Authority demonstrates that structured risk identification, mitigation, and monitoring significantly influence supplier selection outcomes. These findings advance procurement literature and offer practical guidance for embedding risk-aware decision-making in public sector governance.

Collectively, these studies illuminate the interdependence of valuation techniques, capital structure optimization, regulatory compliance, and risk governance. As financial ecosystems evolve, the convergence of rigorous quantitative models and comprehensive risk frameworks will remain central to sustaining institutional integrity and long-term performance.

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Alternative Stochastic Binomial Trees for Quantitative Analysis of Convertible Bonds

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Article submitted to double-blind peer review, received on 21st September 2025 and accepted on 10th November 2025

Abstract
The objective of the present study is to implement the alternative stochastic binomial trees for the evaluation and estimation of the main sensitivity measures of convertible bonds, thus filling a gap in scientific literature. The paper proposes the implementation of the Haahtela, Jarrow-Rudd and Tian numerical schemes and explores the characteristics, convergence properties and reliability of these evaluation tools. A comprehensive case-study considering the German market, which is an extremely active market in the issuance and trading of these hybrid instruments, is also illustrated.

Key Words: Convertible bonds, alternative stochastic binomial trees, Cox-Ross-Rubinstein, Haahtela, Jarrow-Rudd, Tian

JEL codes: C53, C63, G12, G32

1) Introduction
The valuation of many complex financial instruments, such as convertible bonds, typically requires numerical methods due to their option-like features.
A convertible bond is a hybrid security that retains most characteristics of straight debt while also offering the upside potential associated with underlying common stock. A convertible bond is a corporate security giving the bondholder the right, but not the obligation, to convert the bond into another security, typically the ordinary shares of the issuing company, under specific conditions. Once converted into ordinary shares, they cannot revert into bonds. Due to their structure and their option feature, convertibles show characteristics of both debt and equity instruments, leading to their classification as hybrid instruments. A similar instrument is convertible preferred stock, which means preference shares that can be converted into ordinary shares based on specific terms. Convertibles play an important role in corporate finance and have benefited from advanced valuation models originally created for option markets. Their hybrid nature initially presented challenges in analysis and valuation, but modern techniques have largely solved these issues. As a result, issue volumes increased steadily through the 1990s, particularly during rising stock markets. Convertible bonds are fixed-coupon securities issued with the option to convert into equity, at the bondholder's discretion, under pre-determined terms. These bonds are usually subordinated, and only companies with strong credit ratings can issue them. The market perception of the issuer's stock performance is also critical, since investors are purchasing the right to subscribe for shares at a future date, potentially at a premium over the market price. Consequently, the price of a convertible bond fluctuates following changes in the underlying stock price and in interest rates. Convertibles are typically medium to long-term instruments with maturities of 10 to 20 years, and their coupons are typically lower than those on non-convertible bonds from the same issuer. In addition to basic fixed-coupon convertibles, various other instruments are also available. They include zero-coupon convertibles, issued at a deep discount with a low probability of conversion, and discount convertibles. Some convertibles are callable by the issuer, allowing them to force conversion under certain conditions. They are called convertible calls and reduce the bondholder's discretion, potentially leading to unfavourable terms. Conversely, puttable convertibles allow bondholders to redeem the bond or convert it at their pleasure, providing downside protection. Premium put convertibles can only be converted on a single date, while rolling put convertibles offer multiple conversion dates and are generally issued with a lower coupon.
Another variant is the exchangeable security, a bond issued by one company that is convertible into the shares of another company in which the issuer holds a significant interest. Bonds with warrants are convertible bonds issued with an attached warrant that can be traded individually. Step-up convertibles and preference shares are newer innovations, offering a fixed coupon for the first few years before increasing the coupon until maturity or conversion.
Convertible bonds have a long history in capital markets, with the first issuances by U.S. utility companies in the 19th century. In 1997, the global convertibles market was valued at over $360 billion, with the U.S. as the largest issuer, historically dominated by utility and transport companies. Unlike domestic and international securities, convertibles are primarily exchange-traded, offering more liquidity and transparency compared to OTC bonds. However, liquidity still depends on the number of market makers and the volume of the issue, making some convertibles less liquid than conventional bonds.

Contingent Convertible bonds (CoCos), also referred to as enhanced capital notes (ECNs) constitute another variant. While both instruments involve the conversion of debt into equity, they differ fundamentally in terms of trigger mechanisms, investor control, and intended function within the financial system.
CoCos are designed primarily as regulatory capital instruments. They automatically convert into equity when a specific trigger is met, usually when the issuing bank's capital ratio falls below a defined threshold. Unlike standard convertibles, CoCos do not offer conversion at the investor's discretion. Instead, conversion is imposed under adverse conditions, often resulting in the receipt of equity at depressed valuations. The main function of CoCos is to enable financial institutions, particularly banks, to absorb losses and maintain solvency without external support. These instruments gained prominence following the 2007–2008 financial crisis, when they were introduced as a mechanism to strengthen bank capital structures and reduce the need for taxpayer-funded bailouts (De Spiegeleer and Schoutens, 2014).
From an investment perspective, standard convertibles offer a more favourable risk-return profile under normal market conditions, combining steady cash flows with upside equity participation. CoCos, on the other hand, carry a higher level of risk, as their conversion is typically triggered in times of financial distress. If the trigger condition is never met, CoCos may be redeemed at maturity in the same way as conventional bonds. The convertible bond market represents a significant and growing segment in the landscape of fixed-income financial instruments. In recent years, the global convertible bond market has seen a substantial expansion. According to Bloomberg® data and other market sources, the total amount of convertible bond issuances reached $150 billion in 2021, an increase from previous years, driven by favourable market conditions and growing demand for hybrid instruments. S&P Global reports that 2021 was a particularly dynamic year for new issuance, fuelled by market volatility that incentivised companies to seek flexible financing arrangements. Among the most active sectors are technology and healthcare, where companies often need capital to support innovation and long-term growth. On the investor side, convertible bonds are valued for their ability to mitigate risk. With the bond component, investors enjoy downside protection while maintaining a fixed return, while the conversion component offers upside potential if the share price rises. Morningstar points out that funds specialising in convertible bonds have been able to outperform traditional fixed-income funds in good market times, thanks to the ability to participate in stock market rises.
Geographically, the convertible bond market is dominated by the US, Europe and Asia, with the US accounting for the largest share of issues (Calamos Investments, 2021). In Europe, issuances are mainly driven by the banking and technology sectors, where CoCo (Contingent Convertible Bonds) instruments play a key role in banks' regulatory capital. In Asia, countries such as Japan and Hong Kong have developed well-regulated and growing convertible bond markets, with strong interest from both local issuers and global investors.

2) Fair value determination of a Convertible Bond

Convertible bonds embed characteristics of both debt and equity, making their valuation challenging. The presence of multiple embedded option and early-exercise features often makes closed-form solutions unsuitable, especially within realistic market conditions.
In early works by Ingersoll (1977), such solutions rely on assumptions of market completeness and continuous-time trading, which severely limit their applicability. Further critiques by Nyborg (1996) highlighted the inability of analytical models to incorporate market frictions, non-linear payoff structures, and conditions such as early calls or forced conversions.
This complexity led to the development of numerical approaches, particularly within two dominant theoretical frameworks: the structural and the reduced-form approaches.
The structural approach, developed by Ingersoll (1977) and Brennan and Schwartz (1980), models the bond as a contingent claim on the firm's value, assuming the default occurs when the firm's asset value falls below a given value. This method assumes a simplified capital structure, continuous monitoring and perfect observability of firm value, greatly limiting its practical relevance. Furthermore, estimating volatility on a firm-level asset base is commonly unfeasible, especially when other senior claims coexist (Brennan and Schwartz, 1980).
The reduced-form approach, theorized by McConnell and Schwartz (1986), assumes the convertible bond as a contingent claim on the underlying stock price, where credit risk is modeled exogenously. Early implementations, such as Ho and Pfeffer (1996), used a risk-adjusted constant credit spread to reflect default risk, but this assumption has been criticized for failing to capture time-varying credit conditions.
A key contribution in this space was the Tsiveriotis and Fernandes (1998) model, which decomposes a convertible bond into a fixed-income part (subject to credit risk) and an equity-linked part (considered default-free), each discounted with different rates. This model became widely adopted due to its tractability and alignment with market practice.
Refinements by Ayache, Forsyth and Vetzal (2003) and Gushchin and Curien (2008) introduced endogenous modeling of default probability and recovery rates. The former in particular embedded credit risk within both the equity and debt components and incorporated partial recovery, producing more accurate pricing compared to the Tsiveriotis and Fernandes

(1998) and Brennan and Schwartz (1980) models in empirical comparisons. Gushchin and Curien (2008) also demonstrated that modeling credit spreads as stochastic processes, rather than constant, improved calibration and reduced errors.
Given the multidimensional risk profile of convertible bonds, numerical methods are nowadays the dominant paradigm.
Among these, lattice-based methods (or stochastic tree models) play a central role due to their intuitive structure and ability to capture discrete events and decision points such as coupon payments, callability, or conversion. These methods discretize time and the evolution of relevant state variables across a grid, where the bond value is recursively computed from maturity to the present, reflecting the expected payoff under each possible path.
Hung and Wang (2002) developed a binomial tree model incorporating stochastic interest rates via the Ho and Lee (1986) model and default risk through the Jarrow and Turnbull (1995) framework, assuming the stock price jumps to zero upon default. They assigned probabilities to stock price, interest rate, and default transitions, concluding that such integration improved pricing accuracy. Chambers and Lu (2007) refined this approach by introducing a non-zero correlation between stock returns and interest rates, an often-overlooked dependency, using the same interest rate and credit models. Their results underscored the importance of jointly modeling correlated risks in convertible bond valuation.
Das and Sundaram (2007) incorporated stochastic volatility using the Constant Elasticity of Variance (CEV) model, along with stochastic interest rates (modeled à la Heath, Jarrow, Morton, 1990) and default risk. Their framework allowed for correlation between equity and term structure dynamics and showed that the sensitivity of bond value to credit risk is magnified under high volatility conditions.
Ho and Pfeffer (1996) used a binomial model with deterministic volatility and interest rates but introduced credit risk through an option-adjusted spread. Their study on US callable convertible bonds highlighted systematic underpricing in the market.
Trinomial trees have also been adopted to improve convergence and computational accuracy. Gushchin and Curien (2008) used a trinomial tree within the Tsiveriotis-Fernandes framework and found their approach capable of handling a wider range of instruments and market conditions.
Rotaru (2006) applied another trinomial method and modeled callable convertibles using deterministic volatilities and credit spreads and observed consistent underpricing across different market segments.
Overall, tree models have proven to be among the most effective methods in handling the discrete features and early-exercise optionalities typical of convertible bonds. Their flexibility allows for the modeling of complex structures. While simulation methods, such as Monte Carlo approaches, are often preferred for higher-dimensional problems and deeply path-dependent features, tree-based methods remain a robust and intuitive alternative, that balances accuracy and computational feasibility.
Despite these studies, significant gaps remain.
Most tree models rely on simplifying assumptions such as deterministic volatility, constant credit spreads, or uncorrelated risk factors, which reduce realism. Relatively few contributions have succeeded in jointly integrating stochastic equity, interest rates, and credit spreads within a single computationally tractable lattice framework. The consistent empirical evidence of underpricing across various markets suggests that the valuation of convertible bonds remains an open research area. These observations point toward the need for hybrid or adaptive numerical schemes that can reflect complex market imperfections while maintaining robustness and interpretability.
The pricing of convertible bonds has traditionally relied on established numerical methods such as finite difference models, finite element methods, binomial or trinomial trees and Monte Carlo methods. However, the adoption of alternative tree models remains relatively limited, indicating a potential gap in the research. Recent studies started to explore these alternative models.
Hu, Li and Liu (2022) introduced a Jarrow-Rudd model that incorporates asymmetry and kurtosis in the returns of the underlying asset, using an asymmetric random walk process.
This approach allows for more accurate calibration to implied volatility surfaces and includes hedging costs, bringing the model closer to real market scenarios. The foundation for this line of modeling can be traced back to Jarrow and Rudd (1986), who proposed a simplified binomial tree based on equal probabilities for upward and downward price movements. Although computationally efficient, their approach lacked the risk-neutral valuation framework, which limited its practical application in derivative pricing, Jarrow and Rudd (1986). Despite its applicability, the use of the JR model is not yet widespread in this context. Similarly, Tian proposed a flexible binomial tree that matches not only the mean and variance but also the skewness of the underlying asset return distribution. This enhancement improves convergence and accuracy in option pricing, particularly for instruments sensitive to higher-order moments (Tian, 1993a and Tian, 1993b). Haahtela (2006) extended this line of development by introducing a trinomial tree framework designed to incorporate informed trading and asymmetric information, offering a structure more aligned with realistic market behaviour.
Milanov and Kounchev (2012) developed a binomial tree model for pricing convertible bonds that accounts for credit risk, demonstrating convergence to the model of Ayache, Forsyth and Vetzal (2003). This model integrates both reduced and synthetic approaches for modeling default risk, offering an alternative to traditional models.

Despite these developments, the adoption of alternative tree models in convertible bond pricing remains marginal. Most studies continue to focus on traditional lattice models, which have limitations in incorporating more complex market features. There is a significant opportunity to further explore and develop alternative tree models in the context of convertible bonds. These models could offer greater flexibility and accuracy in valuation, especially in complex market scenarios or when dealing with non-standard optional features.

In the next section, we discuss the most widely used valuation methodology for convertible bonds, namely the Cox, Ross and Rubinstein stochastic binomial tree (CRR Tree). The theoretical foundation for this approach was initially proposed by Cox and Ross (1976), who explored the valuation of options under alternative stochastic processes. In the fourth section, we then review the three most popular alternative binomial trees: Haahtela, Jarrow-Rudd and Tian. We first explain the theoretical principles and peculiarities of each approach, then implement them in pricing European and American options.

European options allow the holder to exercise the option only at expiration, while American options offer greater flexibility by allowing exercise at any time up to expiration. This flexibility makes American options more complex to price, as it requires taking advantage of early exercise opportunities.This step is considered to be preparatory to the application of these techniques to the quantitative analysis of convertible bonds because it provides evidence of the correct implementation of the method. Once we verify that the alternative stochastic binomial trees converge correctly, we can implement them for the valuation of a convertible bond. In the fifth section, we then show how to adapt the traditional CRR numerical scheme to alternative techniques: after illustrating the step-by-step procedure and highlighting implementation differences, we provide proof of correct implementation through a convergence analysis. In order to conduct a complete quantitative analysis, we also numerically estimate the sensitivity of the models to the main risk parameters, i.e., the change in the underlying (Delta and Gamma) and volatility (Vega). In this analysis, the Jarrow-Rudd tree has shown a different sensitivity than the other approaches on Vega.

In the sixth section, we provide empirical evidence for our findings by offering numerous market cases suitable for confirming the analysis performed. Robustness in the estimation of the main quantitative measures that can be associated with a convertible bond for all the different alternative stochastic binomial trees discussed in the study is thus proven.

3) The most widespread pricing methodology for pricing a Convertible Bond: the CRR Tree

We provide a short explanation of the working principle of the Cox, Ross and Rubinstein Tree in Appendix A, together with a proof of the derivation of the three main parameters (the up factor: u, the down factor d and the probability $\prod$) that rule the projections of the underlying. We assume the up and down parameters to be constant throughout the paper.

The mathematical notation used for the description is the standard notation adopted by (Haug, 2007) and briefly described in the same Appendix. Consequently, this paragraph focuses exclusively on the implementation of this technique in the convertible bond pricing.

As we have already explained, a convertible bond can be viewed as a combination of a traditional bond and a stock option. When the stock price is significantly lower than the conversion price, the convertible bond behaves like a simple bond. Conversely, when the share price is much higher than the conversion price, it behaves more like a stock. This dynamics should influence the discounting of cash flows.

In a risk-neutral setting, this does not mean arbitrarily changing the discount rate; rather, it reflects the fact that the appropriate discount rate depends on the nature of the payoff being replicated. For a deeply out-of-the-money convertible, future cash flows should be discounted at a rate that includes the credit spread k above the bond's Treasury rate.

If the convertible is deeply in-the-money, the conversion is almost certain, and the cash flows should be discounted at the risk-free rate. Bardhan et al. (1994) incorporated these considerations by applying a discount rate based on a variable conversion probability.

The CRR Tree model begins with a standard binomial stock price tree.

The convertible bond price is then calculated by working back from the final nodes of the stock price tree, ensuring that the value of the convertible at each end node equals the greater of the conversion value or the face value plus the final coupon. To roll backward through the tree, backward induction is used.

If it is optimal to convert the bond, the value is set equal to the conversion value at that node, or else the convertible bond value $P_{n,i}$ is set equal to:

$$P_{n,i} = \max\left[mS, \prod P_{n+1,i+1} e^{-r_{n+1,i+1}\Delta t} + (1-\prod) P_{n+1,i} e^{-r_{n+1,i}\Delta t}\right] \quad (1)$$

where *m* represents the conversion ratio. Certain convertible bonds include an initial lockout period during which conversion is not permitted. At these nodes, the convertible bond value can be simplified to:

$$P_{n,i} = \prod P_{n+1,i+1} e^{-r_{n+1,i+1}\Delta t} + (1-\prod) P_{n+1,i} e^{-r_{n+1,i}\Delta t} \quad (2)$$

Rather than applying a constant discount rate r, the discount rate $r_{n,i}$ is adjusted to vary with the conversion probability $q_{n,i}$ at each node. The conversion probabilities $q_{n,i}$, where *n* is the time step and *i* is the number of up moves (state), are determined by working backward from the end of the stock price tree.
If conversion is optimal at a given end node, the conversion probability is set to 1; otherwise, it is set to 0. For earlier time steps, the conversion probability is also set to 1 if it is optimal to convert at that node; otherwise, it remains at:

$$q_{n,i} = \prod q_{n+1,i+1} + (1 - \prod) q_{n+1,i} \quad (3)$$

The credit-adjusted discount rate is set equal to a conversion probability weighted mixture of the risk-free rate and the credit-adjusted rate. This gives a discount rate for up moves equal to:

$$r_{n,i} = q_{n,i} r + \left(1 - q_{n,i}\right)(r + k) \quad (4)$$

The discount rate is therefore set to the constant risk-free rate r when the conversion probability is 1, and to $r + k$ (the risk-free rate plus the credit spread) when the conversion probability is 0. For conversion probabilities between 0 and 1, the discount rate transitions smoothly between the risk-free and credit-adjusted rates.

Example

In this subsection, we analyse a traditional pricing of a Convertible bond, Haug (2007): we generated the following trees using the data provided in the book and implemented it in a more efficient numerical environment (Python).
Let us consider a convertible corporate bond with five years to maturity. The continuously compounding yield on a five-year treasury bond is 7%, the credit spread on the corporate bond is 3% above treasury, the face value is 100, the annual coupon is 6, the conversion ratio is 1, the current stock price is 75, and the volatility of the stock is 20%.
Consequently, the inputs of the model are: $S = 75$, $T = 5$, $r = b = 0.07$, $k = 0.03$, $m = 1$, and $\sigma = 0.20$.
To price the convertible bond, we need to build a standard binomial stock price tree. With the number of time steps $n = 5$, we obtain Δt = 1 and up and down factors are:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2\sqrt{1}} = 1.2214 \qquad d = \frac{1}{u} = 0.8187$$

The probability of an increase in price is thus given by:

$$\prod = \frac{e^{b\Delta t} - d}{u - d} = \frac{e^{0.07\times 1} - 0.8187}{1.2214 - 0.8187} = 0.6302$$

and we obtain the binomial stock price tree in Figure 1.

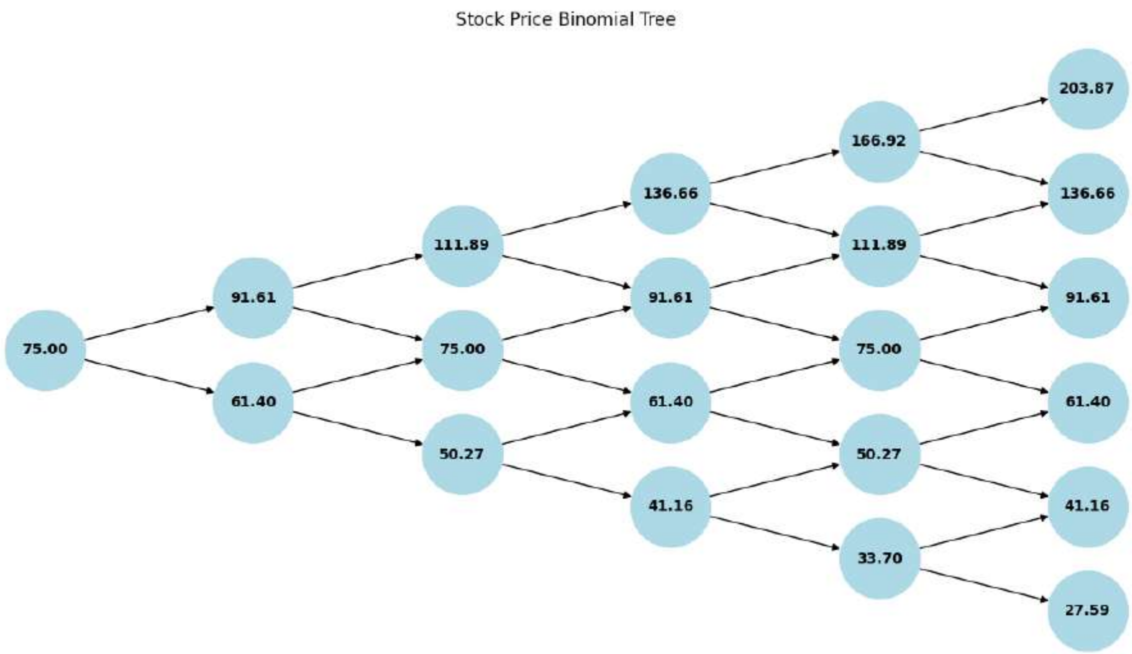

Figure 1: 5-steps Stock Price Binomial Tree: numeric example

The next step is to find the convertible bond values and the conversion probabilities at each node in the tree. Firstly, let us look at the calculation of several nodes.
At the end node with stock price 203.87, it is better to convert the bond into one stock and receive the stock price 203.87 rather than get the notional plus the coupon (100 + 6). The probability of conversion at this node, $q_{5,5}$, is 100%, which we write as 1.00 in the conversion probability tree.
At the end node, with a stock price of 91.61, it is better not to convert the bond and receive the face value plus the coupon of 106. The probability of conversion is $q_{5,3} = 0$. For the node at year four (n = 4) with stock price 111.89, the convertible bond value of 121.77 is found using equation (1):

$$P_{4,4} = \max[1 \times 111.89, 0.6302 \times 136.66e^{-r_{n+1,i+1} \times 1} + (1 - 0.6302)106.00e^{-r_{n+1,i+1} \times 1}]$$

The credit-adjusted discount rates are found using equation (4):

$$r_{n+1,i+1} = 1 \times 0.07 + (1 - 1)(0.07 + 0.03) = 0.07$$

$$r_{n+1,i} = 0 \times 0.07 + (1 - 0)(0.07 + 0.03) = 0.1$$

The conversion probability of 0.63 at this node is given by equation (3):

$$q_{4,4} = 0.6302 \times 1 + (1 - 0.6302) \times 0 = 0.6302$$

The same procedure can be used to find any convertible bond value and conversion probability.

The previous section outlined the basic principles of how to incorporate a convertible bond model. In practice, there are many other aspects to consider. Some convertible bonds allow the issuer to force investors to convert the bond if the share price reaches a certain predetermined level (barrier).
To include a barrier in the convertible binomial model, the number of time steps must be chosen so that the barrier falls exactly on the nodes.
The conversion probability is then set to 1 if the share price is greater than or equal to the barrier. The issuer of the convertible bond often has the right to call the bond, while the investor has the right to sell the bond. Figures 2 and 3 in this paragraph illustrate the 5-steps stock price and convertible bond value trees, respectively.

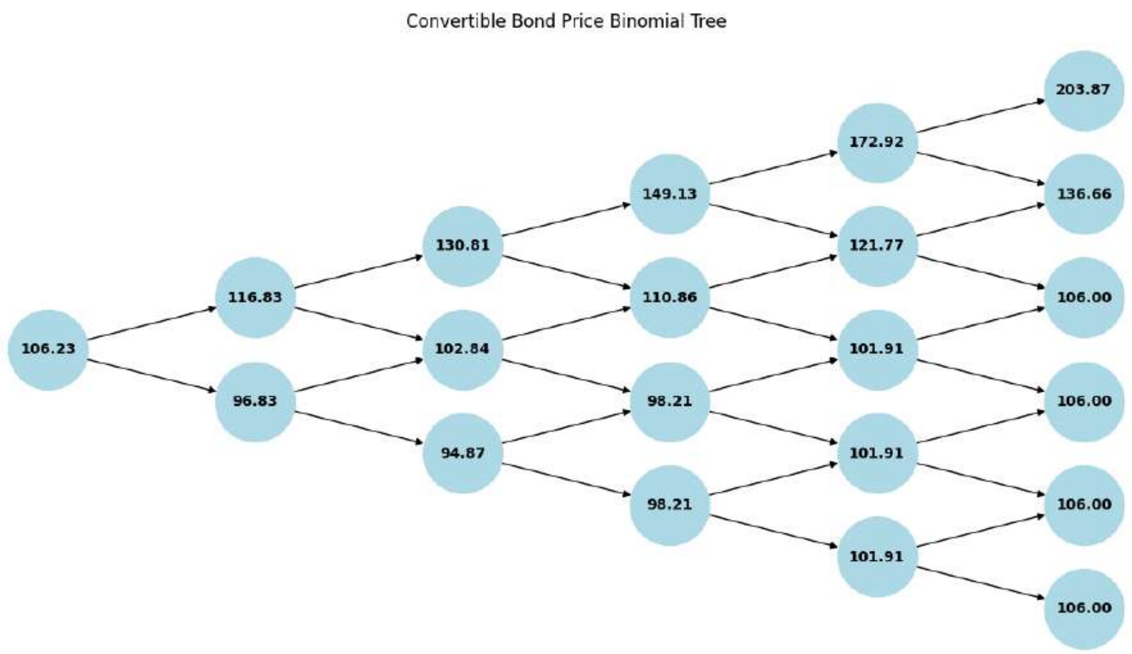

Figure 2: 5-Steps Convertible Bond Price binomial Tree: numeric example

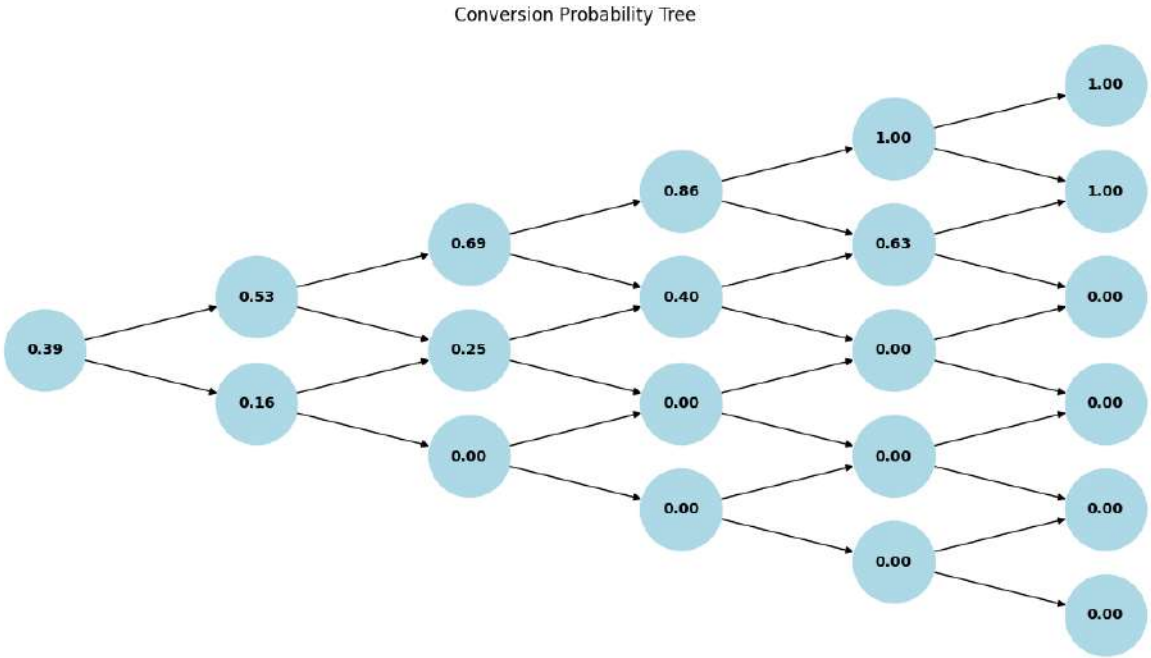

Figure 3: 5-Steps Conversion Probability Tree: numeric example

Convergence of Binomial Trees for CRR model

Having introduced the valuation of European and American options using the binomial tree method, we now examine the accuracy of this technique.

The convergence analysis of the binomial tree is fundamental to understand its effectiveness in option pricing. Our analysis examines how the binomial tree approximates the actual option price as the number of time steps increases.

When analysing convergence, we need to consider the error of a numerical scheme.

If V_{exact} represents the correct option value and V_n is the value from a binomial tree with n steps, the error can be expressed as:

$$Error_n = V_{exact} - V_n \quad (5)$$

To formally define convergence, there exists a constant k such that, for all time steps n:

$$Error_n = O\left(\frac{1}{n^c}\right) \quad (6)$$

where c is the order of convergence. As long as $c > 0$, V_n will converge to V_{exact}. Mathematical proof that shows the convergence of the binomial lattice to the true option price is described in (Giribone and Ventura, 2011).

For European options, we can empirically examine this convergence since we have an analytic expression for V_{exact} (the Black-Scholes price).

We know that the binomial distribution will eventually converge to the lognormal distribution, which underlies the Black-Scholes model.

Empirical evidence shows that, for all basic binomial models (e.g., CRR, RB) $c = 1$, meaning V_n converges to the Black-Scholes price at a rate of $\frac{1}{n}$.

In general, to halve the error, the number of time steps needs to be doubled (Leisen and Reimer, 1996).

In our case, as shown in Figure 4, the convergence chart represents the price behaviour of a convertible bond using a binomial tree approach with the Cox-Ross-Rubinstein model.

The x-axis denotes the number of steps (N) and the y-axis represents the computed price of the convertible bond.

We can see there is a rapid initial convergence from step 3 to around 15 steps. At a low number of steps, the bond price shows significant changes, rising sharply to around 130.

After reaching around 20 steps, the bond price begins to stabilise near 130, indicating convergence.

Beyond this point, minor oscillations persist, fluctuating slightly above and below the convergence value. The plot in Figure 4 suggests that the model reaches practical convergence after about 20 steps.

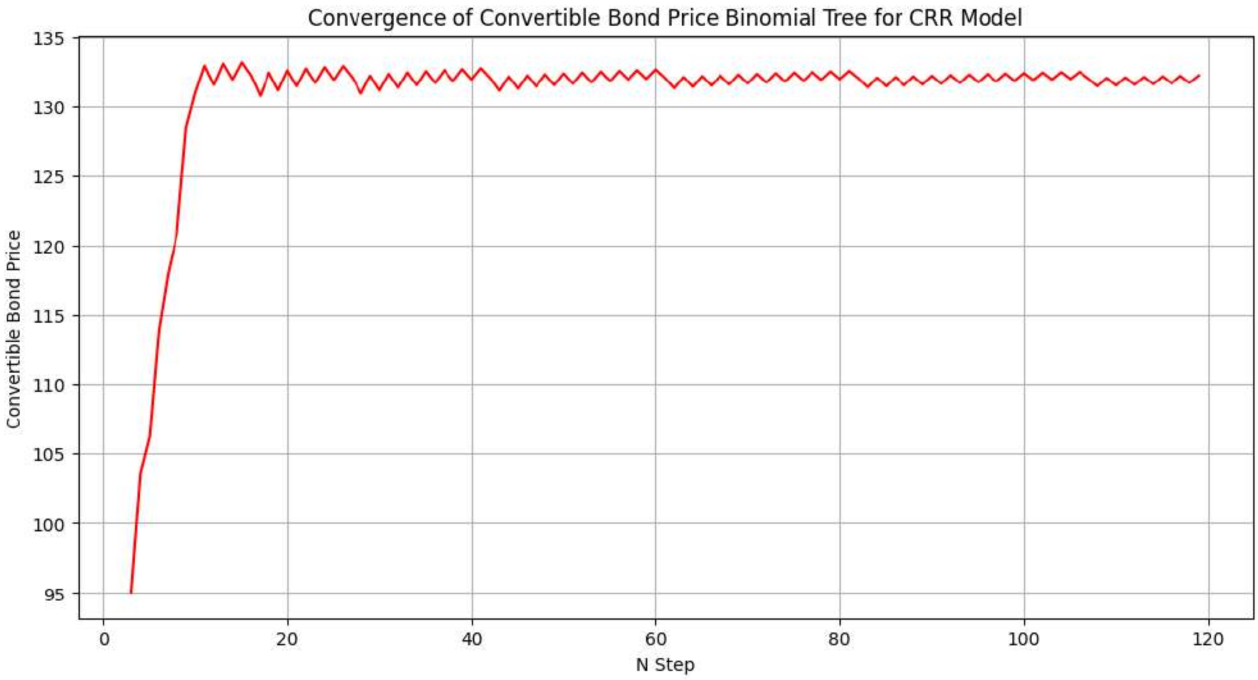

Figure 4: Convergence of convertible bond price for the CRR binomial tree model

4) Alternative binomial trees

We now conduct a convergence analysis of the CRR, Tian, Jarrow-Rudd, and Haahtela models. Our goal is to evaluate the stability and accuracy of each model when pricing convertible bonds by systematically increasing the number of time intervals. As described in the Introduction, all models are calibrated to ensure consistency with the statistical properties of the underlying asset, such as first and second moments.

Tian Binomial Tree Model

The Tian model improves the classic binomial lattice for option pricing by modifying parameters to achieve greater accuracy in approximating the continuous-time stochastic process of an underlying asset. Specifically, the Tian approach adjusts the up and down factors in the lattice so that the lattice corresponds to the first three moments of the continuous-time distribution: mean, variance and skewness. This additional moment constraint is a key distinction from traditional models such as the Cox-Ross-Rubinstein model, which typically only corresponds to the first two moments (mean and variance).

The stock price in a binomial lattice can either increase by a factor u or decrease by a factor d in each time period Δt, where $\Delta t = \frac{T}{N}$ denotes the discrete time step (also referred to as a bump). T is the time to maturity, with N discrete steps. Tian defines the up and down factors using the growth factor $M = e^{r\Delta t}$ which corresponds to the drift under a risk-neutral measure, and the volatility adjustment $v = e^{\sigma^2 \Delta t}$. Then, the model proposed by Tian matches the first three moments of the log-normal distribution followed by the underlying:

$$u = \frac{1}{2} e^{b\Delta t} v \left(v + 1 + \sqrt{v^2 + 2v - 3} \right) \quad (7)$$

$$d = \frac{1}{2} e^{b\Delta t} v \left(v + 1 - \sqrt{v^2 + 2v - 3} \right), \qquad v = e^{\sigma^2 \Delta t} \quad (8)$$

$$\prod = \frac{e^{b\Delta t} - d}{u - d} \quad (9)$$

With these values, the binomial model distribution converges to the lognormal distribution of continuous-time stock prices more accurately by accounting for skewness (third moment). This skewness correction helps the Tian model better approximate the behaviour of asset prices, especially in fewer steps compared to traditional binomial models.

Below, we show the values obtained relative to the auxiliary variables u, d and $\prod$ for the Tian model with respect to the example shown in section 3.

$$Model{:}\,Tian \;\longrightarrow\; u = 1.3657, \qquad d = 0.9124, \qquad \prod = 0.3532$$

The stock price tree generated by the Tian model reflects a wider, more skewed distribution of stock prices over time, accounting for realistic price volatility and potential skewness (see Figure 5). Each node shows a potential stock price at each time interval, with branches indicating upward or downward movements. By incorporating skewness into the Tian model, we observe a wider range of stock prices compared to simpler models, particularly at the terminal nodes, which capture more extreme highs and lows.

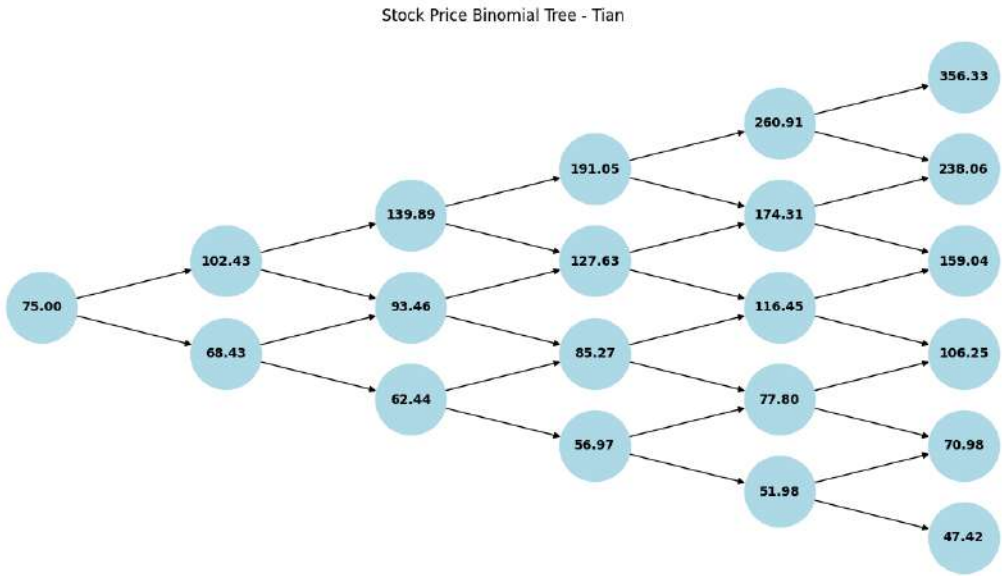

Figure 5: 5-Steps Stock Price Binomial Tree – Tian Model

Jarrow-Rudd Tree Model

The Jarrow-Rudd model is an extension of the traditional binomial approach, and it introduces modifications to the tree model structure that can more accurately adapt to market conditions. Like other binomial option pricing models, Jarrow-Rudd binomial trees are characterized by the sizes of up and down moves and their associated probabilities. The main feature of the Jarrow-Rudd model is that up and down moves are chosen to have a probability of $\frac{1}{2}$.

Beyond these specific formulas, the rest of the model aligns with the Cox-Ross-Rubinstein and other binomial option pricing models.

The up and down multipliers u and d are used to construct the underlying price tree, starting at the current underlying price S and extending to the option expiration. At each node in the tree, the price can either move up $S \cdot u$ or down $S \cdot d$ at either node in the next step.

The underlying prices at expiration (the last nodes in the tree) are then used to determine the option payoffs, forming the last layer of the option price tree. From there, the option price tree is calculated backwards, working toward the first node, which gives the current option price.

$$u = e^{\left(b-\frac{\sigma^2}{2}\Delta t+\sigma\sqrt{\Delta t}\right)} \quad (10)$$

$$d = e^{\left(b-\frac{\sigma^2}{2}\Delta t-\sigma\sqrt{\Delta t}\right)} \quad (11)$$

$$\prod = \frac{1}{2} \quad (12)$$

Where $b = r - q$ is the so-called “cost-of-carry” (see Appendix A).

The equal probability assumption simplifies calculations and aligns with certain market conditions where no bias exists in price movements.

Below, we show the values obtained relative to the auxiliary variables u, d and p for the Jarrow-Rudd model:

$$Model: Jarrow - Rudd \longrightarrow u = 1.2840, \qquad d = 0.8607, \qquad \prod = 0.5000$$

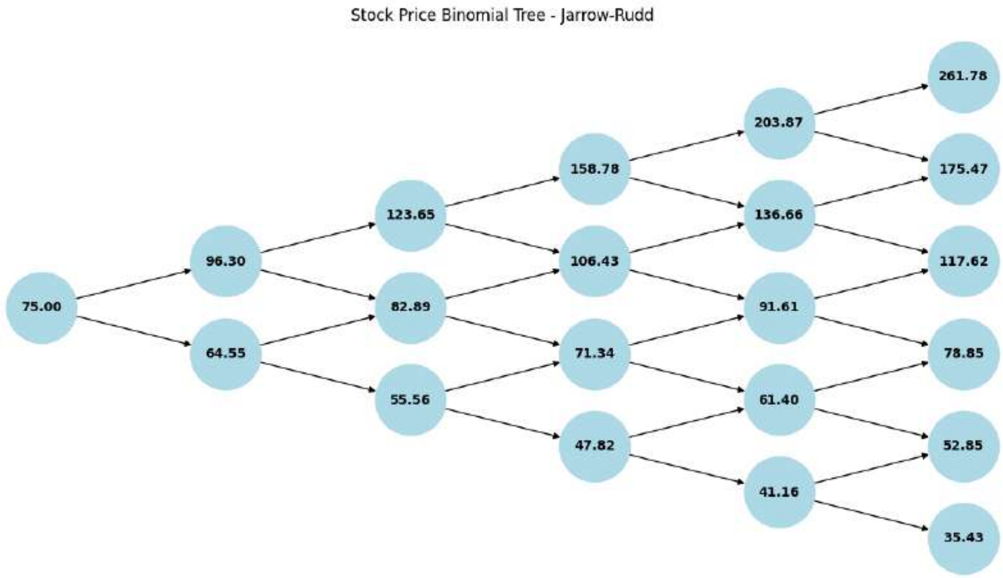

Figure 6: 5-Steps Stock Price Binomial Tree – Jarrow Rudd

In Figure 6 representing the stock price tree, we observe a symmetrical progression of prices as we proceed through the time steps. This symmetry stems from the structure of the Jarrow-Rudd model, which adjusts the risk-neutral drift by maintaining a normal-type distribution around the expected stock price.

Haahtela Tree Model

The Haahtela model addresses option pricing where the underlying asset distribution diverges from the traditional lognormal shape by introducing flexibility for distributions that can incorporate both positive and negative values. This model is particularly useful when the underlying asset value exhibits dynamics that are not well modeled by Geometric Brownian Motion.

In traditional Geometric Brownian Motion based binomial trees, the up and down factors for each step are calculated using the volatility parameter to approximate the future asset price distribution as lognormal.

However, the Haahtela model introduces a shifted diffusion process different than the one of the Geometric Brownian Motion binomial trees, which allows the underlying value to follow a path between normal and lognormal distributions.

This is achieved by introducing a "shift" or "displacement" parameter, which allows the model to capture the skewness and better handle negative values, when necessary.

The Haahtela model adapts the up and down factors and the probability $\prod$ as follows:

$$u = e^{b\Delta t}\left(1 + \sqrt{e^{\sigma^2 \Delta t} - 1}\right) \quad (13)$$

$$d = e^{b\Delta t}\left(1 - \sqrt{e^{\sigma^2 \Delta t} - 1}\right) \quad (14)$$

$$\prod = \frac{e^{(r-q)\Delta t} - d}{u - d} \quad (15)$$

These values are adjusted to account for the shifted distribution by setting the increase factor u and the decrease factor d based on both the traditional volatility and the shift parameters.

The probability of an upward move $\prod$, is modified to align with the risk-neutral valuation principle while accommodating the shifted process.

This formulation maintains no-arbitrage conditions within the model while reflecting the altered dynamics of the displaced process. Below, we show the values obtained relative to the auxiliary variables u, d and $\prod$ for the Haahtela model:

$$Model: Haahtela \;\longrightarrow\; u = \; 1.2892, \qquad d = \; 0.8558, \qquad \prod = \; 0.5000$$

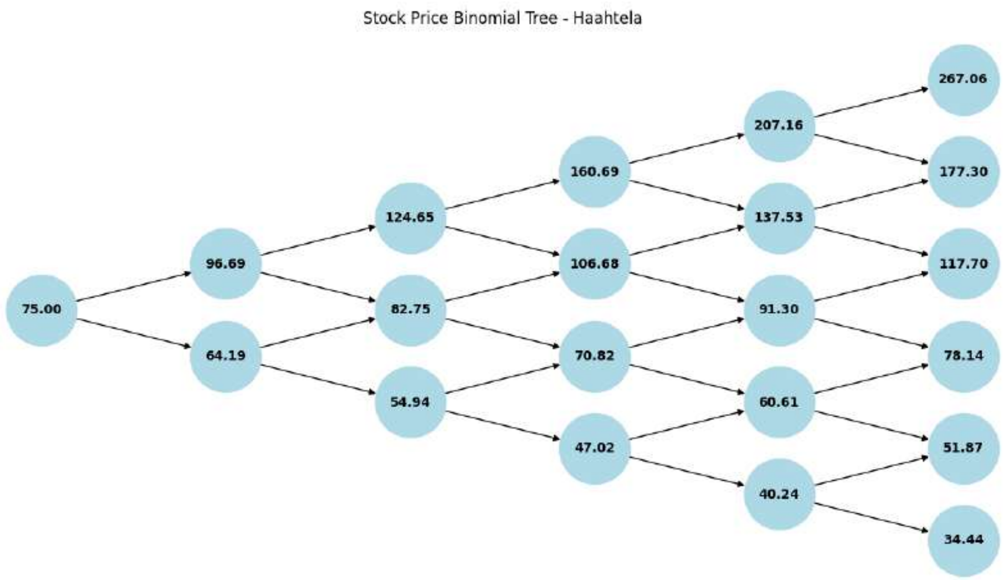

Figure 7: 5-Steps Stock Price Binomial Tree – Haahtela Model

Figure 7 illustrates the evolution of share prices using the specific parameters of the Haahtela model, which tends to allow a wider range of potential outcomes for each node. This tree structure suggests a higher level of implied volatility, as shown by the wide gap between the highest and the lowest final price.

Indeed, the initial stock price starts at 75.00. Moving up the tree, there are rapid increases, reaching a high of 267.06 at the highest final node, indicating strong upside potential according to the assumptions of the Haahtela model. In contrast, the lowest end node ends at 34.44, showing significant downside potential.

This large gap between the highest and the lowest possible price indicates that the parameters of the Haahtela model could incorporate a higher volatility factor, allowing for more pronounced stock price fluctuations over time.

Such a model might be suitable for activities characterised by greater uncertainty or speculative behaviour, where price fluctuations are expected to be more extreme.

The structure reflects the model flexibility in capturing upside and downside risks, which could be useful in the valuation of volatility-sensitive options or derivatives.

Convergence Analysis of the Alternative Binomial Trees

This section aims to compare an alternative pricing model with the CRR model in order to validate its accuracy and reliability for the pricing of both European and American call and put options.

The CRR model has been extensively tested and refined for both types of options, making it a suitable benchmark for assessing the convergence and accuracy of other pricing models.

In this paragraph we analyse the performance of our model by comparing the prices of European and American call and put options with those generated by the CRR model.

Using the same parameters for both models, we examine the degree of convergence between the models, as the number of steps in the binomial tree increases.

We perform the computation using the following parameters: $S_0 = 100, K = 95, T = 0.5$ years, $r = b = 0.01$, and $\sigma = 0.20$. We use a European call option because it has a simple structure and can only be exercised at maturity, making it easier to compare with the theoretical Black-Scholes price.

For the American option, we chose a put since its value is more affected by early exercise, especially when the stock price decreases.

This combination of a European call and an American put allows for a comprehensive convergence analysis. The European call serves to validate the numerical model's accuracy and reliability under simpler conditions, while the American put tests the model's ability to handle the additional complexity of early exercise.

This benchmarking and convergence analysis will provide insights into the robustness and potential limitations of our model with respect to an established standard.

First, we calculate the theoretical price of a European call and put option using the Black-Scholes. The BS framework provides a closed form solution based on inputs like the stock price, strike price, time to maturity, risk-free rate, and volatility. These results serve as a benchmark for assessing the accuracy of the numerical methods.

Then we move on to numerical techniques, specifically binomial tree models, to approximate the option prices.

To study the performance of these models, we conducted a convergence analysis. We applied the numerical methods to both European and American options.
For European options, we calculated the value through backward induction, starting from the terminal payoff at maturity and moving step by step to the present.
For American options, the code incorporates the possibility of early exercise so we checked, at each step, whether exercising the option was more favourable than holding it. This involves comparing the intrinsic value of the option with its continuation value.
We recorded and plotted the results from each model, showing that the estimated prices converge toward the theoretical Black-Scholes price. This provides valuable insights into the speed and accuracy of each model (see Figure 8).

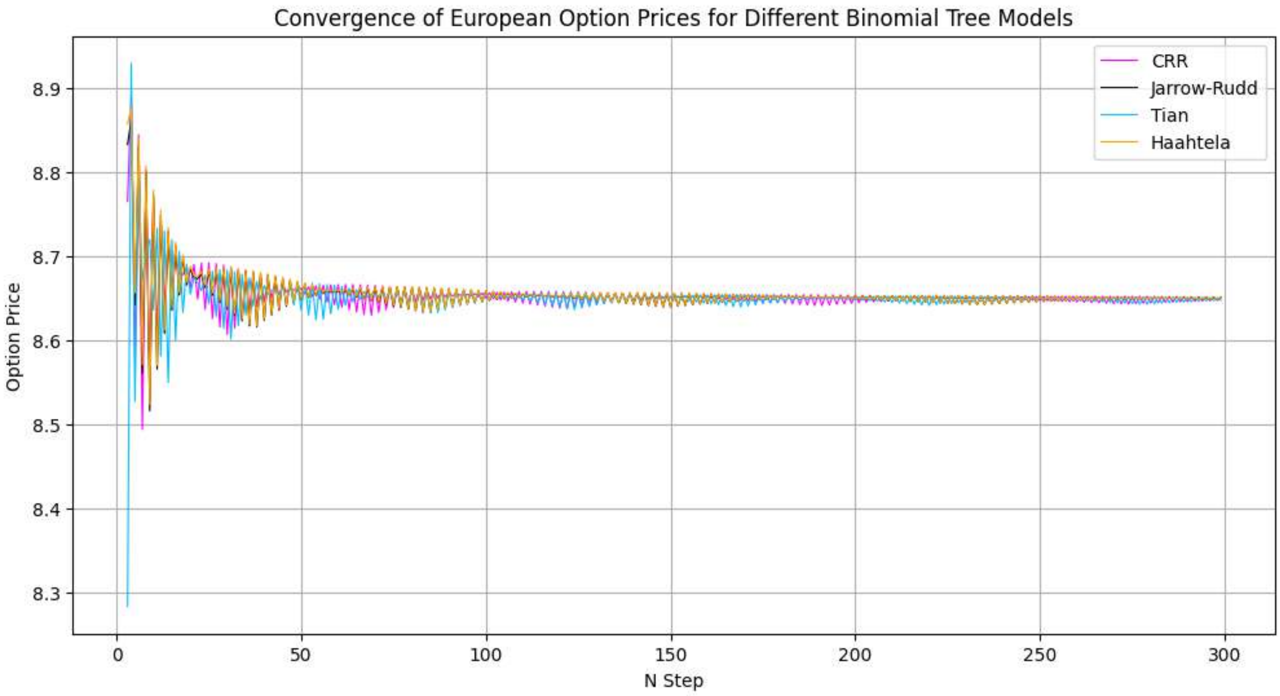

Figure 8: Convergence of European Option Prices for Different Binomial Tree Models

The plot illustrates the convergence of European option prices calculated with different binomial tree models as the number of steps (N Steps) in the tree increases.
Each plot represents a different model: CRR, Jarrow-Rudd, Tian and Haahtela. The y-axis shows the option price, while the x-axis shows the number of steps in the binomial tree.
Initially, there is a significant fluctuation in option prices with a low number of steps, which is particularly noticeable in models such as Tian.
As the number of steps increases, these fluctuations reduce, and the models begin to converge towards a stable price. Around 150-200 steps, all models begin to align closely, indicating that they are approaching the theoretical value predicted by the Black-Scholes formula.
This pattern of convergence suggests that a higher number of steps leads to greater accuracy in binomial tree models. However, the speed of convergence and initial stability vary depending on the model.
For example, the Tian model exhibits greater initial fluctuations, while the CRR shows relatively more uniform convergence.
The plot shows that all models eventually stabilise around the same price, validating the numerical methods against the analytical benchmark, as the tree size increases.
Additionally, we analysed the impact of the number of steps on the precision of American options, which are more complex due to the early exercise feature.
The visualization offers a clear comparison between the models. For instance, the CRR model is expected to provide consistent results as the number of steps increases, but alternative methods like Tian or Haahtela may show faster convergence or better accuracy under certain conditions. These observations help highlight the trade-offs between computational complexity and precision.
Figure 9 shows the convergence of American option prices calculated using different binomial tree models, as the number of steps (N Step) in the tree increases.

American option prices are slightly higher than the corresponding European prices due to the possibility of early exercise inherent in American options.
The possibility of early exercise introduces additional complexity, making the convergence process slightly more variable compared to European options.
Overall, the plot shows that although the models behave differently at a lower number of steps, they all converge to a similar value as the number of steps increases.
This validates the robustness of these binomial models in pricing American options, while highlighting the patterns of convergence and the impact of early exercise characteristics.
Consistent results at higher numbers of steps reinforce the reliability of numerical methods for accurate pricing.

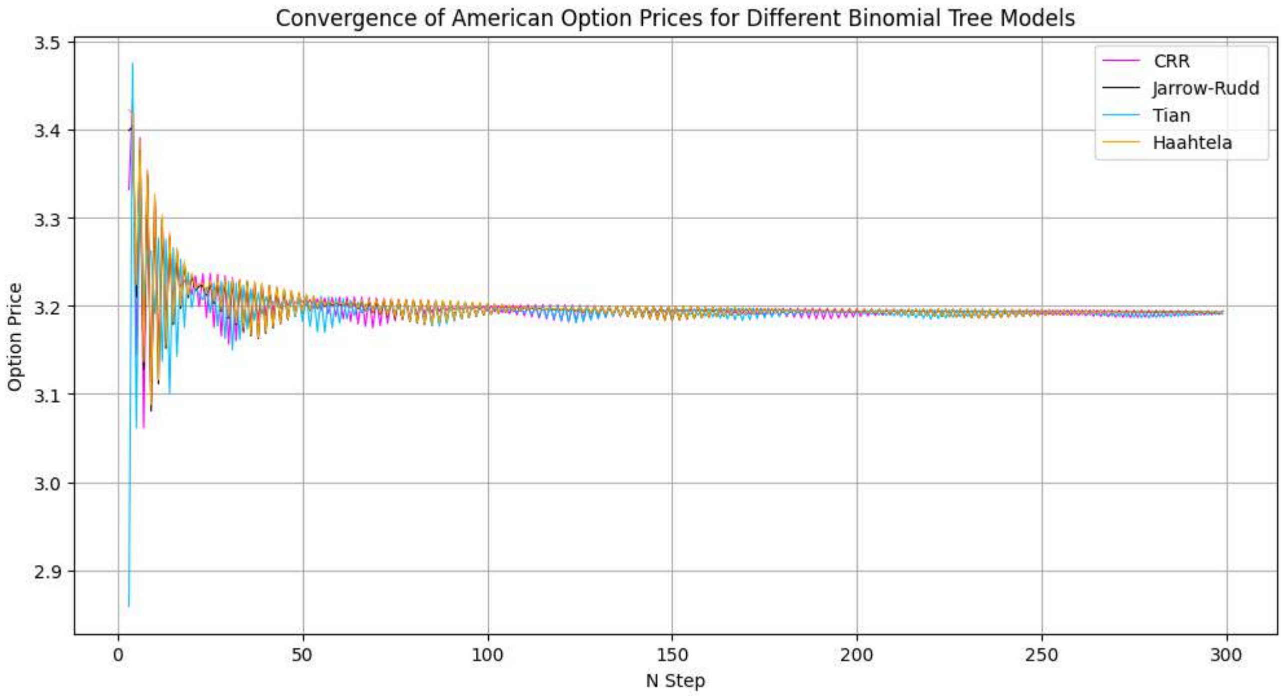

Figure 9: Convergence of American Option Prices for Different Binomial Tree Models

5) Convertible bond valuation and sensitivity analysis

The previous paragraph explored the versatility of alternative binomial tree models in convertible bond pricing, highlighting how variants such as the Tian, Jarrow-Rudd and Haahtela models address specific market dynamics and offer potential enhancements over the classic Cox-Ross-Rubinstein scheme.
These models improve pricing accuracy by capturing distinctions in the behaviour of the underlying asset, including skewness, volatility structure and other real-world characteristics that a standard binomial tree might overlook.
We conducted an in-depth analysis of alternative binomial trees, comparing them with traditional European and American options and verifying their proper convergence to the expected theoretical value. This analysis served as an important validation of the alternative approaches, demonstrating that models such as Jarrow-Rudd, Tian and Haahtela not only preserve theoretical consistency, but they are also capable of converging to reliable results within the context of fair value evaluation.
By analysing the structural differences of these models and conducting convergence analysis, we highlighted their potential for improving price accuracy, especially for complex instruments such as convertible bonds.
These improvements underline the flexibility and adaptability of binomial trees for modelling hybrid securities. Building on this foundation, in this section we move from theoretical considerations to practical implementation, focusing on the application of the CRR model and its alternatives to the valuation of convertible bonds.
Then, based on this theoretical validation, the present section focuses on the practical application of these alternative approaches in a more complex context: the valuation of convertible bonds. By leveraging the base structure of the code implemented for the traditional CRR model, we integrated the parameters u, d, and p specific to the alternative trees.
The objective is to address the same valuation problem previously solved with CRR but using the anticipated variants to explore how these models influence results in the context of hybrid instruments, such as convertible bonds.
Furthermore, in this section we introduce a comparative sensitivity analysis, assessing how the choice of the binomial model influences pricing outcomes and conversion probabilities. This analysis sheds light on the practical implications of model

choice, demonstrating the impact of factors such as volatility, interest rates and share price fluctuations on the valuation process.
Presenting the results of different binomial schemes, the section highlights the strengths and limitations of each approach, guiding practitioners in choosing the most appropriate model based on specific market conditions and bond characteristics.

Pricing Assessment with 5-steps

We now delve into the practical application of the Jarrow-Rudd, Tian and Haahtela models by analysing the Convertible Bond Price Binomial Tree and the Conversion Probability Binomial Tree. These visual tools not only depict the pricing structure but also illustrate the evolving decision-making process of bondholders at each node, highlighting the differences introduced by the alternative models.
The Convertible Bond Price Binomial Tree represents the recursive valuation process where the convertible bond value is determined at each node. At the terminal nodes, the bond value is derived by comparing the payoff of immediate conversion with the bond face value and accrued coupon payments.
Moving backward through the tree, intermediate nodes incorporate the discounted expected value of future payoffs, weighted by the risk-neutral probabilities. The results reveal how the unique assumptions of each alternative model influence the valuation process. The Jarrow-Rudd model, grounded in a symmetric framework, produces a straightforward, smooth progression of values across the tree. This regularity makes it an effective and computationally efficient alternative in stable market conditions. The Tian model, by accounting for skewness in the underlying stock price distribution, yields a wider range of values, particularly evident in extreme upward or downward scenarios. This skewness enables it to better reflect real-world asset behaviour, especially in volatile markets.
The Haahtela model, with its flexibility to move beyond the lognormal assumption, demonstrates the broadest range of values. This adaptability allows it to capture irregular market dynamics, making it particularly suited to environments with atypical price behaviours.
The bond price was evaluated, as shown in the trees below, using 5-steps.
The speed of convergence is a crucial metric in numerical methods, especially for financial modeling. It refers to how quickly a model approaches the theoretical fair value as the resolution of the tree increases. Faster convergence means that fewer steps are required to achieve a given level of accuracy.
In the context of convertible bond pricing, where valuation must account for both equity-like and bond-like features, a model with a high speed of convergence ensures that complex features such as conversion probabilities and early exercise decisions can be captured accurately without excessive computational overhead. This analysis provided an opportunity to examine the initial price structures generated by these models under minimal tree resolution, emphasizing their practical differences.
Our results showed that, even with only 5 steps, the alternative models delivered fair value estimates that were consistent with the CRR benchmark (see Figures 10, 11 and 12). Despite their distinct methodologies - such as Tian's skewness adjustment, Jarrow-Rudd's symmetric structure and Haahtela's flexible distribution - all models demonstrated reasonable accuracy, indicating their robustness. Our analysis also revealed how these structural differences influence the distribution of bond prices at the node level, providing insights into their potential strengths under varying market conditions.

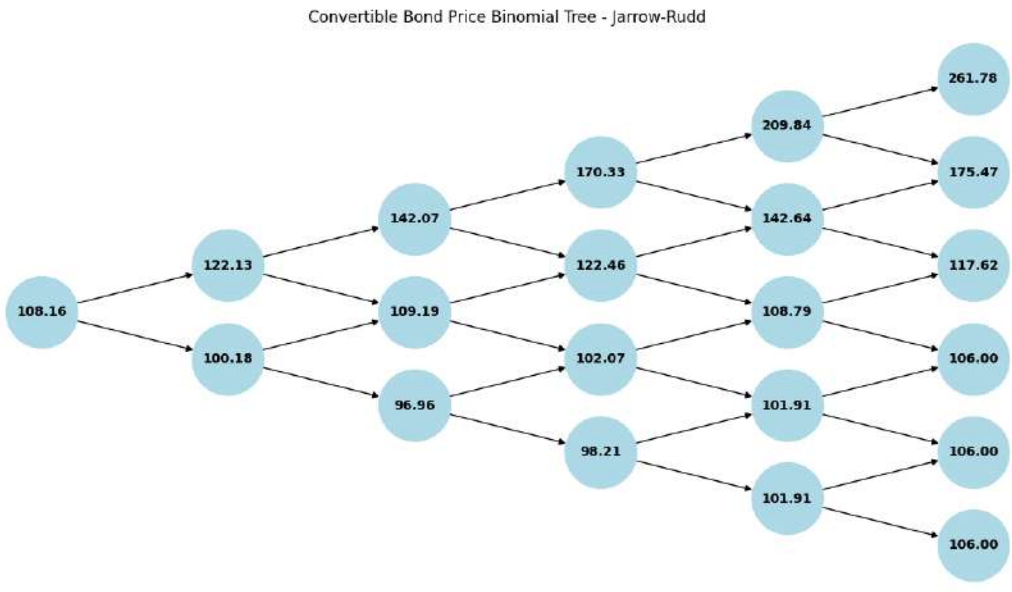

Figure 10a: 5-Steps Convertible Bond Price Binomial Tree – Jarrow Rudd Model

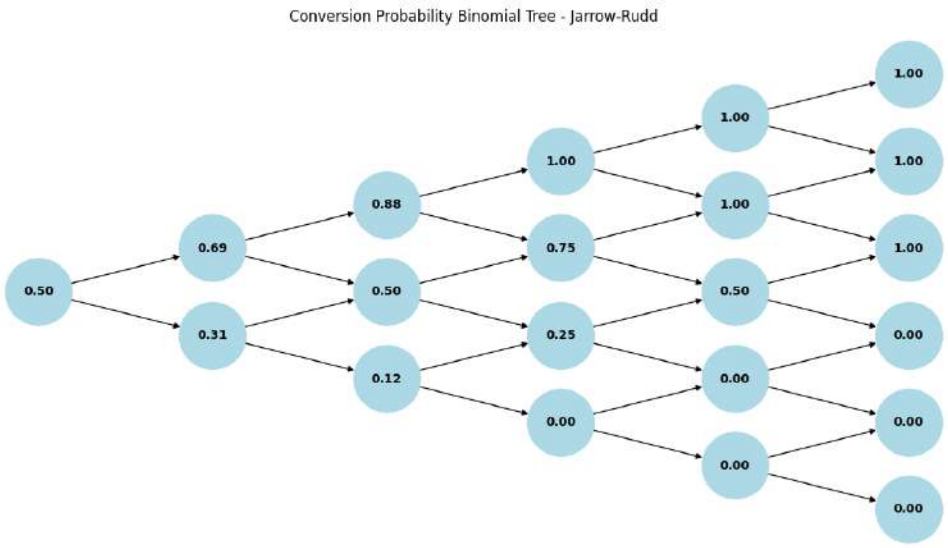

Figure 10b: 5-Steps Conversion Probability Binomial Tree – Jarrow Rudd Model

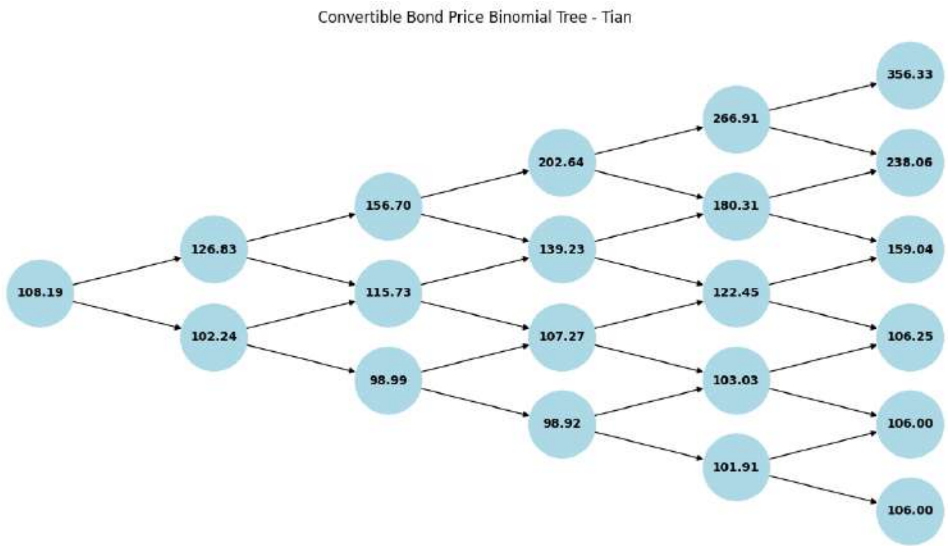

Figure 11a: 5-Steps Convertible Bond Price Binomial Tree – Tian Model

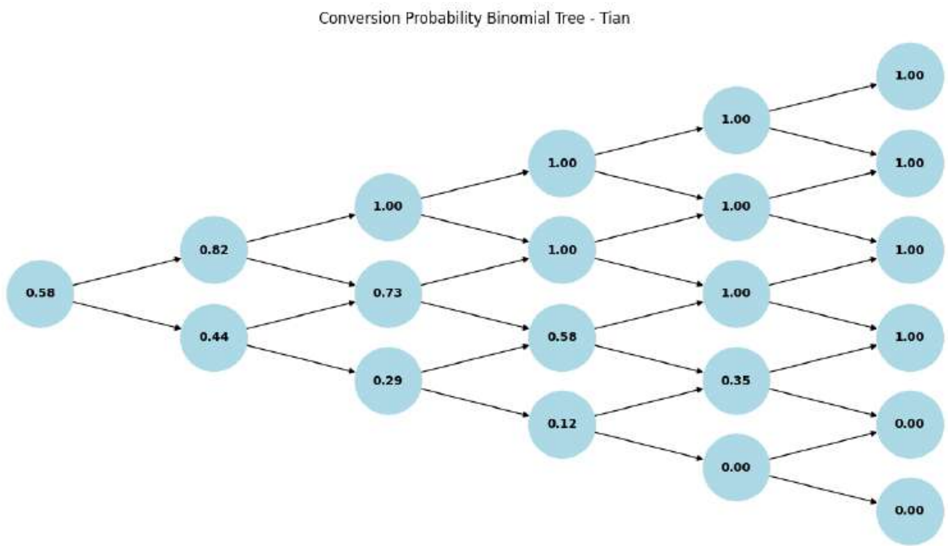

Figure 11b: 5-Steps Conversion Probability Binomial Tree – Tian Model

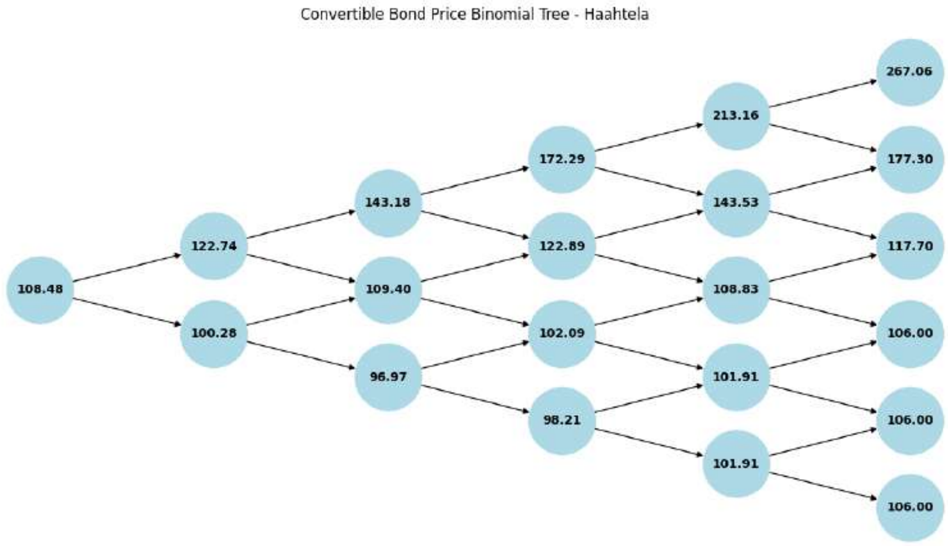

Figure 12a: 5-Steps Convertible Bond Price Binomial Tree – Haahtela Model

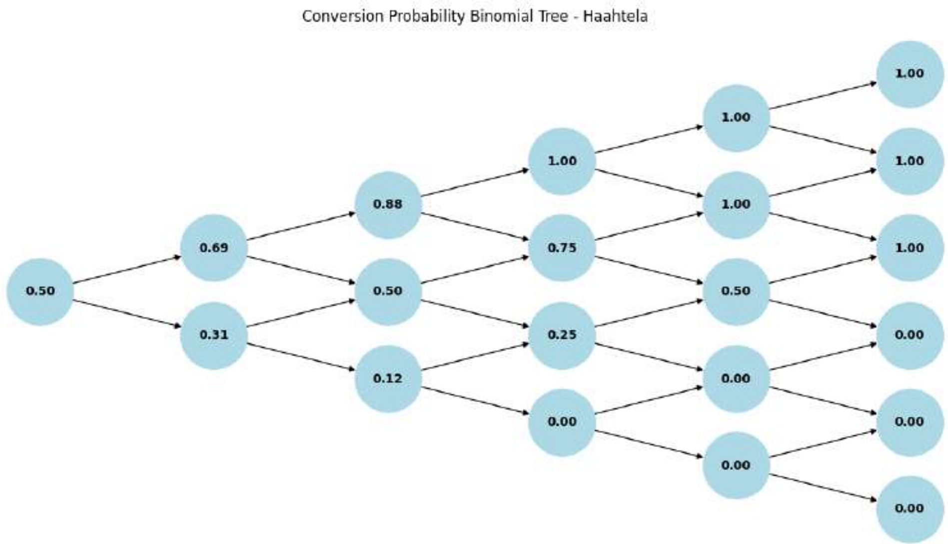

Figure 12b: 5-Steps Conversion Probability Binomial Tree – Haahtela Model

Convertible Bond Price Convergence of Alternative Binomial Trees

In this part, we expanded the analysis by increasing the number of steps in the binomial trees to 120, allowing for a comprehensive study of convergence behaviour.

In Figure 13, we provide a representation of the **convergence of alternative binomial models** in the valuation of convertible bonds. Examining the lines corresponding to the various approaches, it is evident that all models - Jarrow-Rudd, Tian and Haahtela - converge toward the same fair value limit, aligning with the results of the traditional CRR model. This observation is significant because it demonstrates that alternative trees, despite their structural differences, maintain theoretical consistency in the calculation of convertible bond values.

The convergence behaviour among the models suggests that these alternative approaches are as robust as the CRR model, with negligible differences in terms of convergence speed. Specifically, all models achieve stability in their values after a certain number of steps in the tree construction. The initial oscillations, visible mainly in the early steps of the graph, quickly diminish as the values converge toward a stable and reliable price.

This behaviour indicates that, in this specific case, the differences in the formulations of u, d, and p parameters do not significantly affect the model's ability to deliver accurate valuations.

One key implication of our analysis is that alternative models are not only theoretically valid but also practical and useful in real-world applications. The comparability in convergence speed between alternative models and the CRR highlights that they can be implemented without compromising computational efficiency. Therefore, models such as Jarrow-Rudd, Tian or

Haahtela can be confidently used in practical applications, especially in scenarios where their unique characteristics provide additional advantages.

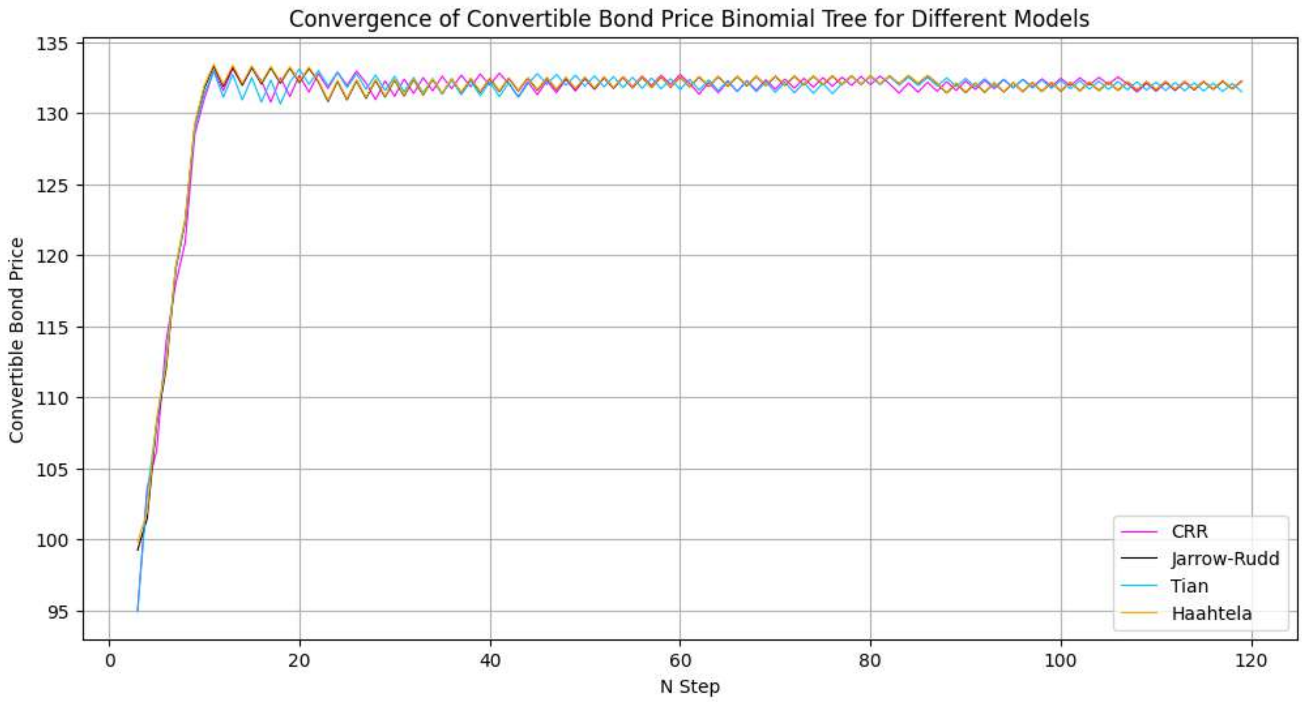

Figure 13: Convergence of Convertible Bond Price of the Alternative Binomial Trees

Moreover, it is interesting to observe that, in the specific case under consideration, the convergence of the models follows a similar pattern even in finer details (see Figure 13). For example, although the Tian model introduces the management of skewness in price distributions and the Haahtela model offers flexibility beyond lognormal assumptions, both exhibit a convergence behaviour that is almost indistinguishable from that of the other models. This result reinforces the idea that the choice between models can be guided not so much by their ability to converge - which is guaranteed - but rather by the specific market requirements or input conditions.

Another significant aspect concerns the interpretation of the plot's stable and aligned behaviour. It demonstrates that regardless of the model chosen, consistent results can be achieved in determining the fair value of the convertible bond. This is particularly important for complex instruments like convertible bonds, which require models capable of handling both equity-related characteristics (conversion option) and bond-related features (coupons and nominal value) simultaneously.

In short, the graph provides solid confirmation of the robustness of alternative approaches, highlighting their ability to converge to the expected theoretical value with a convergence speed comparable to that of the CRR model. This observation makes the alternative models not only a valid choice for convertible bond valuation but also flexible tools that can be employed to address a variety of market scenarios.

In the specific case we analysed, the behaviour of the models confirms that, despite introducing theoretical and structural differences, the alternative trees maintain a level of reliability and stability comparable to the CRR, making them practical and well-suited for financial decision-making processes.

Sensitivity Analysis

In this section, we analyse the sensitivities, known as "Greeks", which are key risk measures in option pricing, that quantify how the value of an option responds to changes in various market factors. These sensitivities are usually calculated as partial derivatives of the Black-Scholes-Merton (BSM) formula. These partial derivatives show the option price change to a small change in the parameters of the formula.

The Greeks include Delta, Gamma, Vega, Theta, and Rho, allowing us to break down complex price movements into measurable components. This analysis is essential for effective hedging strategies, risk management, and for understanding the behaviour of options under various market conditions.

In numerical methods, like the Cox-Ross-Rubinstein binomial tree model, Greeks are approximated using finite-difference techniques.

Unlike analytical approaches, these numerical methods involve discretization, where the underlying asset price path is divided into a finite number of steps. The lack of a closed-form solution means Greeks must be estimated by bumping input parameters and observing the resulting changes in the bond price.

Delta ($\boldsymbol{\Delta}$) is calculated by changing the underlying asset price incrementally:

$$\Delta \approx \frac{f(S+\Delta S)-f(S-\Delta S)}{2\Delta S} \quad (16)$$

Gamma ($\boldsymbol{\Gamma}$) requires two levels of bumping to assess the change in Delta:

$$\Gamma \approx \frac{f(S+\Delta S)-2f(S)+f(S-\Delta S)}{\Delta S^2} \quad (17)$$

Vega ($\boldsymbol{\nu}$) approximates sensitivity to volatility:

$$\vartheta \approx \frac{f(\sigma+\Delta\sigma)-f(\sigma-\Delta\sigma)}{2\Delta\sigma} \quad (18)$$

In addition to the computation of the Greeks, we conducted a further validation to confirm that the computation of the CRR has been conducted correctly.
The inclusion of Taylor approximations further validates the reliability of the binomial tree models, offering a quick and efficient method to estimate.
The Taylor series expansion provides a framework to approximate the change in the convertible bond price as a function of multiple variables, such as the underlying asset price, volatility and time. Using a first-order Taylor approximation, the convertible bond price change is expressed as a linear combination of partial derivatives (Greeks), while the inclusion of second-order terms, such as Gamma, accounts for curvature effects.
Then, given that the Greeks of financial derivatives are computed, we performed an approximate valuation of the instrument. By utilizing the first-order derivatives of the option price with respect to the relevant risk parameters, we applied the general Taylor formula in this context.
The larger the shock applied to the reference parameter, the less accurate is the approximation.
Comparing these approximations with the exact prices calculated for each model highlights the alignment between the numerical sensitivities and the bond price behaviour (see Table 1).

Greeks	CRR	Tian	Jarrow-Rudd	Haahtela
Delta	0.4076	0.4176	0.429	0.4291
Gamma	0.001	-0.001	-0.001	0.001
Vega	69.7186	43.1465	62.4383	62.5352

Table 1: Results of the Greek sensitivities

For the CRR model, the Taylor approximations demonstrate good agreement with exact prices across all Greeks, with minor deviations observed for Δ and ϑ, likely reflecting slight non-linearities in the stock price and volatility sensitivities.
The Taylor estimate of 131.765 for Δ closely approximates the exact price of 131.967.
The Tian model reveals similar trends but shows a slightly different sensitivity structure. For Δ, the Taylor approximation 131.981 again tracks the exact price 132.188 well, though some deviations emerge for ϑ, reflecting model-specific nuances in interest rate and volatility sensitivities.
In the Jarrow-Rudd model, the Taylor approximation for Δ of 132.204 again aligns well with the exact price 132.416, but the ϑ sensitivity shows reduced deviations compared to the CRR model.
Finally, the Haahtela model performs similarly to Jarrow-Rudd, with a high degree of alignment between Taylor approximations and exact prices. For instance, Δ and Γ produce approximations within a tight range of the exact values, while ϑ again shows minor discrepancies due to higher-order effects. These comparisons illustrate that the Taylor approximation acts as a diagnostic checkpoint, confirming the validity of Greek calculations and the numerical implementation of each binomial tree model. Moreover, the observed differences between the approximations and the exact prices highlight the unique sensitivities of each model, offering deeper insights into their structural characteristics and applicability to convertible bond pricing. This further validates the convergence properties explored earlier, as Taylor approximations consistently approach exact prices as the binomial tree models stabilize with increasing time steps. By integrating analytical and numerical methods, the Taylor approach reinforces confidence in the robustness and accuracy of the alternative models studied.

6) Market Case Study

This section presents an experiment in pricing Convertible bonds having different shares as the underlying to the conversion option. We considered the forty stocks in the German DAX index because currently the most active market for these hybrid

instruments is Germany (source: Bloomberg®, module: Fixed Income Search). The reference date for the valuations is December 31, 2024.
The EOY (End-Of-Year) closing prices together with the description of the underlyings are shown in Table B.1 in the Appendix. These constitute the spot prices, S, of the model.
As for the estimation of dividend yields, we employed the term structures of continuous dividend yield $q(t)$ implied by the call-put parity or by forward contracts quotations. If no implied values are contributed, due to the absence of actively traded derivatives written on an underlying, the value considered remains constant for all maturities and is set equal to the ratio of the cash dividend paid, divided by the spot, as is traditionally defined in corporate finance. Table B.2 in Appendix shows these estimates for eight different maturities (6M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, and 10Y) for all forty German stocks.
The implied volatilities are lognormal, $\sigma(K,t)$, and they are calculated from three different strike prices (moneyness: 80%, 100% and 120%) and eight different maturities (6M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y and 10Y).
A two-dimensional linear interpolation (strike price - time) is conducted in the pricing routines in order to choose the most suitable volatility for the valuation.
If there are no vanilla European options, the volatility surface is flat, characterized by a single value of volatility estimated historically by the close-to-close method. Table B.3 shows the three sections of moneyness for all German stocks considered in the experiment.
The risk-free term structure we used for the analysis has one day as tenor. The ESTER curve is shown in Figure B.1 in the Appendix. Under the assumption that the issuer of the convertible bond is the same as the stock, to take into account the correct creditworthiness, we rely on CDS premiums traded in the market. If this information is not available due to the absence of active quotes, a single value computed from the Z-spread closest to the maximum maturity considered in the experiment, i.e., 10 years, is used instead of the entire CDS term structure. The CDS curves are shown in Table B.1.
To estimate the impact on price and the main measures of option sensitivity, namely Delta Δ, Gamma Γ and Vega ϑ, we assumed different convertible bonds for the different alternative binomial trees discussed. These scenarios are designed to test whether the specific results reported in Table 1 are confirmed with different market scenarios.
We now describe the financial characteristics common to all hybrid instruments considered in the different scenarios: the coupon rate is 3% paid semi-annually, the face amount of the bond is 100, the discretization interval of the numerical scheme is one day, $\Delta t = 1/360$. The type of exercise considered for the option is American, i.e., the right can be exercised at any instant in time.
Here are the parameters which define the scenarios for all 40 stocks:

- A structured product maturity from six months to 10 years in six-month steps.

- A moneyness of the option ranging from 50% to 150% with a 10% step.

All interpolations of market term structures are made by dynamically considering the analyzed maturities/strikes.
These simulations are conducted for all four binomial approaches considered (CRR, Jarrow-Rudd, Tian and Haahtela) and for the quantitative measures of price, Delta, Gamma and Vega.
Thus, we produced 140,800 scenarios, stored in tensors of 5 dimensions each: 40 stocks ×20 maturities ×11 strike prices×4 trees×4 quantitative measures.
In terms of price, the values of the alternative binomial trees produced extremely aligned results; in fact, the maximum valuation gap for all scenarios is no more than one cent. Regarding the calculated CRR Greeks, Jarrow-Rudd and Haahtela produced aligned values, while the Tian approach again showed different sensitivity with respect to volatility (ϑ) especially in experiments where the convertible bond had a longer maturity and an ATM strike. The first- and second-order sensitivities with respect to spot (Δ and Γ) showed alignment with those estimated with the other numerical approaches. The Matlab code written for these experiments is available upon request.

7) Conclusions

This study set out to explore and evaluate the applicability of alternative stochastic binomial tree models in the valuation and risk analysis of convertible bonds, a complex class of hybrid securities that combine features of both equity and debt instruments. While the Cox-Ross-Rubinstein binomial tree remains the most widely adopted method in practice and literature for this purpose, we identified a gap in research concerning the implementation and effectiveness of alternative binomial models in this context.
To address this, we systematically examined three well-established binomial models (Haahtela, Jarrow-Rudd and Tian) and compared them against the traditional CRR approach. We began by analyzing the theoretical foundations and structural differences of each model, particularly their assumptions regarding drift, volatility, and node recombination. These differences can have meaningful implications for pricing accuracy and computational behavior, especially when applied to the valuation of instruments with embedded optionality such as convertible bonds.

As a preparatory step, we implemented each model in the pricing of standard European and American options. This served both as a validation of the correct numerical implementation and as a benchmark for convergence behavior. All alternative models demonstrated satisfactory convergence to theoretical option values, thereby confirming their numerical soundness and suitability for more advanced applications.

We then extended the analysis to convertible bonds, adapting the CRR-based lattice framework to each alternative method. This involved step-by-step reconstruction of the pricing trees. Sensitivity analyses were conducted to measure the responsiveness of each model to changes in key risk parameters: Delta, Gamma and Vega. While Delta and Gamma sensitivities remained broadly consistent across models, the Jarrow-Rudd tree exhibited a noticeably different Vega profile, suggesting that its underlying assumptions about return distributions may influence how volatility risk is captured.

Finally, we validated the real-world applicability of these models through a series of empirical tests using market data. These case studies confirmed that the alternative binomial trees not only provide robust estimates of convertible bond values but also yield consistent risk metrics across varying market conditions. This reinforces the view that, when correctly implemented, these models can serve as reliable tools in both academic research and practical financial engineering.

In addition to serving as a comparative study, this research contributes a practical framework for implementing, testing, and validating alternative lattice methods in the pricing of convertible bonds. The structured approach provides a replicable path for practitioners and researchers aiming to extend valuation models beyond conventional techniques.

Looking forward, a natural continuation of this research involves extending the analysis to trinomial stochastic trees, which offer greater flexibility and can better capture skewness, kurtosis, and complex early-exercise features in convertible bond contracts. Adapting both traditional and alternative trinomial approaches - such as the Boyle and Tian trinomial models - using the same rigorous methodology outlined in this study could provide further insight into the comparative strengths of different lattice-based techniques in modeling hybrid financial instruments.

Ultimately, this work enhances the quantitative toolkit available for convertible bond valuation and risk assessment, offering more nuanced and potentially more accurate modeling alternatives that can adapt to various market scenarios and investor requirements.

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Appendix A – Cox Ross Rubinstein Tree for option pricing

The Cox-Ross-Rubinstein (CRR) binomial model provides a discrete-time approximation of the dynamics of asset prices and serves as a foundational tool for derivative pricing. A crucial aspect of the model construction lies in the derivation of its parameters: the up and down factors (*u* and *d*) and the risk-neutral probability (*p*).

These parameters must be chosen to ensure no-arbitrage conditions and to allow convergence of the binomial model to the continuous-time Black-Scholes model as the number of time steps increases (see Figure A.1). This section presents a formal derivation of the CRR parameters, grounded in the principles of risk-neutral valuation and probabilistic convergence.

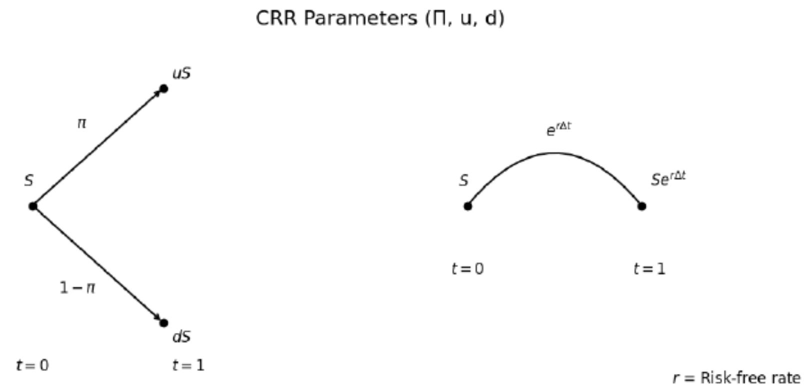

Figure A.1: A graphical representation of CRR parameters

The Black-Scholes analytical formulas (BS closed formulas) cannot provide accurate valuations for all types of options traded in financial markets. They are unable to fairly value options with non-standard features, such as those that allow early exercise (Bermuda/American options) or have complex payoffs (exotic options). In such cases, numerical methods must be used to value the derivative.

The literature offers numerous mathematical techniques that align with the principles of the Black-Scholes framework. We discuss and implement one of the stochastic binomial trees which is most used by quantitative analysts: the CRR model.

To align with the stochastic dynamics assumed by the Black-Scholes framework, Cox, Ross, and Rubinstein proposed selecting the parameters *u* and *d* such that, for each time interval Δt the projected future asset values match the theoretical mean and variance of the continuous model (Cox, Ross and Rubinstein, 1979). Assuming a risk-neutral environment, the expected rate of return of the stock is equal to the risk-free interest rate r. Therefore, the expected stock price at the end of interval Δt is $S \cdot e^{r\Delta t}$ which is the stock price at the beginning of the interval. This leads to match the expected value of the asset in the binomial model to the one implied by the BSM model.

First moment matching $\mathbb{E}(S)$:

$$S \cdot e^{r\Delta t} = \Pi\, uS + (1-\Pi)dS \quad \text{(A.1)}$$

Dividing both sides by S:

$$e^{r\Delta t} = \Pi u + d - \Pi d$$

$$\Pi(u-d) = e^{r\Delta t} - d \rightarrow \Pi = \frac{e^{r\Delta t}-d}{u-d} \quad \text{(A.2)}$$

The stochastic process assumed by the GBS framework (i.e. a Geometric Brownian motion) implies that the variance (VAR) of its rate of change in a short interval of length Δt is $\sigma^2 \Delta t$.

Since the variance of a random variable S is defined as $E(S)^2 - E(S)^2$, where $E(\cdot)$ represents the expected value, we can derive the second equation that connects the second moment of the stochastic process to the dynamics of the binomial tree.

Second moment matching: $VAR(S) = \mathbb{E}(S^2) - \mathbb{E}(S)^2$ (A.3)

$$\underbrace{\Pi\, u^2 + (1-\Pi)d^2}_{\mathbb{E}(S^2)} - \underbrace{[\Pi\, u + (1-\Pi)d]^2}_{\mathbb{E}(S)^2} = \sigma^2 \Delta \mathrm{t} \quad \text{(A.4)}$$

True under BSM framework, where:

$$\sigma^2 \Delta \mathrm{t} = \Pi\, u^2 + (1-\Pi)d^2 - \Pi^2 u^2 - 2\Pi(1-\Pi)ud - (1-\Pi)^2 d^2$$

$$\sigma^2 \Delta t = u^2(\Pi - \Pi^2) + [(1-\Pi) - (1-\Pi)^2]d^2 - 2\Pi(1-\Pi)ud$$
$$\sigma^2 \Delta t = u^2\Pi(1-\Pi) + (1-\Pi)[1-(1-\Pi)]d^2 - 2\Pi(1-\Pi)ud$$
$$\sigma^2 \Delta t = \Pi(1-\Pi)[u^2 - 2ud + d^2] = \Pi(1-\Pi)(\mathrm{u}-\mathrm{d})^2 \quad \text{(A.5)}$$

Substituting Π from eq. (A.2):

$$\Pi(1-\Pi) = \Pi - \Pi^2 = \frac{e^{r\Delta t} - d}{u-d} - \frac{e^{2r\Delta t} - d + 2d \cdot e^{r\Delta t} - d}{(u-d)^2}$$
$$= \frac{e^{r\Delta t}u - ud - e^{r\Delta t}d + d^2 - e^{2r\Delta t} + 2d \cdot e^{r\Delta t} - d^2}{(u-d)^2}$$
$$= \frac{e^{r\Delta t}(u - d + 2d) - ud - e^{2r\Delta t}}{(u-d)^2}$$
$$= \frac{e^{r\Delta t}(u+d) - ud - e^{2r\Delta t}}{(u-d)^2} \quad (\mathrm{A.6})$$

Then, solving for $\prod$ from the first moment equation and substituting this value into the second moment equation, we obtain:

$$\sigma^2 \Delta t = \frac{e^{r\Delta t}(u+d) - ud - e^{2r\Delta t}}{(u-d)^2}(u-d)^2$$
$$\sigma^2 \Delta t = e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} \quad \text{(A.7)}$$

Recalling that Cox, Ross and Rubinstein assumed that $u = \frac{1}{d'}$, we obtain a 3×3 system which allows to express the parameters Π, u, d in terms of $r, \sigma, \Delta t$:

$$\begin{cases} \Pi = \dfrac{e^{r\Delta t} - d}{u-d} & \rightarrow \alpha \\ \sigma^2 \Delta t = e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} & \rightarrow \beta \quad (\mathrm{A.8}) \\ u = \dfrac{1}{d} & \rightarrow \gamma \end{cases}$$

$$\gamma \rightarrow \beta$$

$$e^{r\Delta t}\left(u + \frac{1}{u}\right) - u\frac{1}{u} - e^{2r\Delta t} = \sigma^2 \Delta t$$
$$u + \frac{1}{u} = \frac{\sigma^2 \Delta t + 1 + e^{2r\Delta t}}{e^{r\Delta t}}$$
$$u + \frac{1}{u} = e^{-r\Delta t}[\sigma^2 \Delta t + 1 + e^{2r\Delta t}]$$
$$u + \frac{1}{u} = e^{-r\Delta t}\sigma^2 \Delta t + e^{-r\Delta t} + e^{r\Delta t} \quad (\mathrm{A.9})$$

Under the hypothesis that Δt is very small:

$$e^{-r\Delta t} \approx (1 - r\Delta t), e^{+r\Delta t} \approx (1 + r\Delta t), r\sigma^2 \Delta t^2 \rightarrow 0 \quad (\mathrm{A.10})$$

Thus, the quadratic equation becomes:

$$u + \frac{1}{u} \approx (1 - r\Delta t)\sigma^2 \Delta t + 1 - r\Delta t + 1 + r\Delta t =$$
$$= \sigma^2 \Delta t - r\sigma^2 \Delta t^2 + 2 = \sigma^2 \Delta t + 2$$
$$u + \frac{1}{u} = \sigma^2 \Delta t + 2 \rightarrow u^2 + 1 = \sigma^2 u \Delta t + 2u.$$
$$u^2 - (\sigma^2 \Delta t + 2)u + 1 = 0 \quad (\mathrm{A.11})$$

Solving for u:

$$u = \frac{\sigma^2 \Delta t + 2 \pm \sqrt{(\sigma^2 \Delta t + 2)^2 - 4}}{2} = \frac{\sigma^2 \Delta t + 2 \pm \sqrt{\sigma^4 \Delta t^2 + 4\sigma^2 \Delta t + 4 - 4}}{2}$$
$$= \frac{\sigma^2 \Delta t + 2 \pm \sqrt{\sigma^4 \Delta t^2 + 4\sigma^2 \Delta t}}{2} = \frac{\sigma^2 \Delta t}{2} + 1 \pm \sigma\sqrt{\Delta t} \quad (\mathrm{A.12})$$

For a very small Δt, $\sigma^2 \Delta t^2$ tends to 0. Since $\sqrt{\Delta t}$ is far larger than Δt for small Δt, and σ^2 is relatively smaller than σ, we can ignore the first term $\frac{\sigma^2 \Delta t}{2}$.

$$u \approx 1 \pm \sigma\sqrt{\Delta t} \quad \text{(A.13)}$$
$$u \approx \exp(\sigma\sqrt{\Delta t}) \text{ because } u > 1 \Longrightarrow u \neq \exp(-\sigma\sqrt{\Delta t})$$

Hence, the below set of parameters enables the construction of a binomial stochastic tree that fully aligns with the Black-Scholes pricing framework. Thus, if the number of time intervals approaches infinity $N \to \infty$, the model theoretically converges to the closed-form valuation formula for European vanilla options.

$$\begin{cases} e^{r\Delta t} = \prod u + (1 - \prod)d \\ e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} = \sigma^2 \Delta t \\ u = \frac{1}{d} \end{cases} \rightarrow \quad \begin{cases} \Pi = \frac{e^{r\Delta t} - d}{u-d} \\ u = e^{\sigma\sqrt{\Delta t}} \\ d = e^{-\sigma\sqrt{\Delta t}} \end{cases} \quad \text{(A.14)}$$

The numerical formulas of the binomial approach can be extended to more underlyings by introducing the parameter b called cost-of-carry (Haug, 2007).
Based on the value of the parameter b, we reach a pricing framework applicable to a wide range of underlying assets for which call or put options can be written. The adjustment needed is in the definition of the risk-neutral probability Π, equation (A.2).

If $b = r$ the definition is suitable to be used for the pricing of options written on shares that pay no dividend.

If $b = r - q$ the definition is suitable to be used for the pricing of options written on shares/indexes with a continuous dividend yield q.

If $b = 0$ the definition is suitable to be used for the pricing of options on futures.

If $b = r - r_{FOR}$ the definition is suitable to be used for the pricing of currency options.

Binomial Option Pricing: The Cox-Ross-Rubinstein

Building on the derivation of equation (A.14) in the previous section, which defines the up and down factors and risk-neutral probabilities aligning with the Geometric Brownian Motion framework, this section extends these principles to practical applications. It explores the implementation of the CRR binomial tree model to price European and American options, emphasizing its adaptability for different payoff structures. This transition from theoretical parameter derivation to numerical application demonstrates that the CRR model bridges discrete and continuous approaches to option valuation, providing a robust framework for pricing various financial derivatives.
The Binomial Option Pricing is a numerical method for pricing options and derivative securities. Unlike analytical solutions, this numerical method is versatile and can handle a broader range of options for which no closed-form solutions exist.
The binomial method is the most widely used numerical approach for pricing American options on stocks, futures, and currencies. Originally developed by Cox, Ross, and Rubinstein (1979) and Rendleman and Bartter (1979), this method approximates Geometric Brownian Motion with a recombining binomial tree.
When the number of time steps is large, the binomial tree converges to the continuous Black-Scholes-Merton model for European options. The binomial model is especially suited for pricing American options, where no closed-form solution exists, and for many exotic options.
In a binomial tree, the asset price can increase by a factor u with probability p or decrease by a factor d with probability $(1 - \prod)$ over each time step Δt. The number of time steps is n. Each node is represented by (j, i) where j is the number of time steps to a node in the tree, and i represents the number of upward moves.
The first node $(j = 0, i = 0)$ of the tree progresses with each step. If the asset price goes up at the second node, it will be assigned $(j = 1, i = 1)$. If the asset price goes down at the first-time step, we have $(j = 1, i = 0)$, as shown in Figure A.2.
The number of paths leading to a node (j, i) is $\frac{j!}{i!(j-i)!}$, and the equivalent probability of reaching node (j, i) is $\frac{j!}{i!(j-i)!}\prod^i(1 - \prod)^{j-i}$ (Giribone, 2024).

To price European call or put options, we only need the end nodes at time n, such that (with X denoting the strike price):

$$c = e^{-rT}\sum_{i=0}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^i(1-\prod)^{n-i}\max[Su^i d^{n-i} - X, 0] \quad (\text{A}.15)$$

$$p = e^{-rT}\sum_{i=0}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^i(1-\prod)^{n-i}\max[X - Su^i d^{n-i}, 0] \quad (\text{A}.16)$$

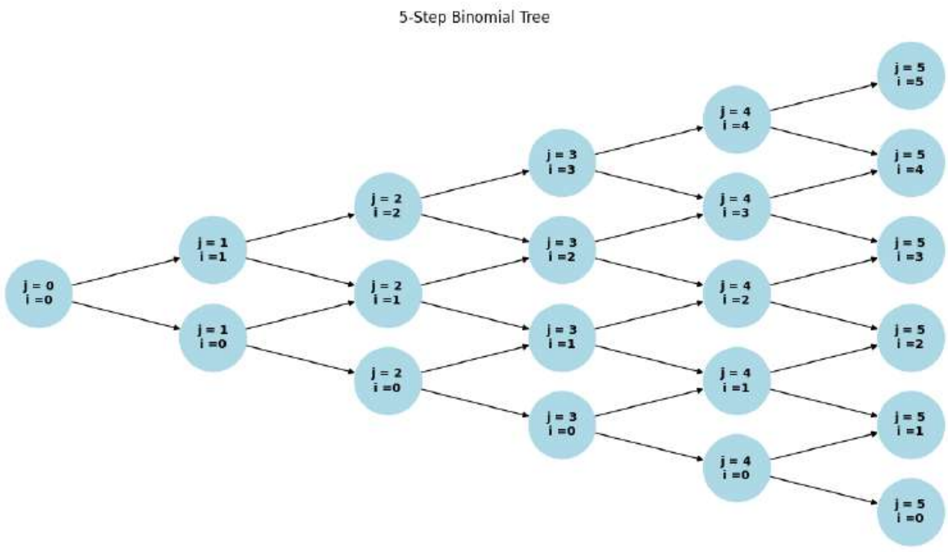

Figure A.2: Standard indexing for a 5-Step Binomial Tree

Many nodes will be out-of-the-money, so instead of starting the count from the lowest node $(i = 0)$, we can improve the algorithm efficiency by beginning at a (for a call option), which is the smallest non-negative integer greater than $\frac{\ln\left(\frac{X}{Sd^n}\right)}{\ln\left(\frac{u}{d}\right)}$. This gives:

$$c = e^{-rT}\sum_{i=a}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^{i}(1-\prod)^{n-i}(Su^i d^{n-i} - X) \quad (A.17)$$

$$p = e^{-rT}\sum_{i=a}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^{i}(1-\prod)^{n-i}(X - Su^i d^{n-i}) \quad (A.18)$$

Generalized European Binomial

The European binomial model in its more general form is:

$$c = e^{-rT}\sum_{i=0}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^{i}(1-\prod)^{n-i}g[S(T),X] \quad (A.19)$$

$$p = e^{-rT}\sum_{i=0}^{n}\left(\frac{n!}{i!\,(n-i)!}\right)\prod^{i}(1-\prod)^{n-i}g[S(T),X] \quad (A.20)$$

where $S(T) = Su^i d^{n-i}$ and $g[S(T),X]$ represents any specified payoff function at maturity.
This highlights the versatility of the simple binomial model, as it can price any European option on a single asset with a payoff that is not path dependent.
For example, to determine the value of a power option with a payoff of $\max[S^2 - X, 0]$ at maturity, we simply replace $g[S(T),X]$ with $\max[(Su^i d^{n-i})^2 - X, 0]$. The variable *z* equals 1 if the contract is a call, and -1 if it is a put.

Cox-Ross-Rubinstein American Binomial Tree

Here, we examine how to apply the Cox-Ross-Rubinstein binomial tree to value American-style options. At each node, the asset price is given by:

$$Su^i d^{n-i}, \qquad with\ i = 0, 1, \dots, j \quad \text{(A.21)}$$

where u and d are the up and down jump factors for each time interval $\Delta t = \frac{T}{n}$, with n being the number of time steps, as previously defined (see Figure A.3). The probability of the stock price increasing by factor u is given by equation (A.17).
Since the probabilities must add up to one, the probability of the stock price decreasing by d is $(1 - \prod)$.

The up and down factors and probabilities are selected to match the first two moments of the stock price distribution, ensuring that as Δt approaches zero, the probability distribution generated by the binomial tree converges to Geometric Brownian motion. A key benefit of this numerical pricing technique consists in its ability to compute the fair value of options with the possibility of early exercise, such as American and Bermudan options. This flexibility, however, introduces additional complexity compared to standard European-style valuation. Rather than simply calculating the payoff at maturity and propagating the value backwards using a conventional algorithm, it becomes necessary to evaluate, at each time step, whether exercising the option immediately yields a higher value than holding it. Consequently, at every node in the binomial tree, the option value is determined as the maximum between its immediate exercise payoff and its continuation value:

$$C_t = \max[C_{dead}; C_{alive}] = \max\left[S_t - K; \frac{C_u \cdot \Pi + C_d \cdot (1-\Pi)}{1+R}\right] \quad \text{(A.22)}$$

$$P_t = \max[P_{dead}; P_{alive}] = \max\left[K - S_t; \frac{P_u \cdot \Pi + P_d \cdot (1-\Pi)}{1+R}\right] \quad \text{(A.23)}$$

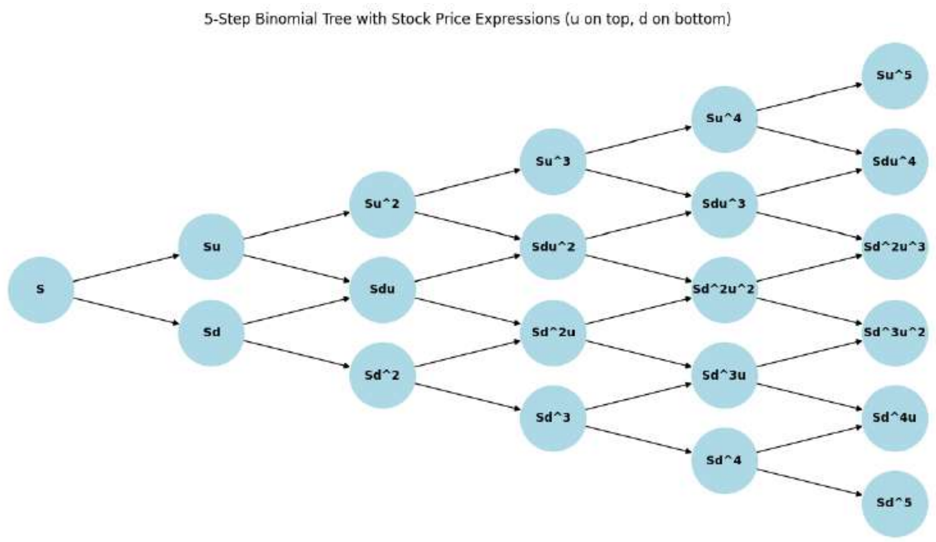

Figure A.3: 5-Step Binomial Tree with stock price movements

Appendix B – Market Inputs

This appendix reports the market data used for the numerical experiment of pricing and sensitivity estimation with the different alternative binomial models. Market values are downloaded from Bloomberg® at closing market values on December 31, 2024.

Underlyings			CDS							
Ticker	Name	Price	0.5	1	2	3	4	5	7	10
ENR GY Equity	Siemens Energy AG	50.38	9.4	13.7	19.4	25.2	29.6	31	44	64.5
SY1 GY Equity	Symrise AG	102.65	36	36	36	36	36	36	36	36
PAH3 GY Equity	Porsche Automobil Holding SE	36.35	48.5	53.1	68.5	83	110.6	146.35	182.1	218.9
MTX GY Equity	MTU Aero Engines AG	322	49	49	49	49	49	49	49	49
RHM GY Equity	Rheinmetall AG	614.6	50.4	58.2	75.6	96.3	117.1	137.1	166.2	185.3
DTG GY Equity	Daimler Truck Holding AG	36.85	97	97	97	97	97	97	97	97
SHL GY Equity	Siemens Healthineers AG	51.2	9.4	13.7	19.4	25.2	29.6	31	44	64.5
ZAL GY Equity	Zalando SE	32.39	71	71	71	71	71	71	71	71
QIA GY Equity	QIAGEN NV	43.045	44.65	44.65	44.65	44.65	44.65	44.65	44.65	44.65
SRT3 GY Equity	Sartorius AG	215.2	47	47	47	47	47	47	47	47
BNR GY Equity	Brenntag SE	57.88	63	63	63	63	63	63	63	63
AIR GY Equity	Airbus SE	154.46	20	20	27	34	41	48	68.5	92.5
ALV GY Equity	Allianz SE	295.9	9.05	11.5	14.6	20.5	26.36	32.5	43.6	53.6
RWE GY Equity	RWE AG	28.83	7.2	8.5	12.3	18	25.14	33.3	46.6	59.9
BAYN GY Equity	Bayer AG	19.314	34.1	39	52	68	86	103.5	135	170
BMW GY Equity	Bayerische Motoren Werke AG	78.98	16.8	19	26.9	34.7	48.1	61.5	84.5	108
CBK GY Equity	Commerzbank AG	15.725	18.6	23.2	30.2	36.6	44.785	53	69.3	90
DBK GY Equity	Deutsche Bank AG	16.64	13.7	16.9	28.8	39.1	49.73	61.5	82.3	102
BAS GY Equity	BASF SE	42.46	13.9	15.6	21.5	27.4	39.7	52	71	92
HEN3 GY Equity	Henkel AG & Co KGaA	84.7	6.8	8	10.6	13.3	17.1	21	29	39.5
SIE GY Equity	Siemens AG	188.56	9.4	13.7	19.4	25.2	29.6	31	44	64.5
VOW3 GY Equity	Volkswagen AG	89.04	35.2	39	50	66.3	89.2	112.5	152	183.5
EOAN GY Equity	E.ON SE	11.245	9.6	11	15.8	20.6	34.5	27.5	52.5	70.5
BEI GY Equity	Beiersdorf AG	124	14	14	14	14	14	14	14	14
HEI GY Equity	Heidelberg Materials AG	119.3	14.4	18.2	27.9	38.2	53.1	68	99	129
MUV2 GY Equity	Muenchener Rueckversicherungs-Gesellschaft AG in Muenchen	487.1	8.2	10.5	14.2	20.3	27.07	34.5	45.6	56.1
FRE GY Equity	Fresenius SE & Co KGaA	33.54	6.8	10.6	18.6	27.5	35.27	40	56.1	62.8
SAP GY Equity	SAP SE	236.3	26	26	26	26	26	26	26	26
MRK GY Equity	Merck KGaA	139.9	5.9	8.4	11.7	16.6	22.88	30	40.7	52.5
ADS GY Equity	adidas AG	236.8	24.3	27.1	31.9	37.8	45.19	52.9	65.2	78.7
DTE GY Equity	Deutsche Telekom AG	28.89	13.7	16.7	22	27.2	32.6	39.5	50.5	69.5
DHL GY Equity	Deutsche Post AG	33.98	7.5	9.1	12.5	15.5	20.9	26	36.5	46.5
FME GY Equity	Fresenius Medical Care AG	44.16	6.8	10.6	18.6	27.5	35.27	40	56.1	62.8
MBG GY Equity	Mercedes-Benz Group AG	53.8	17.2	19.9	25.3	34.4	47.5	60.5	83.5	107.5
IFX GY Equity	Infineon Technologies AG	31.4	72.7	78.7	87.1	95.3	101.73	107.3	116.1	125.1
DB1 GY Equity	Deutsche Boerse AG	222.4	18.4	20	22.2	25.6	29.75	34.3	42.2	49.9
VNA GY Equity	Vonovia SE	29.32	107	107	107	107	107	107	107	107
P911 GY Equity	Dr Ing hc F Porsche AG	58.42	67	67	67	67	67	67	67	67
HNR1 GY Equity	Hannover Rueck SE	241.4	8.4	10.9	14.7	21.4	27.79	35	48	58.5
CON GY Equity	Continental AG	64.82	13.6	16.4	31.2	46	65.7	85.5	122.5	146.5

Table B.1 Close prices for the Equity shares in EUR and CDS premiums for the Issuers in bps. Reference Date: EOY 2024

T/Equity	ENR GY Equity	SY1 GY Equity	PAH3 GY Equity	MTX GY Equity	RHM GY Equity	DTG GY Equity	SHL GY Equity	ZAL GY Equity	QIA GY Equity	SRT3 GY Equity
0.5	0	2.718	5.998	0	0.546	7.394	2.907	0	0.556	1.531
1	0	1.355	2.991	0	0.272	3.687	1.449	0	0.277	0.763
2	0.94	0.968	4.385	0.614	0.6	3.332	1.722	0.108	0.277	0.837
3	1.156	0.837	4.581	0.592	0.714	3.183	1.71	0.131	0.269	0.859
4	1.257	0.77	4.608	0.647	0.767	3.084	1.704	0.15	0.276	0.868
5	1.313	0.73	4.576	0.679	0.797	3.009	1.689	0.158	0.276	0.872
7	1.369	0.682	4.435	0.712	0.827	2.887	1.659	0.165	0.272	0.872
10	1.386	0.643	4.163	0.731	0.842	2.733	1.616	0.174	0.274	0.865

T/Equity	BNR GY Equity	AIR GY Equity	ALV GY Equity	RWE GY Equity	BAYN GY Equity	BMW GY Equity	CBK GY Equity	DBK GY Equity	BAS GY Equity	HEN3 GY Equity
0.5	3.898	1.68	7.583	7.339	0.212	8.687	6.065	5.357	6.65	4.64
1	1.944	1.68	3.781	3.659	0.106	4.332	3.024	2.671	3.316	2.314
2	2.214	1.68	3.619	2.977	0.354	3.896	2.697	3.36	3.626	2.35
3	2.284	1.68	3.526	2.717	0.434	4.503	2.556	3.558	3.673	2.358
4	2.301	1.68	3.449	2.573	0.473	4.715	2.47	3.595	3.655	2.356
5	2.301	1.68	3.386	2.477	0.496	4.778	2.41	3.589	3.616	2.336
7	2.281	1.68	3.259	2.347	0.52	4.718	2.323	3.517	3.514	2.289
10	2.215	1.68	3.08	2.21	0.535	4.464	2.211	3.357	3.335	2.21

T/Equity	SIE GY Equity	VOW3 GY Equity	EOAN GY Equity	BEI GY Equity	HEI GY Equity	MUV2 GY Equity	FRE GY Equity	SAP GY Equity	MRK GY Equity	ADS GY Equity
0.5	3.645	7.257	6.204	1.603	4.178	4.969	0	1.23	1.943	0.583
1	1.817	3.618	3.094	0.799	2.083	2.478	0	0.614	0.969	0.291
2	1.866	3.503	3.698	0.506	2.127	2.651	0	0.738	0.998	0.526
3	1.869	3.458	3.836	0.409	2.126	2.682	0	0.776	1.002	0.603
4	1.861	3.403	3.875	0.362	2.112	2.676	0	0.793	1	0.639
5	1.85	3.36	3.851	0.332	2.095	2.668	0	0.805	0.998	0.66
7	1.821	3.245	3.751	0.298	2.057	2.612	0	0.811	0.993	0.681
10	1.772	3.075	3.557	0.272	1.992	2.513	0	0.808	0.978	0.693

T/Equity	DTE GY Equity	DHL GY Equity	FME GY Equity	MBG GY Equity	IFX GY Equity	DB1 GY Equity	VNA GY Equity	P911 GY Equity	HNR1 GY Equity	CON GY Equity
0.5	0	7.709	3.534	10.268	1.39	2.64	6.386	0	3.239	4.084
1	0	3.844	1.762	5.12	0.693	1.317	3.184	2.527	1.615	2.036
2	4.199	3.815	1.813	4.622	0.571	1.292	3.388	2.73	0.938	1.975
3	7.428	3.755	1.831	4.396	0.529	1.279	3.46	2.768	0.712	1.93
4	8.51	3.703	1.829	4.247	0.507	1.267	3.459	2.764	0.598	1.906
5	8.842	3.633	1.822	4.118	0.496	1.257	3.433	2.746	0.53	1.879
7	8.546	3.496	1.799	3.905	0.479	1.24	3.348	2.691	0.453	1.832
10	7.535	3.296	1.754	3.634	0.465	1.213	3.191	2.589	0.393	1.771

Table B.2 Dividend Yield term structures for the German stocks in [%]. Reference Date: EOY 2024

ENR GY Equity	80	100	120
0.5	55.56	51.77	50.21
1	54.65	51.53	49.75
2	53.23	51.15	49.83
3	52.52	50.84	49.73
4	51.99	50.54	49.55
5	51.55	50.25	49.34
7	50.9	49.79	49
10	50.2	49.27	48.58

SY1 GY Equity	80	100	120
0.5	25.65	20.63	19.99
1	24.44	21.22	20.22
2	23.57	21.37	20.09
3	23.31	21.58	20.46
4	23.29	21.81	20.79
5	23.33	22	21.07
7	23.41	22.28	21.45
10	23.24	22.26	21.52

PAH3 GY Equity	80	100	120
0.5	32.13	26.7	25.27
1	29.91	25.69	24.1
2	33.44	26.95	23
3	31.32	26.09	22.73
4	30.15	25.65	22.69
5	29.25	25.21	22.51
7	28.04	24.61	22.28
10	26.68	23.77	21.76

MTX GY Equity	80	100	120
0.5	34	28.05	25.18
1	31.3	27.65	25.51
2	29.98	27.51	25.61
3	29.51	27.49	25.8
4	29.28	27.51	26
5	29.13	27.53	26.15
7	29	27.63	26.43
10	28.66	27.48	26.47

RHM GY Equity	80	100	120
0.5	39.35	36.78	36.38
1	37.71	36.13	35.81
2	36.83	35.73	35.41
3	36.37	35.57	35.31
4	36.1	35.47	35.25
5	35.93	35.4	35.21
7	35.7	35.3	35.15
10	35.49	35.2	35.09

DTG GY Equity	80	100	120
0.5	31.83	27.24	26.98
1	30.81	27.09	26.01
2	29.3	26.39	25.23
3	28.55	26.29	25.27
4	28.44	26.24	25.31
5	27.87	26.21	25.34
7	27.54	26.16	25.39
10	27.24	26.11	25.43

SHL GY Equity	80	100	120
0.5	27.23	22.79	21.96
1	26.16	23.31	21.76
2	25.24	23.27	21.96
3	24.96	23.32	22.16
4	24.83	23.38	22.32
5	24.77	23.45	22.46
7	24.72	23.57	22.68
10	24.73	23.73	22.94

ZAL GY Equity	80	100	120
0.5	43.34	39.92	39.52
1	42.93	40.61	39.61
2	41.66	40.26	39.55
3	41.26	40.2	39.61
4	41.09	40.21	39.7
5	41.05	40.29	39.83
7	41.04	40.44	40.06
10	40.72	40.23	39.91

QIA GY Equity	80	100	120
0.5	29.43	24.29	24.88
1	26.13	23.24	22.88
2	25.26	23.07	21.97
3	24.96	23.22	22.24
4	24.83	23.34	22.44
5	24.83	23.49	22.65
7	24.84	23.7	22.94
10	24.78	23.81	23.11

SRT3 GY Equity	80	100	120
0.5	48.6	44.82	42.28
1	47.48	44.51	42.16
2	46.49	44.33	42.59
3	45.96	44.16	42.7
4	45.57	43.99	42.7
5	45.25	43.82	42.64
7	44.71	43.47	42.45
10	44.01	42.94	42.06

BNR GY Equity	80	100	120
0.5	25.53	23.85	21.81
1	25.11	22.63	20.09
2	26.1	23.25	21.13
3	25.54	23.14	21.35
4	25.31	23.18	21.62
5	25.06	23.12	21.72
7	24.81	23.14	21.95
10	24.19	22.78	21.77

AIR GY Equity	80	100	120
0.5	38.385	38.385	38.385
1	38.385	38.385	38.385
2	38.385	38.385	38.385
3	38.385	38.385	38.385
4	38.385	38.385	38.385
5	38.385	38.385	38.385
7	38.385	38.385	38.385
10	38.385	38.385	38.385

ALV GY Equity	80	100	120
0.5	24.26	16.97	15.87
1	22.73	17.69	16.17
2	20.62	18.11	16.85
3	19.98	18.02	17.16
4	19.81	18.25	17.54
5	19.52	18.26	17.69
7	19.34	18.41	17.96
10	18.96	18.23	17.88

RWE GY Equity	80	100	120
0.5	27.95	24.4	24.92
1	27.2	25.07	24.7
2	26.05	24.98	24.45
3	26.19	25.12	24.58
4	26.03	25.13	24.64
5	25.84	25.06	24.63
7	25.6	24.94	24.57
10	25.32	24.83	24.58

BAYN GY Equity	80	100	120
0.5	40.02	38.24	38.54
1	39.77	38.19	38.28
2	38.32	37.47	37.36
3	36.77	35.94	35.84
4	35.78	35.03	34.87
5	34.88	34.17	33.99
7	33.61	33.02	32.82
10	33.04	33.23	31.9

BMW GY Equity	80	100	120
0.5	32.77	27.69	26.22
1	30.77	26.68	24.81
2	27.44	25.29	24.08
3	26.96	25.33	24.43
4	26.4	25.3	24.75
5	26.11	25.24	24.79
7	25.91	25.25	24.88
10	25.55	25.04	24.75

CBK GY Equity	80	100	120
0.5	37.86	31.97	29.46
1	37.2	32.44	29.65
2	35.35	32.82	31.07
3	34.32	32.45	31.2
4	33.91	32.39	31.37
5	33.54	32.26	31.41
7	33.13	32.16	31.49
10	32.58	31.78	31.23

DBK GY Equity	80	100	120
0.5	34.75	30.04	28.56
1	33.88	30.74	29.07
2	32.1	30.16	29.27
3	31.29	30.18	29.79
4	31.29	30.45	30.15
5	31.19	30.51	30.25
7	31.26	30.77	30.57
10	31.05	30.66	30.51

BAS GY Equity	80	100	120
0.5	30.65	25.61	23.92
1	28.67	24.56	22.54
2	25.45	23.35	22.04
3	25.05	23.28	22.22
4	24.76	23.39	22.57
5	24.49	23.36	22.68
7	24.27	23.37	22.83
10	23.87	23.13	22.69

HEN3 GY Equity	80	100	120
0.5	23	17.6	17.35
1	21.53	18.1	17.25
2	20.14	18.06	17.42
3	19.61	18.05	17.48
4	19.34	18.09	17.59
5	19.12	18.08	17.64
7	18.88	18.1	17.73
10	18.54	17.88	17.53

SIE GY Equity	80	100	120
0.5	29.63	25.31	25.18
1	29.42	25.62	24.7
2	27.93	25.84	24.59
3	27.35	25.53	24.63
4	26.95	25.45	24.62
5	26.65	25.39	24.64
7	26.27	25.32	24.73
10	25.83	25.09	24.64

VOW3 GY Equity	80	100	120
0.5	31.04	27.63	26.57
1	29.57	26.43	25.22
2	27.71	25.33	24.25
3	27.15	25.48	24.96
4	27.07	25.67	25.24
5	26.88	25.7	25.32
7	26.71	25.76	25.47
10	26.61	25.8	25.56

EOAN GY Equity	80	100	120
0.5	26.36	20.77	19.31
1	24.49	20.45	18.38
2	22.55	20.11	18.76
3	21.74	19.74	18.99
4	21.39	19.81	19.22
5	21.09	19.79	19.28
7	20.76	19.85	19.47
10	20.43	19.7	19.38

BEI GY Equity	80	100	120
0.5	24.79	19.48	18.56
1	23.02	19.4	18.07
2	21.97	19.19	18.03
3	21.25	19.09	18.05
4	20.91	19.14	18.2
5	20.7	19.18	18.34
7	20.48	19.27	18.54
10	20.13	19.14	18.49

HEI GY Equity	80	100	120
0.5	30.72	26.36	25.34
1	29.8	26.35	25.04
2	28.33	26.14	25.28
3	27.72	26.05	25.36
4	27.39	26.05	25.47
5	27.17	26.03	25.52
7	26.91	26.03	25.62
10	26.61	25.89	25.54

MUV2 GY Equity	80	100	120
0.5	26.42	21.83	21.09
1	24.62	21.59	20.3
2	22.9	20.42	19.24
3	22.41	20.41	19.64
4	22.12	20.58	19.87
5	21.86	20.61	20.02
7	21.73	20.73	20.2
10	21.33	20.54	20.12

FRE GY Equity	80	100	120
0.5	29.17	24.02	22.64
1	27.38	23.77	22.21
2	26.14	24.04	22.57
3	25.7	23.99	22.85
4	25.51	24.12	23.18
5	25.36	24.21	23.43
7	25.25	24.36	23.77
10	24.96	24.28	23.85

SAP GY Equity	80	100	120
0.5	30.5	25.74	25.08
1	30.2	26.16	25.09
2	28.19	25.8	24.85
3	27.34	25.59	24.72
4	26.93	25.46	24.7
5	26.65	25.37	24.67
7	26.24	25.21	24.62
10	25.84	25.03	24.59

MRK GY Equity	80	100	120
0.5	30.23	25.01	24.4
1	28.54	25.32	24.57
2	27.12	25.32	24.73
3	26.68	25.28	24.79
4	26.44	25.29	24.85
5	26.28	25.29	24.88
7	26.04	25.28	24.94
10	25.88	25.18	24.84

ADS GY Equity	80	100	120
0.5	33.75	28.92	27.95
1	32.31	29.1	27.73
2	30.34	28.23	26.98
3	29.74	27.97	27.26
4	29.58	28.05	27.32
5	29.44	28.11	27.42
7	29.15	28.15	27.58
10	28.95	28.19	27.76

DTE GY Equity	80	100	120
0.5	22.92	17.27	17.53
1	21.39	16.42	16.52
2	17.74	15.63	16.32
3	16.83	16.65	17.54
4	17.04	17.35	17.82
5	17.45	17.76	18
7	18	18.01	18.02
10	18.29	18.18	18.17

DHL GY Equity	80	100	120
0.5	27.95	22.66	21.66
1	26.76	22.47	20.75
2	24.37	21.87	20.34
3	23.73	21.74	20.45
4	23.35	21.66	20.52
5	23.11	21.6	20.57
7	22.78	21.51	20.63
10	22.49	21.43	20.67

FME GY Equity	80	100	120
0.5	34.24	30.5	28.77
1	32.56	29.62	27.95
2	31.08	27.86	25.96
3	30.05	27.12	25.25
4	29.68	27.03	25.27
5	29.42	27.05	25.45
7	29.26	27.26	25.87
10	28.29	26.55	25.36

MBG GY Equity	80	100	120
0.5	32.64	26.85	24.7
1	30.82	26.05	23.96
2	29.64	25.95	24.22
3	29.95	27.1	25.86
4	29.6	27.31	26.23
5	29.45	27.51	26.55
7	29.44	27.93	27.12
10	29.74	28.58	27.91

IFX GY Equity	80	100	120
0.5	38.57	35.08	33.58
1	37.48	34.71	33.4
2	36.87	34.44	32.82
3	35.62	34.7	34.25
4	35.91	34.59	33.74
5	35.59	34.47	33.74
7	35.26	34.36	33.74
10	34.77	33.99	33.44

DB1 GY Equity	80	100	120
0.5	24.63	19.02	17.4
1	22.9	18.87	17.49
2	21.65	19.02	17.78
3	31.1	19.06	17.98
4	20.9	19.17	18.2
5	20.79	19.25	18.35
7	20.7	19.4	18.59
10	20.41	19.28	18.54

VNA GY Equity	80	100	120
0.5	31.72	29.26	28.07
1	31.46	28.91	27.75
2	30.48	28.66	27.12
3	30.37	28.83	27.56
4	30.36	29	27.9
5	30.34	29.1	28.11
7	30.37	29.29	28.45
10	30.14	29.22	28.5

P911 GY Equity	80	100	120
0.5	35.4	29.63	29.85
1	32.52	30.22	29.66
2	31.55	30.07	29.5
3	31.12	29.94	29.41
4	30.85	29.84	29.35
5	30.66	29.77	29.3
7	30.4	29.64	29.21
10	30.11	29.47	29.09

HNR1 GY Equity	80	100	120
0.5	25.58	20.24	19.45
1	24.62	20.63	19.4
2	23.65	20.91	19.7
3	23.4	21.16	20.02
4	23.34	21.4	20.32
5	23.38	21.63	20.6
7	23.57	22.07	21.13
10	23.98	22.71	21.86

CON GY Equity	80	100	120
0.5	34.91	30.2	28.29
1	34.04	30.67	28.74
2	31.05	28.87	27.35
3	30.77	29.02	27.74
4	30.68	29.18	28.05
5	30.59	29.24	28.22
7	30.52	29.38	28.5
10	30.31	29.34	28.58

Table B.3 Implied Volatility sections for 80% 100% 120% Moneyness. Volatility expressed in [%]. Reference Date: EOY 2024

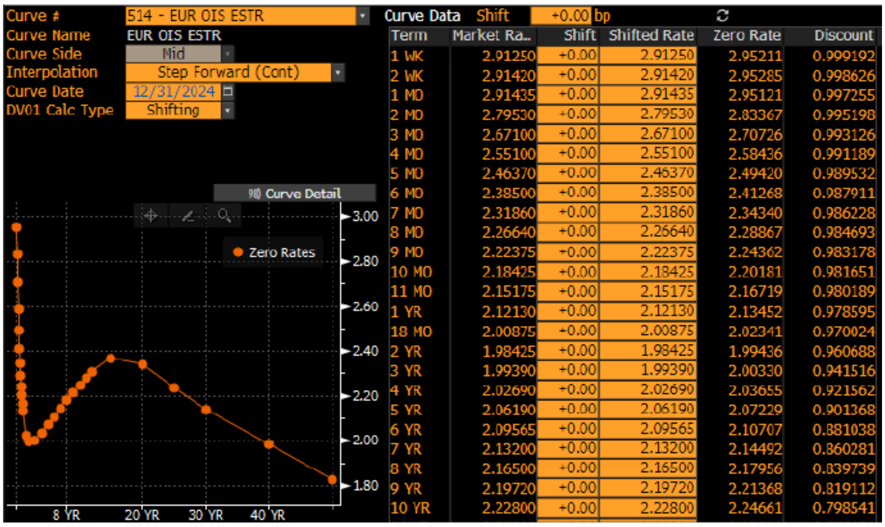

Figure B.1 Interest rates term structure – Tenor: 1 day (ESTER curve). Rates expressed in [%]. Reference Date: EOY 2024

Firm Performance and capital structure: does liquidity matter?

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Article submitted to double-blind peer review, received on 25th August 2025 and accepted on 10th November 2025

Abstract

This research examines how capital structure and liquidity impact the financial performance of South African firms from 2012 to 2023. Using panel data methodologies and the Generalised Method of Moments (GMM) estimation, the study addresses potential endogeneity issues and inaccuracies in the dataset. The findings reveal a negative and significant relationship between the proportion of long-term debt ratio (LTDR) and corporate profitability, measured by return on assets (ROA) and return on equity (ROE). This suggests that a higher reliance on long-term borrowing negatively affects corporate outcomes. In contrast, liquidity metrics, represented by the current ratio (CUR) and quick ratio (CR), show a direct and significant positive effect on return on asset (ROA), return on equity (ROE) and Tobin's Q. These results imply that firms with stronger liquidity positions are better able to meet their immediate financial obligations and capitalise on growth opportunities, thereby enhancing their financial performance. The study provides valuable insights for corporate finance policies and suggests directions for future research on corporate financial strategies in developing economies.

Keywords: Capital Structure, Liquidity, Financial Performance, Trade-off Theory, Pecking Order Theory, and Emerging Markets.

1. Introduction and background

This article provides a critical analysis of the impact of capital structure and liquidity on company performance, with a specific focus on the South African corporate landscape. Within this context, liquidity appears as a vital factor, intricately linked with capital structure in determining performance results. As noted by Miglo (2016) and Hannyama, Kabwe, and Zulu (2025), capital structure pertains to the proportional combination of debt and equity employed by firms to finance their operations. Traditionally, this concept was regarded as a highly technical and peripheral matter, managed by only a limited number of finance experts within companies. Cornett, Adair, and Nofsinger (2018) propose that capital structure was frequently considered either static or irrelevant to wider financial decision-making.

In contrast, liquidity is characterised by a firm's capability to fulfill its financial commitments as they arise, and to maintain its operational and expansion activities (Nguyen, Phan, & Hang, 2024; Nguyen & Dao, 2022; Adebiyi, 2021; Bodie, Kane & Marcus, 2017). This analysis situates liquidity as a fundamental component in the interaction between capital structure and performance. The core proposition is that decisions regarding capital structure cannot independently lead to optimal company outcomes if restrictions in liquidity exist. In this context, liquidity functions as a facilitating factor that guarantees the operational success and efficacy of a company's financial framework.

On the other hand, firm performance is viewed as a vital tool for determining whether a firm is thriving or struggling. According to Suwaidan, Al-Khoury, Areiqat, and Cherrati, (2021), financially strong companies are more likely to report openly and gain the trust of investors. Many elements, such as technological progress, employee alignment, communication quality, and how well a company responds to customers, have an impact on performance. However, profitability is the most commonly accepted measure of performance. Profitability not only indicates a firm's ability to manage expenses and earn revenue (Nassar, 2016) but also acts as a crucial standard for assessing sustainability (Etale, Ochuba & Sawyer, 2021). Therefore, firm performance is the main perspective through which this study examines how effectively capital structure and liquidity are managed.

This study also highlights the intricate two-way link between how a firm is financed and how well it performs. On one side, having the right mix of debt and equity can boost a firm’s success (Doan, 2020; Amare, 2021). On the flip side, better financial outcomes might enable a firm to secure loans more easily and on better terms (Abdullah & Tursoy, 2021). According to Yusuf, Al Attar, and Al Shattarat (2015), if financing choices are poor, they can lead to financial troubles, but smart financial structuring can improve both value and efficiency. Yet, as noted by Marozva (2019) and Brunnermeier and Oehmke (2012), there are still gaps in how we measure overall liquidity, making it tricky to evaluate its impact on a firm’s performance.

The academic literature examining the relationship between liquidity, financial gearing, and firm performance has produced inconclusive and often conflicting results. Some research highlights a positive link between liquidity, leverage, and firm performance (Sharma & Sarin, 2024; Abubakar, 2023; Jihadi, Vilantika, Hashemi, Arifin, Bachtiar, & Sholichah, 2021; Zaitoun & Alqudah, 2020), while other studies find a negative connection (Daryanto, Samidi, & Siregar, 2018; Källum & Sturesson, 2017). However, other studies suggest that increased liquid assets and debt financing can enhance corporate performance by facilitating investment and optimising capital allocation (Abubakar, 2023; Jihadi, Vilantika, Hashemi, Arifin, Bachtiar, & Sholichah, 2021). In contrast, other research indicates that excessively high levels of liquid assets and debt may exacerbate financial instability and hinder positive outcomes (Daryanto, Samidi, & Siregar, 2018). These inconsistencies are further complicated by external factors, such as the macroeconomic environment, industry characteristics, and legal frameworks, all of which can significantly influence these relationships (Källum & Sturesson, 2017).
Despite extensive research across various international contexts, there remains a notable gap in understanding the effects of liquidity and financial gearing on firm performance in specific developing market situations like South Africa, which has unique economic and regulatory characteristics. This highlights the need for further empirical analysis to clarify these relationships and provide insights that can inform corporate governance and policy development.
Thus, this study seeks to explore the relationship between liquidity, financial leverage, and firm performance among firms in South Africa, providing a deeper understanding of these dynamics in developing economies. Developing economies are argued to be structurally, fundamentally, and technically different from the developed world (Marozva, 2020).
This research presents various contributions to the extant scholarly knowledge. Firstly, it addresses a significant gap in the South African academic discourse by concurrently investigating the influences of capital structure and liquidity on firm performance domains frequently analysed separately. Secondly, the study posits liquidity as an intervening variable that either amplifies or restricts the effects of capital structure choices on corporate results. This approach thus reconceptualises liquidity not just as an auxiliary element, but as a crucial aspect of financial planning and viability.
Thirdly, the study enriches the theoretical conversation by shedding light on the direction and intricacies of how capital structure, liquidity, and a company's performance interact, especially in growing markets. Fourthly, it offers valuable advice for corporate finance managers by emphasising the dangers of overlooking liquidity management. Lastly, the real-world evidence from South African companies provides insights tailored to the context, which could guide policymaking, financial strategy, and risk management in comparable economic environments.
The article unfolds in the following manner: First, in Section 2, we delve into the theoretical concepts surrounding capital structure, liquidity, and company performance. Next, Section 3 guides us through the methodological approach, discussing the study's design, where the data comes from, an explanation of the variables used, and the econometric model applied. Then, Section 4 highlights and explains the results we observed, leading us to the wrapping up in Section 5, where we draw conclusions and explore the implications for policy.

2. Literature Review

2.1 Theoretical literature

The relationship between a firm's capital structure and its performance has been extensively studied, starting with the foundational work of Modigliani and Miller in 1958. They demonstrated that, in a perfect market, one without taxes, bankruptcy costs, or information asymmetry, the mix of debt and equity does not impact firm value. However, when considering more realistic assumptions, various theories emerge that explain financing behaviour from different perspectives. One of these theories, known as the Trade-off Theory, was further developed by Miller in 1977. It argues that firms weigh the tax advantages of debt against the costs of potential financial distress. By balancing interest tax shields with the risks of insolvency and managerial constraints, firms are expected to determine an optimal debt-equity mix (Myers, 2015). This framework portrays capital structure as a balancing act between benefits and risks.
Another perspective emphasizes the role of information asymmetry between managers and investors. The Pecking Order Theory, proposed by Myers and Majluf in 1984, suggests that firms have a hierarchy of financing preferences: they prefer to use internal funds first, then debt, and will only issue equity as a last resort. This order reflects the managerial understanding of firm value compared to that of outside investors, where issuing equity may be viewed negatively by the market. Empirical evidence, such as that from Bui et al. (2023), indicates that financially weaker firms often rely more on debt due to limited internal resources. Similarly, Signaling Theory, introduced by Ross in 1977, posits that financing choices convey information to the market. Issuing debt can serve as a positive signal, demonstrating managerial confidence in future cash flows, as interest obligations must be met regardless of a firm's performance (Kerongo, 2022).
Another important area of research focuses on conflicts of interest within firms. The Agency Cost Theory, articulated by Jensen and Meckling in 1976 and later expanded by Myers in 1977, highlights tensions among managers, shareholders, and creditors, where agency costs arise from differing objectives. These inefficiencies can be mitigated through monitoring, bonding costs,

or aligning incentives (Mabandla, 2023). Building on this, Jensen (1986) introduced the Free Cash Flow Hypothesis, which argues that firms with excess liquidity, but limited investment opportunities, may face overinvestment or wasteful spending. Debt can act as a disciplinary mechanism, obligating managers to meet fixed financial commitments. Thus, free cash flow serves as both a measure of financial capacity and a potential source of governance challenges (Suciani & Setyawan, 2022). Together, these perspectives illustrate that capital structure decisions are influenced not by a single principle but rather by a combination of trade-offs, information dynamics, and agency considerations.

2.2 Empirical studies and hypothesis development

This section reviews empirical studies that examine the effects of capital structure and liquidity on firm performance.

2.2.1 Firm Performance and Capital Structure

Many studies have delved into how a company's financial structure affects its success, yielding different results. Abubakar (2020) found that leverage indicators, short-term debt ratio (STDR), and long-term debt ratio (LTDR) had no significant influence on the return on equity (ROE) in Nigerian oil and gas firms. On the other hand, Kalash (2021) noted that leverage negatively impacted business performance in Turkey, especially during currency crises. Ahmed, Nugraha, and Hágen (2023) observed both negative and positive effects, depending on which measures were used, while Ronoowah and Seetanah (2024) identified complex relationships and the mediating role of agency costs. Conversely, Moradi and Paulet (2019), along with Abdullah and Tursoy (2023), presented conflicting findings showing positive links between leverage and performance in specific situations. Overall, academic literature shows there's no clear agreement, suggesting that these relationships are nuanced and dependent on context.

H1: There is a significant relationship between capital structure and firm performance.

2.2.2 Liquidity and Firm Performance

The research on liquidity and company performance shows varied results across different studies. Sharma and Sarin (2024) found that liquidity significantly and positively influences Return on Assets (ROA), though it has a weaker effect on Return on Equity (ROE); meanwhile, leverage negatively impacts ROA. Nguyen and Dao (2022), using meta-analysis, concluded that certain liquidity measures may harm business performance, yet robust corporate governance boosts firm value. Abubakar (2020) discovered positive and meaningful connections between liquidity indicators and ROA for Nigerian companies. Furthermore, Zaitoun and Alqudah (2020), along with Alfawareh *et al.* (2021b) and Alhassan and Islam (2021), identified positive links between liquidity and company profitability. In summary, these studies suggest that managing liquidity effectively is vital for company success, though its exact influence depends on specific contexts and methods used.

H_2 There is a significant relationship between liquidity and firm performance.

3. Research Methodology

This study utilises a balanced panel dataset consisting of firms from the Top 40 companies listed on the Johannesburg Stock Exchange (JSE). After accounting for delistings, mergers, and incomplete financial. Also, financial institutions were excluded; thus, 31 firms were retained for analysis. The exclusion of nine firms ensured the consistency and reliability of the panel throughout the study period (2012–2023). Audited financial statement data were sourced from the IRESS database and used to calculate relevant financial ratios. As a result, the final sample comprises firms that were continuously listed and had complete and dependable data throughout the study period. This selected timeframe allows for a comprehensive examination of the effects of capital structure and liquidity on corporate performance within the South African context. Given the focus on JSE-listed firms, the sectoral classification aligns with the South African Standard Industrial Classification (SIC). The sectors typically represented among the Top 40 JSE companies in this study include: Materials/Resources (for example, mining, metals), Consumer Discretionary (for example, retail, automotive), Industrials (for example, manufacturing, logistics), and Energy (for example, oil and gas, energy utilities). The dependent variables, namely Return on Equity (ROE), Return on Assets (ROA), and Tobin's Q, are extensively utilised in scholarly research to evaluate corporate success (Nguyen, Le, & Nguyen, 2023; Ramadan & Hassan, 2022 & Ullah, 2020).

The explanatory variables include leverage and liquidity. Leverage metrics, such as the debt-equity ratio, long-term debt to total assets, and total debt to total assets, are incorporated due to their acknowledged linkage with corporate performance, risk, and cost of capital (Abubakar, 2020). Liquidity measures, such as the current ratio and cash flow ratio, are integrated as they indicate companies' capacities to meet short-term liabilities and adjust to market variations, which are crucial for sustaining performance (Ndugbu et al., 2024).

Table 1: Summary of variables and proxies

Name of variables	Measurements	Use in literature	A priori expectation
		Dependent variables	
Return on Equity (ROE)	Net profit ÷ Owner's equity	Marozva and Makina (2020) and Omokore, Njogo, Omankhanlen, Islaka, and Akinjare (2024)	
Return on assets (ROA)	EBIT÷ total assets	Antwi, (2021), and Nguyen and Nguyen, (2020).	
Tobin's Q (TQ)	(Total market value of company +Liquidity) ÷ (Total asset value + Liabilities)	*Nguyen, Phan, and Hang (2024)*	
		Independent variables	
$LEV_{B_{it}}$ Leverage:			
Debt- equity (DE)	Debt ÷ Equity	Kerongo, (2022) and Siaf - Alyousfi *et al.,* (2020).	+/-
Long-term debt (LTD)	LTL÷ TA	Mabandla and Marozva (2024).	
Total debt ratio (TDR)	TD ÷ TA	Mabandla and Marozva (2025).	
$LIQ_{B_{it}}$ Liquidity:			
Current ratio (CUR)	Current assets ÷ current liabilities	Kalash (2023); Nam, and Tuyen, (2024)	+/-
Cash ratio (CR)	(Cash + marketable securities) ÷ current liabilities	Surachman, and Ningsih, (2023).	+/-
		Control variables	
Firm size (FS)	LnTA	Alodat *et al., (*2021) *and* Pourmansouri *et al., (*2022).	+/-
Gross domestic product (GDP)	(GDPn – GDPn-1)	*Khan, Bashir, Attuwaijri, and Khalid (2023)*	+
Interest rate (INT)	(i – P) ÷ (1 + P)	Ullah, (2020)	-
Inflation (INFL)	$(P_t) \div (P_{t-n})$	Maria & Hussain, 2023 and Ahmad *et* al., (2022)	+/-
Covid-19	Dummy variable, 1 for the Covid period, 0 for the non-Covid period.	Mabandla and Marozva (2025)	-

Source: Authors own compilation

3.1 Model Specification

The Generalised Method of Moments (GMM) was used in this research. The generic GMM dynamic technique has the following form:

$$\boldsymbol{Y_{it} = \alpha Y_{i\,t-1} + \beta X_{it} + \beta MEF_t + \mu_i + \varepsilon_{it}} \quad (1)$$

Where:

$\boldsymbol{Y_{it}}$ indicates the financial performance metrics for firm *i* at time *t*; $\boldsymbol{X_{it}}$ is the explanatory variable vector for firm *i* at time *t*, signifying the variable unique to the firm. $\boldsymbol{\alpha}$ represent a coefficient for the lagged financial performance Metrix; *β* is the linear coefficient representing the relationship between each independent variable and the dependent variable; $\boldsymbol{MEF_t}$ is the macro-economic factors at time *t*; $\boldsymbol{\mu_i}$ indicates fixed effects in firms; $\boldsymbol{\varepsilon_{it}}$ it is a random error term; the subscript i indicates the cross-section, and t indicates the time-series scale. This study employed the two-step GMM system prediction model of Arellano and Bover (1995) and Blundell and Bond (1998), with dimension and lag parameters operating as instruments. The one-step GMM system method for forecasting is assumed to supplement the GMM estimate approach of Arellano and Bond (1991).

This study employs the system Generalised Method of Moments (GMM) estimator within a dynamic panel framework to examine the effect of liquidity on financial performance. The methodology addresses endogeneity concerns associated with reverse causality and unobserved firm-specific effects. Lagged values of the current ratio and firm size are used as internal instruments, supported by theoretical justification and temporal relevance. The validity of these instruments is confirmed through the Sargan and Hansen tests, whose non-significant results affirm instrument exogeneity and model robustness. By integrating sound econometric techniques with theoretically informed instrumentation, the study ensures consistent and efficient estimates, thereby reinforcing the credibility of the findings on the liquidity and performance relationship.

This paper solely employed South African data since it was our article's focus. This study investigated the important elements influencing financial performance in the South African firm by regressing financial performance (ROA, ROE, and TQ) against the components in the following questions 2 to 4.

$$\Delta \mathrm{ROA_{it}} = (1-\alpha)\Delta \mathrm{ROA_{it-1}} + \beta_1 \Delta \mathrm{LEV_{it}} + \beta_2 \Delta \mathrm{LIQ_{it}} + \beta_j \sum_{\frac{j}{t}=1}^{n} \Delta MEF_t + \Delta\, \varepsilon_{it} \qquad (2)$$

$$\Delta \mathrm{ROE_{it}} = (1-\alpha)\Delta \mathrm{ROE_{it-1}} + \beta_1 \Delta \mathrm{LEV_{it}} + \beta_2 \Delta \mathrm{LIQ_{it}} + \beta_j \sum_{\frac{j}{t}=1}^{n} \Delta MEF_t + \Delta\, \varepsilon_{it} \qquad (3)$$

$$\Delta \mathrm{TQ_{it}} = (1-\alpha)\Delta \mathrm{ROA_{it-1}} + \beta_1 \Delta \mathrm{LEV_{it}} + \beta_2 \Delta \mathrm{LIQ_{it}} + \beta_j \sum_{\frac{j}{t}=1}^{n} \Delta MEF_t + \Delta\, \varepsilon_{it} \qquad (4)$$

Where:

Δ is a differentiator,

$\boldsymbol{ROA_{B_{it}}}$ represents return on assets measured by EBIT over total assets,

$\boldsymbol{ROE_{B_{it}}}$ indicates return on equity measured by net profit divided by owner's profit,

$\boldsymbol{TQ_{B_{it}}}$ is the Tobin's Q measured by (total market value of company plus liquidity) divided by (Total asset value plus liabilities),

$\boldsymbol{LEV_{B_{it}}}$ Leverage :

D/E: Debt-equity measured by debt divided by equity,

LTD: Long-term debt to total assets measured by the long-term ratio divided by total assets,

TD: Total debt to total assets measured by the total debt ratio divided by total assets,

$\boldsymbol{LIQ_{B_{it}}}$ Liquidity:

CUR: current ratio measured by current assets divided by current liabilities

CR: Cash ratio measured by (cash plus marketable securities) divided by current liabilities,

FS: Firm size measured by the natural logarithm of total assets

ε_{it}: Error term

3.2 Descriptive statistics

The descriptive statistics reveal notable variation in firm-level financial performance and structural characteristics across the sample. The mean ROA was 7.25%, with a standard deviation of 12.04%, indicating moderate dispersion in firms' ability to generate profits from their asset base. The ROA ranged from a minimum of -37.88% to a maximum of 59.52%, reflecting the presence of both underperforming and highly efficient firms within the sample.

Table 2: Descriptive Statistics

Variables	Mean	Median	Maximum	Minimum	Std Dev	Skewness	Kurtosis	Jarque-Bera
ROA	7,25	4,17	59,52	- 37,88	12,04	0,95	5,15	11,95
ROE	15,61	15,05	657,18	- 483,65	46,92	3,85	138,32	26,63
TOBINQ	353,06	186,00	22 708,00	36,00	1 247,42	16,71	298,40	12,81
CUR	1,67	1,17	35,38	0,16	2,98	9,14	96,23	13,09
TDR	0,54	0,52	1,37	0,00	0,28	0,04	2,20	9,33
LTDR	0,36	0,37	0,96	0,00	0,25	0,24	1,97	18,71
RINT	9,29	9,38	11,75	7,00	1,30	- 0,13	2,51	4,36
RGDP'Billions	2 610,00	2 630,00	2 890,00	2 380,00	137,00	0,26	2,69	5,33
TA 'Billions	481,00	143,00	3,05	1,53	611,00	1,57	4,99	200,60
DE	4,20	1,16	288,97	0,00	16,10	16,10	283,51	11,56
INFL	5,33	5,45	7,00	3,20	1,12	- 0,18	2,12	13,20
CR	1,31	0,87	35,38	0,16	2,97	9,50	101,09	14,48

The ROE displayed substantially greater volatility, ranging from -483.65% to 657.18%. This pronounced variability underscores the sensitivity of ROE to changes in net income and equity capital, particularly in firms with thin equity buffers. The consistently higher magnitude of ROE values relative to ROA suggests that firms, on average, generate stronger returns for equity holders than for total assets, potentially driven by leverage. This aligns with theoretical expectations that equity returns are amplified in the presence of debt financing. Tobin's Q, which captures the ratio of market valuation to the replacement cost of assets, ranged from 36.00 to an extreme of 22,708.00, highlighting significant disparities in market-based firm valuation. Such outliers may be attributed to investor expectations of future growth, firm-specific intangible assets not reflected on balance sheets, or speculative market behaviour. In terms of liquidity, the CUR was 1.67, indicating that, on average, firms held current assets sufficient to cover 167% of their current liabilities.

The relatively low standard deviation suggests a narrower distribution of liquidity across firms compared to profitability and valuation measures. The CUR values ranged from a minimum of 0.16, indicative of severe short-term liquidity constraints, to a maximum of 35.38, which may reflect conservative working capital management or potential inefficiencies in asset utilisation. Financial leverage was assessed using three key indicators: the TDR, LTDR, and the DE. The mean TDR was 0.54, with a standard deviation of 0.28, indicating that, on average, 54% of firms' capital structures comprised debt financing. The TDR ranged from 0.00 to 1.37, reflecting substantial variation in firms' overall leverage. The LTDR had a mean value of 0.36 and a standard deviation of 0.25, with values ranging from 0.00 to 0.96, suggesting that long-term debt constituted a notable component of firms' liabilities.

The DE ratio recorded an average of 4.20 with a markedly high standard deviation of 16.10, ranging from 0.00 to 288.97, highlighting significant heterogeneity in the extent to which firms rely on equity relative to debt. These findings suggest that the firms in the sample are generally highly leveraged, albeit with considerable variability across the dataset. The mean for GDP was approximately ZAR2610.00 billion, with a standard deviation of ZAR137.00 billion. GDP values ranged from a minimum of ZAR2,380.00 billion to a maximum of ZAR2,889.00 billion. These figures suggest significant variation in GDP, which may have a substantial impact on firm performance. Firm size was proxied by total assets (TA). The mean TA was approximately ZAR481.00 billion, with a standard deviation of ZAR611.00 billion, ranging from a minimum of ZAR1.526.00 billion to a maximum of ZAR3.050.00 billion. The descriptive statistics indicate a considerable range in firm size, highlighting the potential association between company size and asset base.

The mean value of GDP was ZAR261.00 billion, with a standard deviation of R137.00 billion. GDP ranged from a minimum of ZAR2,380.00 billion to a maximum of ZAR2,889.00 billion. These statistics suggest substantial variation in GDP, which may significantly influence firm performance. Firm size was proxied by total assets (TA), measured using the natural logarithm to account for scale differences. The mean TA was ZAR481.00, with a standard deviation of ZAR611.00 billion. TA values ranged from a minimum of ZAR1,526.00 billion to a maximum of ZAR3,050.00 billion. These descriptive results indicate a wide distribution in firm size, underscoring the potential association between company size and asset base.

The average INFL was 5.33%, with a standard deviation of 1.12. Inflation ranged from a minimum of 3.20% to a maximum of 7.00%. Inflation serves as an important macroeconomic indicator reflecting price stability within a country. The mean for QR was 1.31 with a standard deviation of 2.97. CR values ranged widely from 0.16 to 35.38, indicating considerable variation in firms' liquidity positions. Analysis of skewness and kurtosis revealed the presence of asymmetry and leptokurtosis in the distributions of all variables under study. Furthermore, the Jarque-Bera test for normality confirmed that these variables are not normally distributed. The correlations between the variables are discussed in the subsequent section.

3.3 Correlation analysis

Correlation analysis, as presented in Table 3, illustrates the relationships between the independent and dependent variables utilised to evaluate firm performance.

Table 3: Correlation Analysis

Variables	ROA	ROE	TOBINQ	CR	TDR	LTDR	RINT	RGDP	TA	DE	INFL	QR
ROA	1											
ROE	0.2370***	1										
TOBINQ	0.1063*	-0.5234***	1									
CR	-0,0068	0.3012***	-0,0298	1								
TDR	-0,0568	-0.1025*	0.1204**	-0.3750***	1							
LTDR	-0,0151	-0.1868***	0.0984*	0.1314**	-0.2198***	1						
RINT	-0,0799	-0,0639	-0,0374	-0,0789	-0,0041	-0,0096	1					
RGDP	-0,0233	0,0126	-0.0923*	0,0567	0,0294	0,0271	0.6164***	1				
TA	-0.3697***	-0,0484	-0.1063**	-0.1889***	0.3488***	-0.3250***	0,0148	0.1314**	1			
DE	-0.1018*	-0.5786***	0.9378***	-0,0858	0.2582***	0,0106	-0,0433	-0,0748	0.1259**	1		
INFL	0,03	-0,0643	0,0258	-0,0084	0,0015	-0,0266	0.5148***	0.4039***	0,0141	0,0228	1	
CR	-0,0537	0.3040***	-0,0263	0.9920***	-0.3355***	0.1012*	-0,0791	0,0542	-0.1406***	-0,0621	-0,009	1

Return on Assets (ROA) exhibited a positive and statistically significant correlation with ROE. Additionally, ROA was positively and significantly associated with Tobin's Q, suggesting that the market anticipates improved firm performance due to enhanced supervisory mechanisms arising from the firm's social constraints.This expectation may explain why Tobin's Q responds more rapidly than ROA. Conversely, TA and DE were negatively and significantly correlated with ROA, indicating that larger firm size is associated with lower performance. ROE demonstrated a significant negative correlation with Tobin's Q, reflecting a strong inverse relationship and suggesting efficient market pricing of firm performance. In contrast, ROE was positively correlated, albeit weakly, with the CR. Furthermore, ROE was negatively associated with both the TDR and the LTDR. A similar negative and significant relationship were observed between ROE and DE. Notably, the CR showed a positive and statistically significant correlation with ROE.

Tobin's Q demonstrated a positive and statistically significant association with both the TDR and the LTDR, suggesting that elevated leverage levels are correlated with higher market valuation as captured by Tobin's Q. In contrast, Tobin's Q exhibited a significant negative relationship with Gross Domestic Product (GDP) and TA, implying that increases in macroeconomic output and firm size are associated with lower market valuation ratios. The subsequent section elaborates on the methodology and findings derived from the application of the Two-Step System Generalised Method of Moments (GMM) estimator.

4. Results

The results presented in Table 4 highlight a significant positive relationship between a firm's financial performance, measured by ROA, and its lagged value of ROA. This finding suggests that firms with higher ROA in earlier periods are likely to maintain better financial results in subsequent periods, indicating a strong capacity for performance sustainability. The research indicates a negative relationship, yet statistically insignificant, between DR and ROA. The lack of statistical significance implies that debt levels do not significantly affect immediate financial results within the context of this specific data set. The analysis reveals a negative and significant relationship between the LTDR and ROA. Specifically, the research suggests that a greater reliance on long-term debt financing is generally linked to a decrease in overall firm earnings, as indicated by the return on assets metric. This relationship supports the trade-off theory, which argues that the potential tax benefits of leveraging debt are countered by the risks of financial instability and limitations on management discretion. When long-term borrowing becomes excessive, it can negatively impact overall operational success (DeAngelo, Gonçalves & Stulz, 2021). The results are consistent with the findings of Kalash (2023), who finds a negative and significant association between leverage and firm financial performance. This study's results demonstrate a positive and statistically significant correlation between liquidity, as assessed through both the CUR and CR, and ROA. This suggests that improved liquidity positions empower firms to fulfill their financial commitments and capitalise on lucrative prospects (Bourke, 1989). Furthermore, a positive and significant relationship was observed between firm size and ROA, providing evidence for the economies of scale theory. This indicates that larger institutions may achieve greater operational efficiency and a more diversified risk profile (Athanasoglou, Brissimis & Delis, 2008). In contrast, interest rate levels exhibited a negative and significant association with ROA, implying that increases in interest rates may elevate the cost of funding or suppress the demand for loans, thereby diminishing profitability (Demirgüç-Kunt & Huizinga, 1999).

Table 4: The effect of capital structure on ROA

	System GMM	System GMM	System GMM	System GMM
	Model 1	Model 2	Model 3	Model 4
Variables	**ROA**	**ROA**	**ROA**	**ROA**
L.ROA	0.367^{***}	0.484^{***}	0.646^{***}	0.483^{***}
	(0.0610)	(0.0300)	(0.0263)	(0.0391)
DR	-1.027		-0.690	
	(1.943)		(2.264)	
LTDR		-9.729^{***}		-31.08^{***}
		(1.996)		(1.115)
CUR	0.907^{***}	0.402^{***}		
	(0.0259)	(0.0294)		
CR			0.657^{***}	0.599^{***}
			(0.0189)	(0.0271)
LTA	16.92^{***}	14.13^{***}	22.14^{***}	20.00^{***}
	(1.053)	(0.619)	(0.625)	(0.482)
RINT	-1.742^{***}	-1.696^{***}	-1.462^{***}	-1.360^{***}
	(0.234)	(0.102)	(0.241)	(0.269)
LRGDP	-15.17	-20.81	-49.67^{***}	-55.93^{***}
	(7.827)	(13.37)	(6.005)	(11.89)
INFL	0.529^{***}	0.476^{***}	0.376^{***}	0.481^{***}
	(0.0680)	(0.0425)	(0.0698)	(0.134)
COVID_19	-2.264^{**}	-1.863^{***}	-2.392^{***}	-1.310
	(0.752)	(0.269)	(0.498)	(0.675)
N	341	310	310	310
Groups	31	31	31	31
Instruments	40	44	35	46
R(1)	-3.00	-3.09	-3.25	-3.01
R(2)	-0.37	0.38	0.62	0.51
Sagan Test	176.25	114.06	98.70	112.85
Hansen Test	27.69	26.04	27.61	29.19

Robust Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. In model 1, capital structure is measured as the total debt ratio (DR). Model 2, capital structure is measured as the long-term debt ratio (LTDR), and liquidity is measured as the current ratio (CR). Model 3, capital structure is measured as the total debt ratio (DR), and liquidity is measured as the quick ratio (CR). Model 4, capital structure is measured as the long-term total debt ratio (LTDR), and liquidity is measured as the quick ratio (CR).

Table 5 presents findings that reveal a negative and statistically significant relationship between ROE and its lagged value. This indicates that a high ROE in a prior period tends to predict a lower ROE in the current period, suggesting a trend towards mean reversion in profitability. In addition, the results of the study show a negative and significant correlation between LTDR and ROE. This implies that a greater reliance on long-term debt is associated with reduced returns for shareholders. Contributing factors may include higher interest costs, increase financial risk, and limited operational flexibility.

Table 5: The effect of capital structure on ROE

	System GMM	System GMM	System GMM	System GMM
	Model 1	Model 2	Model 3	Model 4
Variables	ROE	ROE	ROE	ROE
L.ROE	-0.471	-0.406^{***}	-0.506^{***}	-0.405^{***}
	(0.409)	(0.0283)	(0.0164)	(0.0287)
DR	34.88		-49.47^{**}	
	(23.11)		(18.23)	
LTDR		101.0^{**}		114.3^{**}
		(38.15)		(36.34)
CR	13.56^{**}	11.57^{***}		
	(4.702)	(0.358)		
CR			14.47^{***}	11.63^{***}
			(0.701)	(0.373)
LTA	68.49	120.8^{***}	175.0^{***}	120.8^{***}
	(40.29)	(12.26)	(22.11)	(13.19)
RINT	3.542	1.519	2.735	1.724
	(7.564)	(2.656)	(1.623)	(2.680)
LRGDP	-353.0	-330.9^{***}	-536.5^{***}	-334.0^{***}
	(229.5)	(82.16)	(54.50)	(81.52)
INFL	-0.659	0.441	0.185	0.305
	(3.681)	(1.389)	(0.910)	(1.416)
COVID_19	2.836	-4.352	-5.641	-4.102
	(19.25)	(6.056)	(4.336)	(6.158)
N	341	310	310	310
Groups	31	31	31	31
Instruments	28	24	24	24
R(1)	-0.35	-0.81	-0.76	-0.82
R(2)	-0.85	-1.12	-1.06	-1.13
Sagan Test	31.89	22.83	25.27	22.49
Hansen Test	19.37	18.44	27.08	19.07

Robust Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. In model 1, capital structure is measured as the total debt ratio (DR), and liquidity is measured as the current ratio (CR). Model 2, capital structure is measured as the long-term debt ratio (LTDR), and liquidity is measured as the current ratio (CR). Model 3, capital structure is measured as the total debt ratio (DR), and liquidity is measured as the quick ratio (CR). Model 4, capital structure is measured as the long-term total debt ratio (LTDR), and liquidity is measured as the quick ratio (CR).

Theoretically, these findings align with the Pecking Order Theory (Myers & Majluf, 1984), which posits that companies prefer to use internal funding sources instead of debt financing. This preference helps them avoid the costs associated with borrowing and alleviates potential conflicts of interest between debtholders and equity holders. Excessive long-term leverage can hinder profitability, as the obligation to service debt may outweigh the benefits gained from investments financed through borrowing. These results support the findings of Omokore *et al* (2024), who also reported a negative and statistically significant relationship between LTDR and ROE.

The research reveals a significant positive correlation between liquidity metrics and profitability. Specifically, both the CUR, which measures general short-term liquidity, and the CR, a stricter indicator, show a statistically significant positive relationship with ROE. This finding suggests that companies with a stronger short-term financial position, regardless of whether assessed broadly or narrowly, tend to deliver better returns for their shareholders. This outcome may be due to a lower risk of financial difficulties, greater operational reliability, and an improved ability to seize favourable investment opportunities (Lalithchandra & Rajendhiran, 2021). The findings are inconsistent with Nguyen *et al*. (2024), who report a negative relationship between liquidity and firm performance.

Table 6: The effect of capital structure on TobinQ

	System GMM	System GMM	System GMM	System GMM
	Model 1	Model 2	Model 3	Model 4
Variables	**TobinQ**	**TobinQ**	**TobinQ**	**TobinQ**
L.TobinQ	0.00624	0.0614^{***}	0.117^{***}	0.0628^{***}
	(0.0844)	(0.0166)	(0.0179)	(0.0174)
DR	234.8		255.8	
	(266.7)		(262.1)	
LTDR		327.4		335.4
		(351.0)		(343.4)
CUR	42.21	43.72^{***}		
	(41.68)	(11.50)		
CR			41.65^{***}	44.51^{***}
			(7.572)	(11.49)
LTA	655.6	1388.3^{***}	1253.5^{***}	1415.3^{***}
	(512.8)	(359.5)	(213.3)	(353.1)
RINT	-239.1	-202.7^{***}	-162.2^{***}	-203.7^{***}
	(209.2)	(34.34)	(38.44)	(34.16)
LRGDP	-806.7	-1823.1^{*}	-2587.6^{***}	-1763.8^{*}
	(1485.0)	(756.9)	(509.6)	(773.0)
INFL	124.2	106.5^{***}	90.14^{***}	106.2^{***}
	(104.3)	(18.72)	(22.32)	(18.81)
COVID_19	-684.0	-644.4^{***}	-538.6^{***}	-646.4^{***}
	(539.9)	(112.3)	(121.4)	(112.5)
N	341	310	310	310
Groups	31	31	31	31
Instruments	28	24	24	24
R(1)	-0.14	-0.84	-1.18	-0.84
R(2)	-0.86	-0.17	-0.43	-0.17
Sagan Test	13.10	10.82	11.28	10.81
Hansen Test	26.23	20.84	17.18	20.70

Robust Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. In model 1, capital structure is measured as the total debt ratio (DR), and liquidity is measured as the current ratio (CR). Model 2, capital structure is measured as the long-term debt ratio (LTDR), and liquidity is measured as the current ratio (CUR). Model 3, capital structure is measured as the total debt ratio (DR), and liquidity is measured as the quick ratio (CR). Model 4, capital structure is measured as the long-term total debt ratio (LTDR), and liquidity is measured as the quick ratio (CR).

The findings shown in Table 6 indicate a statistically significant positive correlation between Tobin's Q and its lagged value. In addition, the analysis reveals a positive, but statistically insignificant, relationship between the LTDR and Tobin's Q.

These results suggest that companies with higher levels of long-term debt may experience slightly increased market valuations; however, the evidence does not strongly support this relationship.

On the other hand, the results of the study revealed a positive and significant correlation between the current ratio and Tobin's Q. A comparable pattern was identified between the quick ratio and Tobin's Q, with a significant positive association evident. These observations imply that firms demonstrating greater liquidity typically experience elevated market valuations, as indicated by Tobin's Q (Myers, 1984; Almeida, Campello & Weisbach, 2004). This conclusion lends support to the Liquidity Preference Theory, which suggests that investors generally favour firms possessing sufficient liquidity. The rationale is that these entities are better positioned to satisfy immediate financial commitments and exploit potential investment avenues. This, in turn, mitigates the potential for financial instability and ultimately boosts the company's overall worth (Keynes, 1937; Opler, Pinkowitz, Stulz & Williamson,1999). The study's outcomes are, however, at odds with the recent research of Nguyen, Phan, and Hang (2024), who demonstrate a negative and significant connection between liquidity and firm performance.

5. Conclusion and policy implications

This study examined the impact of capital structure and liquidity on the financial performance of South African firms over the period 2012 to 2023. To address potential issues of correlation and bias commonly found in panel datasets, the study employed the Generalised Method of Moments (GMM) for estimation. The results showed a negative and significant relationship between the LTDR and firm profitability, as measured by ROA and ROE. This suggests that a higher reliance on long-term borrowing generally harms a company’s earnings and shareholder returns. Conversely, liquidity represented by the CUR and CR exhibited a positive and significant impact on financial performance across various measures, including ROA, ROE, and Tobin’s Q. These findings highlight that firms with better liquidity are more capable of meeting immediate financial obligations, seizing potential investment opportunities, and ultimately enhancing both their reported profitability and market value.

The results highlight the crucial importance of sound capital structure and liquidity management for firms operating within the South African economic landscape. The government initiatives could focus on enhancing access to immediate liquidity options, enabling firms to maintain operational continuity and pursue growth opportunities. Regulatory bodies and state organisations could also consider establishing programs to help firms improve their liquidity management skills, which could contribute to overall economic stability and a competitive edge. Moreover, the South African Reserve Bank may adopt a more expansionary monetary policy stance, which would lower interest rates and increase the money supply, thereby enhancing overall market liquidity. Such measures are expected to incentivize Johannesburg Stock Exchange (JSE)-listed firms to increase borrowing, thereby strengthening their liquidity positions.

Future research could benefit from examining the diverse aspects of capital structure and liquidity impacts across different sectors and organisational dimensions within the South African economy. By integrating broader economic indicators such as benchmark interest rates, inflation, and overall economic growth, researchers could gain a deeper understanding of how external economic conditions influence corporate financial strategies. Furthermore, studying the role of organisational leadership and executive decision-making in capital and liquidity management could provide valuable insights into the factors that contribute to organisational success. Lastly, evaluating the financial strategies implemented in the aftermath of the global health crisis may yield important observations on how organisations adapt their funding and liquid asset management practices in response to financial instability.

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Operational Risk: New Standard Approach and Impacts on Banks

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Article submitted to double-blind peer review, received on 29th May 2025 and accepted on 3rd October 2025

Abstract

In the increasingly complex and dynamic financial landscape, managing operational risks poses a crucial challenge for financial institutions. Evolving regulations, the rise of cyber threats, and growing stakeholder expectations make a rigorous and systematic approach to quantifying and managing these risks essential. In this context, Basel 4 focuses on a more robust framework for operational risk management, introducing a standardized approach for calculating operational risk capital. This framework aims to encourage greater 'risk sensitivity' in risk assessment and requires an increase in the capital that banks must hold to address losses arising from operational events, such as internal errors, fraud, or natural disasters. Basel IV will have significant implications for financial institutions. The greater capital requirements imposed by the introduction of the new regulations will push banks to revise their processes and strategies in order to contain the higher capital absorptions.

Key Words: financial institutions , Improving operational risk management framework, New Regulation on capital requirements for operational risks, financial stability, new standardised method for operational risk, new impact on capital

1. Introduction

Operational risk is defined as the risk of incurring losses resulting from the inadequacy or malfunction of procedures, human resources, and internal systems, or from external events. This category includes, among other things, losses resulting from fraud, human error, operational disruptions, system unavailability, contractual breaches, and natural disasters. Operational risk includes legal risk but does not include strategic and reputational risk (Bank of Italy, 2006).

Operational risk management for banks is a process that aims to identify, assess, mitigate, and monitor risks that may adversely impact. In particular, it helps prevent losses, protects reputation, ensures financial stability, and meets regulatory requirements. Effective operational risk management allows for process optimization, efficient resource allocation, and reduced waste, thus improving overall efficiency.

Today, effectively managing operational risk in banks is equivalent to enabling the organization to pursue its business objectives. Given the regulatory frameworks and competitive landscape that characterize the financial world, a proper approach to operational risk management is no longer simply about mitigating the damage resulting from the materialization of hypothesized threats: it becomes a true tool for verifying and correcting activities, processes, and strategies to minimize the impact of events that could alter their outcomes (Intonti M. 2012).

The experience of the financial crisis has highlighted that not all types of exposures are suitable for modelling in a sufficiently reliable manner. The crisis has highlighted two main shortcomings. Firstly, the capital requirements for operational risk turned out to be insufficient to cover the operational losses incurred by banks. Secondly, the nature of these losses has highlighted the reduced predictive effectiveness of internal models. Basel IV increases the financial resilience of banks and global operational uniformity with effects on asset management and technological and infrastructural development. EU Regulation 2024/1623 (CRR3) has amended EU Regulation 2013/575 (CRR) with regard to credit risk requirements, credit valuation adjustment (CVA) risk, operational risk, and market risk. The new rules were designed to improve the prudential regulation, supervision and risk management of banks in response to the 2007-2008 global financial crisis, with the aim of increasing the resilience of EU banks to economic shocks and strengthening their supervisory and risk management frameworks. CRR3 is the European translation of the latest revision of the Basel standards for banking supervision published in 2017 and commonly referred to as Basel IV (although the Basel Committee continues to call it Basel III).

In December 2017, the Basel Committee on Banking Supervision (BCBS 2017) has released new rules for measuring the own funds requirement against operational risks. With reference to the assessing the calculation of capital requirements for operational risk, the final text of 19 June provides for a single “non-model based approach” (SA, Standardized Approach) as defined by the Committee reform. With the introduction of the new model in force from January 2025, the Basel Committee has decided to proceed with a clear simplification by replacing the four approaches currently applicable with a single standardized approach.

In particular, for operational risk, the lack of risk sensitivity of standardised methods and comparability of advanced measurement methods resulting from a wide range of internal modelling practices by individual banks was noted. CRR3 in force from 1 January 2025 provides for the elimination and replacement of all existing methods for calculating own funds requirements

[1] The opinions expressed are personal and do not in any way commit the institution to which they belong.

for operational risk (Basic Indicator Approach (BIA), Traditional Standardised/Alternative Standardised Approach (TSA/ASA) and Advanced Measurement Approaches (AMA)) with a single standardised method defined as the Business Indicator Component (BIC).

Basic Indicator Approach (BIA) uses the intermediation margin as a proxy for risk exposure, applying a coefficient to determine the capital absorption;

Traditional Standardized Approach (TSA) for the management of operational risk, provides for the division of a bank's activities into eight business lines, for which specific criteria and documented policies are developed. This approach allows the risk indicator to be distributed among the different business lines, taking into account their specificities and the activities carried out.

Advanced Measurement Approach (AMA) is a approach that banks can use, subject to regulatory approval, to estimate unexpected losses arising from operational risk events. This approach combines internal and external data, scenario analysis and assessment of internal controls to assess the potential impact of such events.

In this work, we will analyze the new standardized method for operational risks and the implications in terms of capital and management determined by the new regulations. Based on the Basel standard, the capital requirement is a function of the operational size of individual banks and, for medium and large banks only, of the historical past of operational risk losses. In this context, the logic introduced by Basile 4 will no longer be predictive, but retrospective: those who have historicized and correctly classified past events will be rewarded; those who have underestimated the processes of loss data collection will find themselves paying a capital premium.

2. Literature Review on the topic of operational risk

Recent literature on operational risks emphasizes the complexity of these risks and the importance of adopting proactive and systematic approaches to their management to ensure the resilience and sustainability of companies in the long term.

In an evolving context, it is clear that operational risk can also arise from the management of environmental issues (for example, it could arise from greenwashing activities carried out by client companies and which reflect on the intermediaries who have provided them with financing, or from incorrect disclosure of ESG reporting) and from the evolution of banks' business models, resulting from the so-called digital transformation.

Given the systemic importance of operational risk, over the years it has increasingly been the subject of study and analysis by regulatory and supervisory authorities as well as by legal scholars who have investigated not only the methodologies that can be implemented to predict and reduce potential losses or imbalances resulting from the emergence of operational risk, but also its correlations with other risk categories.

A first interrelationship is found between legal risk (a component of operational risk) and compliance risk. In its 2011 "Guidelines on Internal Governance"—updated first in 2017 and again in 2021— the EBA developed a nearly identical definition, indicating the similarity and near-perfect fit between these two risk categories. Specifically, compliance risk is defined as "the current or prospective risk to profits and capital arising from violations of, or non-compliance with, laws, rules, regulations, agreements, prescribed practices, or ethical standards that could result in fines, damages, and/or cancellation of contracts and diminish the reputation of an institution" (EBA, 2011).

One of the possible interdependencies between operational risk and credit risk, however, has been studied by McNulty & Akhigbe (2014, 2015, and 2017), whose empirical contribution highlighted how credit risk can easily transform into operational risk. Aggressive lending policies adopted by intermediaries can, in fact, lead to higher short-term profits, but they also likely lead to disputes with customers.

A close link is also found between operational risk and reputational risk, as events of the first type negatively impact a bank's reputation, as well as with cyber risk, whose exposure is accompanied by an operational loss that often entails a legal and reputational impact (Porretta and Santoboni, 2022).

There is a negative correlation between operational risk, the efficiency of internal control systems, corporate culture, and corporate governance. The possible interdependence between these variables is based on the following considerations:

- A bank with an efficient internal control system should experience a reduction in potential litigation arising from its activities;
- bank with a greater number of independent directors should experience fewer litigation events and, consequently, a lower level of operational risk;
- Corporate culture influences the framework within which managers operate and, therefore, the internal control system.

Focusing on the governance structure, it appears crucial not only to have independent directors on the bank's board of directors, but also to focus on identifying specific areas of responsibility to be assigned to each director (Oliveira et al., 2023). This would allow, in the event of a dispute, thanks also to the implementation of so-called "individual accountability regimes," to identify the director responsible for the disputed conduct, rather than placing all responsibility on the intermediary, thus preventing a loss of trust on the part of banking customers.

It should be emphasized that effective internal control systems allow the bank not only to reduce the probability of incurring risks that could compromise its economic, financial and capital stability, but also to strengthen the trust of customers, investors and shareholders for whom the adequacy of internal control systems will represent a guarantee for the protection of their assets (Murè, 2021).
According to the Basel Committee, one of the primary objectives that banks should achieve to absorb the impacts arising from the various possible manifestations of operational risk is "operational resilience," defined as "the ability of a bank to deliver critical operations through disruption" (BCBS, 2021), the tolerance threshold for which can be identified on a case-by-case basis. In this sense, banks should ensure the fulfillment of critical functions, defined by the Financial Stability Board (FSB, 2014) as the set of activities, processes, services, and related support activities whose disruption would be significant for the continuation of the bank's operations or its role in the financial system.
To contain the costs arising from the manifestation of operational risk (for legal fees, fines and sanctions imposed by supervisory authorities and, more generally, any operational loss), Eceiza et al. (2020) propose a shift from a qualitative, manual control of this type of risk to real-time monitoring based on the analysis of data present within each bank. This shift should be accompanied by the use of an interdisciplinary team of professionals, aimed at quickly and promptly addressing the issues, threats and risks emerging from the bank's normal operations.
In this regard, the European System Risk Board (ESRB, 2015) conducted an analysis of so-called misconduct risk in the banking sector – defined, as a subcategory of operational risk (within legal risk), as "the current or prospective risk of losses for an institution".
To contain the costs arising from the manifestation of operational risk (for legal fees, fines and sanctions imposed by supervisory authorities and, more generally, any operational loss), Eceiza et al. (2020) propose a shift from a qualitative, manual control of this type of risk to real-time monitoring based on the analysis of data present within each bank. This shift should be accompanied by the use of an interdisciplinary team of professionals, aimed at quickly and promptly addressing the issues, threats and risks emerging from the bank's normal operations.

3. New rules for measuring the own funds requirement against operational risk

The new single standard model for all banks, for the calculation of capital requirements for operational risks is based on three factors:

1) the Business Indicator (BI) which is a balance sheet-based proxy for operational risk;
2) the Business Indicator Component (BIC), which is calculated by multiplying the BI by a set of marginal coefficients determined by the regulation
3) the Internal Loss Multiplier (ILM), which is a scaling factor based on the average historical losses of a bank and the BIC

The first factor "BI" is a measure that indicates how much a bank is potentially exposed to operational risk. This indicator is given by the sum of three aggregates reported below:

- Income from interests, dividends and financial leases (ILDC);
- Income from services (SC);
- Financial income (FC).

BI = ILDC + SC + FC (art. 312 CRR3) of which:

ILDC (interest leases dividend component) = min (IC, 0.0225*AC) +DC

Where:

IC (Interest and leases Component): interest income from all financial assets and other interest income, including financial income from finance leases, income from operating leases and profits from leased assets, net of interest expenses from all financial liabilities and other interest expenses, including interest expenses from finance and operating leases, depreciation and amortization, and losses on operating leased assets, calculated as the annual average of the absolute values of the differences in the last three financial years.
AC (Asset Component): total sum of gross loans, advances, interest-bearing financial instruments, including government bonds, and leased assets, calculated as the annual average of the last three financial years based on the amounts recorded at the end of each financial year.

DC (Dividend Component): dividend income from investments in shares and funds not consolidated in the institution's balance sheet, including dividends from subsidiaries, associates and unconsolidated joint ventures, calculated as the annual average of the last three financial years.
About SC (Service Component), this is equal to:

SC =Max (OI, OE) + Max (FI, FE)

Where:

OI (Other operating Income): annual average of the last three financial years of the institution's revenues from ordinary banking operations not included in other items of the business indicator but of a similar nature.
OE (Other operating Expenses): annual average of the last three financial years of the institute's expenses and losses arising from ordinary banking operations not included in other items of the business indicator but of a similar nature, as well as from operational risk events.
FI (Fee and commission Income): annual average of the last three financial years of the institution's revenues from the provision of consultancy and services, including revenues received by the institution as an external provider of financial services.
FE (Fee and commission Expenses): annual average of the last three financial years of the expenses incurred by the institution for receiving advice and services, including commissions paid for the outsourcing of financial services, but excluding commissions paid for the outsourcing of non-financial services.

Finally, regarding the Financial component, we can say that:**FC** (Financial Component) = **TC + BC**

TC (Trading book Component): annual average of the absolute values of the last three financial years of net profit or loss, on the institution's trading portfolio, including trading assets and liabilities, hedging accounting and exchange differences.
BC (Banking book Component): annual average of the absolute values of the last three financial years of the net profit or loss, as applicable, on the institution's non-trading portfolio, including financial assets and liabilities measured at fair value through profit or loss, hedging derivatives, exchange rate differences and profits and losses realized on financial assets and liabilities not measured at fair value through profit or loss.

Exclusively for the financial component (FC), institutions are required to report the approach used (Accounting Approach (AA) or Prudential Boundary Approach (PBA).
The use of the AA provides for an alignment between the trading portfolio on which to calculate the TC component and the accounting trading portfolio. The use of the PBA would make it possible to avoid unjustified increases in the TC and BC components resulting from the accounting of specific transactions (i.e. implicit derivatives within hybrid financial instruments) that are closely correlated to each other but of opposite sign in the two portfolios.
The second factor, for the calculation of capital requirements for operational risks, “BIC” is obtained by multiplying BI by the marginal coefficients (table 1): 12% for BI values lower than €1 billion euros, 15% for BI intermediate values (greater than one billion euros and not greater than 30 billion euros), 18% for BI values higher than €30 billion
The third factor, ILM, Internal Loss Multiplier, measures how much the bank has proven to be concretely capable of controlling operational risks in the past and depends on the historical average of the related losses. In this way, virtuous banks, with few losses, are rewarded with a lower capital requirement. It is calculated with the following algorithm that compares past losses («loss component», «LC») with the BIC.

Table 1: Marginal coefficients

Coefficient α		
Bucket	BI range (€/bn)	α coefficient
1	<=1	12%
2	1<BI>=30	15%
3	>30	18%

Source: CRR3 data processing

The new standard method is based on the assumption that the relationship between the Business Indicator BI and the exposure to operational losses is relatively similar for banks that have similar BI values.
The losses recorded by the individual bank affect the calculation of the capital requirement through the Internal Loss Multiplier (ILM) defined by the following formula:

$$ILM=Ln\left(\exp(1)\text{-}1+\left(\frac{LC}{BIC}\right)^{0.8}\right)$$

where the loss component (LC) is equal to 15 times the average annual losses recorded for operational risks over the last 10 years. ILM varies as a function of LC:

1. when LC is equal to the BI component (i.e. the historical average measure, which takes into account the actual operating losses historically recorded by the bank, is exactly equal to the average level of operating losses of the reference bucket), then ILM = 1; consequently, the own funds requirement is exactly equal to the BI component;
2. when LC is lower than the BI component (i.e. the historical average measure, which takes into account the actual operating losses historically recorded by the bank, is lower than the average level of operating losses of the reference bucket), then ILM < 1; consequently, the own funds requirement is lower than the BI component;
3. when LC is higher than the BI component (i.e. the historical average measure, which takes into account the actual operating losses historically recorded by the bank, is higher than the average level of operating losses of the reference bucket), then ILM > 1; consequently, the own funds requirement is higher than the BI component;
4. when LC is equal to 0 (i.e. the bank has not historically recorded any operating loss in the last 10 years), then ILM has a lower limit at the level Log [exp (1)-1] ≈ 0.541; at the same time, the own funds requirement is approximately equal to 11% of the BI.

For loss data, a bank must have documented procedures and processes for the identification, collection and processing of internal operational loss data. These processes and procedures must be validated and checked by internal and external functions of the bank. -Internal operational loss data must capture all banking activities. The minimum threshold for including a loss event in the collection of average annual losses is EUR 20,000. At the discretion of the national supervisory authority, this limit may be increased to EUR 100,000 for banks that fall into buckets 2 and 3 (table 1). In addition to information on gross loss data, the bank must collect additional information, such as the date of occurrence, accounting, etc. It is also required to collect data on recoveries and descriptive information useful for understanding the determinants and causes of the manifestation of the loss event. The level of detail of the descriptive information should be commensurate with the size of the loss.

4. The new impacts in light of the new legislation

The new calculation method will be the same for all credit institutions, regardless of their size and business model. The original proposal of the Basel Committee envisaged the combination of the BIC component and the ILM component for the calculation of the capital requirement for operational risks. The new CRR proposes to place an ILM equal to 1, thus sterilizing the effect on the regulatory capital requirement of historical data on the operational losses of each bank. Furthermore, the new CRR defines specific regulatory requirements for the implementation of the operational risk management framework that were previously not binding for banks that did not use advanced methods. Credit institutions with a BIC higher than €750 million are required to calculate and report the historical loss levels of the last 10 years. The potential organizational and business process impacts will be significant for large banks (over €750 or €1 million) that do not currently use the basic method, which will have to, in addition to the definition and maintenance of a risk management framework, ensure a Loss Data Collection based on articles and high quality standards. For banks that currently adopt the TSA (Traditional Standardized Approach), the interventions to be planned may only have to concern specific refinements to the Loss Data Collection. LSI banks with an indicator below the aforementioned threshold can still refer to the new regulatory provisions to improve overall their operational risk measurement, control and management framework.
On 20 June 2024, the EBA published the final document in the Pillar III disclosure area «Final Draft Implementing Technical Standards on public disclosures by institutions of the information referred to in Titles II and III of Part Eight of Regulation (EU) No 575/2013», which defines the new methods of third pillar disclosure through the introduction, as regards Operational Risk, of a qualitative table (EU ORA) and the following three templates:

- EU OR 1: provides information on the number and amounts of operational risk losses incurred in the last 10 years, based on the accounting date and considering any recoveries and exclusions;
- EU OR 2: provides information on the calculation of the Business Indicator (BI) for the last three financial years and on the value of the Business Indicator Component (BIC)

• EU OR 3: provides information on the minimum capital requirements (Operational Risk Own Funds - OROF) for operational risk.

The revisions to the Operational Risk framework result in an overall increase in the Minimum Required Capital (MRC) for operational risk of 28.4% (table 2), with an increase of 32.0% for Group 1 banks and 10.5% for Group 2 banks.

The cumulative impact analysis uses a sample of 152 banks. The sample is divided into 60 Group 1 banks (large, internationally active banks) and 92 Group 2 banks. Group 1 banks are those with Tier 1 capital above €3 billion and active internationally; all other banks are classified as Group 2 banks.

The impact is greater for Group 1 banks using the AMA model (35.3%) than for Group 2 banks (10.1%). Overall, banks migrating from AMA approaches are more impacted (33.7%) than those using other approaches (23.3%).

15 of the 21 banks using AMA models (90% of the AMA OpRisk MRC) belong to Group 1.

Table 2: Changes in T1 MRC assigned to operational risk only; in % of T1 MRC assigned to operational risk under CRR2/CRD5

Bank group	AMA	Others	Total
All banks	33,7	23,3	28,4
Group 1	35,3	28	32,0
G-SIIs	33,9	45,7	37,2
Group 2	10,1	10,1	10,5

Source: EBA Qis data (december 2023)

The baseline impact assessment (Table 3) quantifies the difference in minimum capital requirements between the Basel (CRR2/CRD5) and the final version of Basel III (CRR3/CRD6) at the time of full implementation in 2033.

The new final capital requirements of Basel III determine an increase for operational risks of 2.8% (table 2). The increase in the MRC for the operational risk is mainly due to the increase in the net interest margin (NIM) which determined the increase in the BIC with a particularly significant impact for AMA banks.

Table 3: Change in total T1 MRC, as a percentage of the overall current T1 MRC, due to the implementation of the final Basel III framework under the EU-specific scenario (including all buffers and P2R capital requirements – frozen); weighted averages in %

Bank group	Credit Risk				Market Risk	CVA	Op risk	Output Floor	Other pillar 1	Total Risk based	Revised LR	Total
	SA	IRB	Sec.	CCPs.								
All banks	1,2	-1,5	0,0	0,0	1,1	0,3	2,8	5,7	-0,8	8,8	-1,0	7,8
Group 1	1,2	-1,7	0,0	0,0	1,3	0,4	3,1	6,4	-0,9	9,7	-1,2	8,6
G-SIIs	1,4	-1,4	0,0	0,0	2,7	0,5	3,8	8,6	-0,5	14,8	-2,6	12,2
O-SIIs	1,0	-2,1	0,0	0,0	-0,2	0,3	2,6	5,2	-1,2	5,5	-0,1	5,5
Other	0,5	0,5	0,0	0,0	4,1	0,4	2,8	0,5	-0,7	8,0	0,0	8,0
Group 2	1,5	-0,5	0,0	0,0	0,3	0,1	0,8	2,0	-0,1	4,0	-0,3	3,6
O-SIIs	1,4	0,0	0,0	0,0	0,3	0,0	0,9	1,4	-0,2	3,7	-0,5	3,2
Other	1,6	-1,2	0,0	0,0	0,5	0,1	0,6	2,9	-0,1	4,3	0,0	4,2
Universal	1,3	-1,1	0,0	0,0	1,3	0,3	2,9	5,4	-0,8	9,2	-1,1	8,2
Retail oriented	1,7	-0,8	0,0	0,0	-0,3	0,3	0,5	2,7	-0,3	3,7	-0,7	3,0
Corporate oriented	-0,1	-6,5	0,0	0,0	0,2	1,1	2,6	9,7	-0,3	6,8	-0,8	6,0

Source: EBA Qis data (december 2023)

There are several reasons why Group 1 banks have a higher MRC than Group 2 banks[2]. Firstly, 15 of the 21 banks using AMA models (90% of the AMA OpRisk MRC) belong to Group 1. On average, these banks manage to significantly reduce their capital requirements compared to the current standardized approaches.

[2] EBA (2023)

Secondly, Group 1 banks, or large Group 2 banks, mainly operate fee-based business models, while the rest of the Group 2 banks tend to offer more diversified banking services, less dependent on fees. For banks operating fee-based business models, the new indicator has been set at a more conservative level to reflect the higher operational risks typically observed in these models. The marginal coefficient increases from 0.12 (band 1) to 0.18 (band 3), leading to an increasing average marginal coefficient as the business indicator increases, with the result that large banks are generally more affected. Finally, banks active in different geographical areas with significant differences in their NIM (Net Interest Margin), could significantly reduce their capital requirements using either the Standardised Approach (TSA) or the Alternative Standardised Approach (ASA). In the new framework, the NIM will be calculated at group level, making such reductions no longer possible. The figure below highlights that the distribution of operational risk capital requirements for AMA Group 2 banks is significantly wider than the corresponding distribution for AMA Group 1 banks, while the simple mean and median are lower than for AMA Group 1 banks. This is because the business models of Group 1 banks offer universal services and therefore have relatively homogeneous operational risk characteristics, while Group 2 banks comprise a variety of business models offering specialised or more diversified types of services. Some Group 2 banks are particularly specialised and do not offer services that would be subject to credit or market risk. Operational risk is therefore the most important risk category for them.

5. Case Studies

In order to verify the capital impacts through the use of the new methodology introduced by Basel 4, an analysis was carried out on 6 banks comparing any differences. In particular, 6 italian banks were analyzed:

- two banks (Annex 1) with assets exceeding one billion that used the BIA model (belonging to Group 2);
- two banks (Annex 2) with assets exceeding 30 billion that used the TSA model (belonging to Group 1);
- two systemic banks (Annex 3) with assets exceeding 30 billion that used the AMA model (belonging to Group 1).

It emerged, in line with the EBA results, that the greatest impacts in terms of capital requirements for operational risks are found in the systemically important banks that used the AMA model.
Below we will verify the average impacts that the new methodology BIC determines compared to the use of the previous models: BIA, TSA and AMA

5.1 Case Study Banks A1, A2 with assets exceeding one billion that used the BIA model

The Italian banks A1 and A2 of Group 2 (Figure 1 and 2- Annex 1), analyzed, have adopted the BIA approach in the years 2021, 2022 and 2023 for the calculation of the regulatory requirement for operational risks. To verify the new impact regarding the replacement of the BIA approach with the new standardized approach of Basel 4, the BI and the BIC (average balance sheet items - Annex 1) were calculated. The BI comprises three components: the interest, leases and dividend component (ILDC); the services component (SC), and the financial component (FC). which is calculated by multiplying the BI by a set of regulatory determined marginal coefficients (figure 1). In the calculation of the capital requirement, an ILM, Internal Loss Multiplier, equal to 1 was considered as proposed by the CRR (Capital Requirements Regulation). The balance sheets and annual reports available online from 2021 to 2023 were used. The calculated BIC value was compared with the value of Own Funds allocated for operational risk present in the Pillar 3 public disclosure. The NIM (Net Interest Margin) and the SC Component (Service Component) have a significant impact on the BI indicator than the other components (Annex 1A, Graphs 1 and 3). The application of the min (IC, 2.25% * Assets) prevents overcapitalization from occurring, ensuring that regulatory capital is adequately balanced with respect to the operational risk faced. The lack of compensation between commission income and expense (Annex 1A, Graphs 2 and 4 has a impact on the calculation of the regulatory requirement operational risks. The results obtained highlighted an average increase in the regulatory requirement for operational risks equal to 10,44%. This result is in line with EBA estimates which an overall increase in the MRC for operational risk equal to 10.6% (table 1). The data analyzed show a very limited increase in own funds compared to the BIA approach. The capital requirement under the BIA method is calculated by applying a regulatory ratio, equal to 15%, to an indicator of the company's operating volume, identified as the three-year average of intermediation margin. This methodology is in line with the BIC estimate which provides for the application of a marginal coefficient equal to 15% for banks with assets exceeding one billion.

5.2 Case Study Banks B1, B2 with assets exceeding 30 billion that used the TSA model

The Italian banks B1 and B2 of Group 1 (Figure 1 and 2- Annex 2), analyzed, have adopted the TSA approach in the years 2021, 2022 and 2023 for the calculation of the regulatory requirement for operational risks. To verify the new impact regarding the replacement of the TSA approach with the new standardized approach proposed by Basel 4, the BI and the BIC (average balance sheet items - Annex 2) were calculated. In the calculation of the capital requirement, an ILM equal to 1 was considered as proposed by the CRR. The balance sheets and annual reports available online from 2021 to 2023 were used. The calculated BIC value was compared with the value of Own Funds allocated for operational risk present in the Pillar 3 public disclosure. The NIM (Net Interest Margin) and the SC Component (Service Component) have a significant impact on the BI indicator than the other components. (Annex 2B, Graphs 1 and 3). The lack of compensation between commission income and expense (Annex 2B, Graphs 2 and 4) generate an increase on the BI and consequently has an impact on the regulatory requirement. The results obtained highlighted an average increase in the regulatory requirement for operational risks equal to approximately 31,5% compared to the TSA method. The estimate obtained is in line with the EBA estimates that show an overall increase in the MRC for operational risk equal to 28% (table 1). The data analyzed show that the new methodology determines a more significant capital impact compared to banks that used the BIA method. The Traditional Standard method (TSA) is characterized by the determination of the capital requirement through the application of coefficients differentiated by business line (which vary between 12% and 18%) to the average of the relevant indicator defined by CRR 2013/575 of the last three financial years divided by business line. While the Base Method is characterized by the highest degree of simplicity. In this approach, a fixed rate of 15% is applied to the intermediation margin, the standard method provides that the bank's activities are divided into eight lines of business. Within each line of business, the intermediation margin represents a general indicator of the size of the activity and the possible operational risk to which it is exposed. The capital requirement for each line of business is calculated by multiplying GI - gross income or intermediation margin - by a factor β assigned to each line of business.

5.3 Case Study Banks C1, C2 with assets exceeding 30 billion that used the AMA model

The Italian systemic banks C1 and C2 of Group 1 (Figure 1 and 2 - Annex 3), analyzed, have adopted the AMA approach in the years 2021, 2022 and 2023 for the calculation of the regulatory requirement for operational risks. To verify the new impact regarding the replacement of the AMA approach with the new standardized approach proposed by Basel 4, the BI and the BIC (average balance sheet items – Annex 3) were calculated. In the calculation of the capital requirement, an ILM equal to 1 was considered as proposed by the CRR. The balance sheets and annual reports available online from 2021 to 2023 were used. The calculated BIC value was compared with the value of Own Funds allocated for operational risk present in the Pillar 3 public disclosure. The NIM (Net Interest Margin) and the SC Component (Service Component) have a significant impact on the BI indicator than the other components. (Annex 3C, Graphs 1 and 3). The NIM has a greater impact on BI than the SC component. The lack of compensation between commission income and expense generate a significant increase on the BI. Commission Income have a significant impact on the regulatory Requirement operational Risk (Annex 3C, Graphs 2 and 4). The results obtained highlighted an average increase in the regulatory requirement for operational risks equal to approximately 40,3% compared to the AMA method. The estimate obtained is in line with the EBA estimates that show an overall increase in the MRC for operational risk equal to approximately 35,3% (table 1). The data analyzed show that the new methodology determines a more significant capital impact compared to banks that used the BIA and TSA method. The result obtained is in line with the EBA estimates which highlight a greater impact in terms of capital requirement for banks that used the AMA method. This is because the new method aims to standardize the calculation of regulatory capital, reducing the discretion and complexity associated with AMA models, which rely on internal data and more elaborate scenario analyses.
Table 4 shows the balance sheet items used to calculate the BI and BIC:

Table 4: Balance sheet items used for the calculation of BI and BIC

<table>
<tr><td rowspan="4">IC</td><td></td><td colspan="2">Interest and similar income</td></tr>
<tr><td></td><td colspan="2">Interest and similar expenses</td></tr>
<tr><td></td><td colspan="2">NIM=(Interest and similar Income - Interest and similar expenses)</td></tr>
<tr><td></td><td colspan="2">Dividend Income</td></tr>
<tr><td rowspan="7">AC</td><td></td><td rowspan="7">Asset Component</td><td>Cash and cash balances</td></tr>
<tr><td></td><td>Financial assets held for trading</td></tr>
<tr><td></td><td>Financial assets mandatorily at fair value</td></tr>
<tr><td></td><td>Financial assets at fair value through other comprehensive income</td></tr>
<tr><td></td><td>Financial assets at amortised cost</td></tr>
<tr><td></td><td>Hedging derivatives</td></tr>
<tr><td></td><td>Tangible assets to functional use (right-of-use assets acquired under leases)</td></tr>
<tr><td>ILDC</td><td></td><td>Income from interests, dividends and financial leases</td><td>ILDC=min (IC, 0,0225*AC)+DC</td></tr>
<tr><td rowspan="5">SC</td><td></td><td colspan="2">Other Operating Expenses (OOE)</td></tr>
<tr><td></td><td colspan="2">Other Operating Income (OOI)</td></tr>
<tr><td></td><td colspan="2">Fee and commissions Income(FI)</td></tr>
<tr><td></td><td colspan="2">Financial and commission expenses(FE)</td></tr>
<tr><td></td><td colspan="2">SC=(Service Component)=MAX (OI,OE) +MAX (FI,FE)</td></tr>
<tr><td rowspan="2">FC</td><td>TB</td><td>Trading Book Component</td><td>Net gain (loss) on trading activities</td></tr>
<tr><td>BC</td><td>Banking Book Component</td><td>Net gain (loss) on hedging activities</td></tr>
</table>

			Net gain (loss) on the disposal or repurchase of:
			Net gain (loss) of other financial assets and liabilities measured at fair value through profit or loss
	FC =Financial Component=TB+BC		
		FC=TB+BC	
BI		Business Indicator	BI=ILDC + SC + FC
BIC		Business Indicator Component	% MARGIN Component *BI

Source: Aifirm[3]

6. Conclusion

With the introduction of Basel IV[4], the phase of uncertainty regarding the regulatory framework of the banking sector finally ended. The reforms enacted after the 2007-2009 financial crisis represent a corpus of appropriate and desirable interventions, set out to remedy some significant gaps in the prudential regulation of banks. The impact of the regulatory changes has been the subject of periodic monitoring for several years by the Basel Committee at global level and at a European Banking Authority (EBA) at European level. Thanks to the reforms implemented in recent years, banks have faced the COVID-19 crisis starting from capital and liquidity conditions that were significantly better than those prevailing on the eve of the global financial crisis of 2007-08 and the European sovereign debt crisis of 2011-12. The need for binding regulation of the banking system lies in the extreme importance that this system has in the economy of each country and, at the same time, in its permeability to risks that can lead banking institutions to collapse, with serious repercussions on citizens and the national economy. Making the banking system more vigorous by increasing capital requirements and liquidity standards, strengthening governance structures and increasing the quality of regulatory capital is absolutely necessary to ensure the stability of a system from which the economy cannot ignore. To make the capitalization system more solid and improve confidence in the banking system, the regulator has implemented a new reform process. The set of reform measures prepared by the Basel Committee on Supervision aim to strengthen the regulation and risk management of the banking sector, as well as to ensure greater uniformity and comparability among financial institutions in relation to the capital requirements required for banking activities. The management of operational risks is an essential component for the stability and sustainability of the financial sector. The changes established, especially with reference to the use of standard models, go in the direction of limiting the excessive advantages associated with the use of these methodologies, in particular for G-SIB banks. Basel IV introduces higher capital requirements, particularly for operational risk. This means that G-SIBs will have to hold larger capital buffers, reducing their profitability and ability to generate profits. The implementation of Basel IV will have significant implications for financial institutions. The increase in capital requirements will first require a reassessment of banks' capital management strategies. Firms will have to increase their capital ratios, which could theoretically lead to an adjustment in lending practices and a potential increase in financing costs for consumers and businesses. Financial institutions will also be forced to better orient their business models to focus on highly specialized areas. Basel 4 provides for an increase in RWA (Risk Weighted Assets) related to operational risk, with the aim of strengthening the stability and resilience of the banking system. The analysis shows that for smaller banks, the impact of the more restrictive treatment of fees is negligible. This is because such institutions, with traditional business models and simplified operational structures, do not extensively rely on commission-based strategies, which are generally associated with a higher operational risk. Larger banks, on the other hand, are more affected by the method, due to the gradual increase in the marginal coefficient, which reaches 18% as the Business Indicator (BI) rises. In the absence of the FINREP report, the BI items have been mapped to the balance sheet items by applying some approximations (e.g. operating losses not included in the calculation of OE). The entry into force of the new regulations requires careful planning and the adoption of targeted strategies to optimize capital requirements and mitigate business impacts. The assessment of operational risk in the future requires a holistic perspective that considers the entire organizational ecosystem. To avoid losses, it is necessary to correct processes in advance. The implementation of Basel IV requires careful planning and the adoption of targeted strategies to optimize capital requirements and mitigate business impacts. Financial institutions must be ready to implement the necessary actions to address these new challenges and seize the opportunities presented by the new framework. It is therefore essential to effectively and efficiently manage, measure, and produce information on operational risks in order to ensure the sustainability of the business over time, improve operational efficiency and competitive positioning, and safeguard the continuity of the organization.

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Effect of Risk Management Practices on Supplier Selection in Lower Benue River Basin Development Authority Makurdi, Nigeria

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Article submitted to double-blind peer review, received on 26th July 2025 and accepted on 10th November 2025

Abstract
This study examines the influence of risk management practices on supplier selection at the Lower Benue River Basin Development Authority (LBRBDA) in Makurdi, highlighting the importance of integrating structured risk management into public procurement processes. A descriptive survey design was employed, with quantitative data collected from 101 procurement stakeholders, yielding 88 valid responses and a response rate of 87.1%. Data were analyzed using SPSS version 26, with multiple regression applied to assess the relationship between risk management practices and supplier selection. Results revealed that risk management practices significantly impact supplier selection decisions ($R = 0.872$, $R^2 = 0.761$, $F = 91.417$, $p < 0.001$). Among the predictors, risk identification emerged as the most influential ($\beta = 0.628$), followed by risk mitigation ($\beta = 0.451$), risk monitoring and control ($\beta = 0.301$), and risk assessment ($\beta = 0.187$). These findings underscore that a comprehensive risk management framework strengthens the supplier selection process, enhancing transparency, efficiency, and effectiveness in public procurement. The study contributes to procurement literature by providing empirical evidence on the distinct roles of risk identification, assessment, mitigation, and monitoring in shaping supplier selection outcomes within a Nigerian river basin development authority. It further offers practical insights for public procurement agencies seeking to embed risk-aware practices into decision-making processes, thereby improving accountability and long-term performance in supplier management.

Keywords: Risk identification, Risk Mitigation, Risk monitoring and control, Risk assessment, Supplier selection.

1 Introduction
1.1 Background to the Study
Integrating risk management into supplier selection is increasingly essential in procurement and supply chain management, particularly in large-scale public infrastructure projects that face uncertainties related to supplier performance, geopolitical factors, and market volatility (Baldwin & Freeman, 2022). Structured approaches enable organisations to select suppliers whose capabilities, financial stability, and ethical standards align with long-term project goals and sustainability objectives. International best practices, such as the European Union's Directive 2014/24/EU and the United States' Federal Acquisition Regulation (FAR), along with corporate strategies by firms like Siemens and Lockheed Martin, demonstrate how systematic frameworks reduce procurement disruptions and enhance supplier reliability (European Union, 2014; Siemens AG, 2021).
In Asia, economies like Japan, South Korea, and China have institutionalised risk-oriented procurement strategies. Toyota's Just-In-Time (JIT) system integrates supplier evaluation with real-time risk response, while China's Belt and Road Initiative employs advanced frameworks to manage political and logistical uncertainties (Ram & Zhang, 2020; Morris, 2020). In Africa, weak regulatory oversight and institutional inefficiencies often hinder procurement performance; nevertheless, countries such as South Africa and Kenya have improved outcomes through public-private partnerships and risk transfer mechanisms (Ameyaw & Chan, 2015). Nigeria has undertaken procurement reforms, notably through the Public Procurement Act of 2007, to address inefficiencies and procurement-related risks. However, implementation remains inconsistent, particularly within agencies such as the Lower Benue River Basin Development Authority (LBRBDA), where unreliable suppliers, political interference, contract underperformance, and bureaucratic delays continue to compromise outcomes. These challenges highlight the need for a context-specific framework that adapts global risk management principles to Nigeria's institutional realities.
Although risk management practices—encompassing the identification, evaluation, mitigation, and monitoring of threats—are widely acknowledged to improve supplier selection (Gurtu& Johny, 2021; Yazdani et al., 2020), their application in Nigeria's public sector remains limited. This gap has resulted in financial losses, reputational risks, and weak compliance with procurement regulations. To address this deficiency, the present study empirically examines how risk management practices influence supplier selection within LBRBDA. By contextualising global best practices within Nigeria's operational environment, the study seeks to develop a tailored framework for enhancing procurement performance, supplier reliability, and institutional accountability.

1.2 Objective of the Study

The main objective of the study is to examine the effect of risk management practices on supplier selection practices in the Lower Benue River Basin Development Authority, Makurdi, Nigeria. The specific objectives are:

i. Determine the effect of risk identification on supplier selection practices in the Lower Benue River Basin Development Authority.

ii. Assess the effect of risk mitigation on supplier selection practices in the Lower Benue River Basin Development Authority.

iii. Evaluate the effect of risk monitoring and control on supplier selection practices in the Lower Benue River Basin Development Authority.

iv. Examine the effect of risk assessment on supplier selection practices in the Lower Benue River Basin Development Authority.

1.3 Research Hypotheses

The following null hypotheses were formulated in line with the study objectives:

$H0_1$: Risk identification has no significant effect on supplier selection in the Lower Benue River Basin Development Authority, Makurdi.

$H0_2$: Risk mitigation has no significant effect on supplier selection in the Lower Benue River Basin Development Authority, Makurdi.

$H0_3$: Risk monitoring and control has no significant effect on supplier selection in the Lower Benue River Basin Development Authority, Makurdi.

$H0_4$: Risk assessment has no significant effect on supplier selection in the Lower Benue River Basin Development Authority, Makurdi.

2 Literary Review

2.1 Theoretical framework

This section outlines the key theories underpinning the study, providing a conceptual basis for understanding how risk management practices influence supplier selection.

2.1.1 Risk Management Theory

Risk management theory has evolved significantly since its inception. One of the foundational works in risk management theory is by Chapman and Ward (2003) in their book, *Project Risk Management: Processes, Techniques, and Insights*. They established a comprehensive framework for understanding risk management in projects, emphasizing the importance of risk identification, assessment, and mitigation strategies. Their framework includes processes for identifying risks, analyzing their potential effects, and implementing response strategies to mitigate these risks. Their work remains a cornerstone in the study and application of risk management theory.Risk management theory assumes that risks can be systematically identified through a structured process. Risk management theory provides a structured approach to integrating risk considerations into procurement practices. By applying the concepts and frameworks discussed, LBRDBA Makurdican enhance its supplier selection process, ensuring that risks are effectively managed and procurement outcomes are optimized.

2.1.2 Supplier selection theory

Supplier selection theory has evolved as a critical component of procurement and supply chain management, influencing how organizations choose suppliers to meet their needs effectively. One of the seminal works in supplier selection theory is the research by Verma and Pullman (1998), which significantly shaped the modern understanding of supplier selection processes. Their research highlighted the importance of a systematic approach to supplier selection, emphasizing that the choice of suppliers can significantly affect an organization's operational efficiency and competitive advantage. The framework proposed by Verma and Pullman includes several key criteria for supplier selection such as cost, quality, delivery time, service, and flexibility. Supplier selection theory is grounded in several assumptions that are crucial for its effective application. The theory assumes that supplier selection is a multi-criteria decision-making (MCDM) process, where various factors such as cost, quality, delivery, and service are considered. This approach acknowledges that no single criterion is sufficient on its own to determine the suitability of a supplier. It assumes that both quantitative metrics (e.g., cost, delivery time) and qualitative aspects (e.g., supplier reputation, flexibility) are essential for a comprehensive evaluation. The theory posits that integrating both types of evaluation criteria leads to a more balanced and informed decision-making process. Ardently, the theory assumes that the selected suppliers should align with the strategic goals and operational needs of the organization. This alignment ensures that suppliers contribute to achieving long-term objectives and provide a competitive edge. Supplier selection theory also assumes that the environment in which organizations operate is dynamic, and therefore, supplier selection processes must be adaptable to changes in market conditions, technological advancements, and organizational needs. These assumptions underscore the complexity of the supplier selection process and the need for a structured and comprehensive evaluation approach. The relevance of supplier selection theory to this study can be understood through several key points. The study examines how risk management practices influence supplier selection decisions. By incorporating insights from the supplier selection theory and its critiques, the research can explore how risk considerations affect the choice of suppliers and how effective risk management can lead to better procurement outcomes. The supplier selection theory provides a framework for evaluating suppliers based on multiple criteria. In the context ofLBRBDA, this framework can be adapted to address local challenges and requirements, ensuring that supplier selection aligns with regional needs and project goals.

2.2 Conceptual framework

The conceptual framework illustrates the dynamic relationship between risk management practices and supplier selection, with a particular focus on the Lower Benue River Basin Development Authority (LBRBDA). This framework is structured around key constructs, where risk management practices serve as the independent variable (risk identification, risk assessment, risk mitigation and risk monitoring and control), influencing the supplier selection process, which functions as the dependent variable(supplier capacity, supplier past performance, supplier risk exposure, and supplier relationship management).

2.2.1 Risk management practices

Risk management practices (RMP) encompass a range of actions designed to identify, assess, respond to, monitor, and control risks. These practices aim to mitigate the adverse effects that risks can have on procurement activities and project outcomes. Effective risk management is crucial for ensuring that suppliers are selected based on their ability to manage potential risks, thereby enhancing project execution and success. According to Hopkin (2018), RMP refers to the systematic application of management policies, procedures, and practices to the tasks of identifying, analyzing, evaluating, treating, and monitoring risks. Similarly, ISO 3001 (2018) defines RMP as a proactive approach to uncertainty, emphasizing the need for structured and repeatable processes to ensure that risks are managed efficiently across all organizational levels. Risk management professionals, such as those certified by the Risk Management Society (RIMS), further emphasize that RMP is not only about addressing potential threats but also about identifying opportunities that may arise in a dynamic environment (RIMS, 2021).
RMP serve as a vital foundation for effective supplier selection and procurement practices. By integrating the 4 independent variables of the study risk identification, risk assessment, risk mitigation, and risk monitoring and control into procurement frameworks, organizations can enhance their ability to manage uncertainties, improve supplier reliability, and achieve project success. This study focuses on four key components of risk management: risk identification, risk assessment, risk mitigation, and risk monitoring and control.

2.2.1.1 Risk identification

Risk identification is the first and foundational step in the risk management process. It involves systematically recognizing potential risks that could disrupt procurement or project outcomes. According toKuang*et al.*(2019), risk identification requires a comprehensive analysis of both internal and external factors that may affect the project. These risks could stem from financial, technical, operational, environmental, or legal issues. The identification process is critical as it ensures that the procurement team is aware of possible threats early in the project lifecycle (Alikhani*et al.*, 2019), enabling them to develop appropriate mitigation strategies. Empirical studies underscore the importance of risk identification in procurement processes. By identifying risks early, procurement managers can include relevant risk-related criteria in the supplier selection process, favoring suppliers who are better equipped to manage these challenges. For example, Huma*et al*. (2020) demonstrated that prioritizing suppliers with robust risk management practices, such as compliance certifications and contingency planning, enhances procurement outcomes. Risk identification thus serves as the foundation for a comprehensive risk management strategy, ensuring that subsequent steps, such as risk assessment and mitigation, are grounded in a thorough understanding of potential threats.

2.2.1.2 Risk mitigation

This refers to the development and implementation of strategies to minimize the likelihood or impact of risks. Schramm*et al*. (2020) highlight the importance of supplier diversification and the use of performance guarantees or other contract clause as risk mitigation measures in public procurement. By selecting multiple suppliers or incorporating specific performance metrics into contracts, procurement entities can reduce their exposure to supplier-related risks. Moreover, risk mitigation also involves supplier pre-qualification, a process where suppliers are vetted based on their technical, financial, and operational capacity to handle potential risks. According to Gurtu and Johny (2021), supplier pre-qualification serves as a filtering mechanism, allowing procurement teams to only engage suppliers who demonstrate strong capabilities in managing risks related to their supply chain, financial stability, and compliance with regulatory requirements. This process not only reduces procurement risks but also ensures that the selected suppliers are better equipped to meet project demands under uncertain conditions.

2.2.1.3 Risk monitoring

Risk monitoring is an ongoing process that ensures risk management practices remain effective throughout the procurement and project lifecycle. Continuous monitoring allows project managers to track identified risks and identify new threats as they emerge. Li *et al.* (2021) stress the importance of regular risk assessments and supplier performance reviews to adjust risk management practices as needed. This process is critical for maintaining alignment between risk management efforts and dynamic project requirements.Risk monitoring also involves the use of software tools that track key performance indicators (KPIs), allowing for real-time adjustments and early interventions. For instance, Rane and Potdar (2021) highlight that the integration of digital technologies, such as Artificial Intelligence (AI) and Blockchain, has revolutionized risk monitoring by enabling real-time data collection and analysis. These technologies allow procurement managers to detect anomalies in supplier performance, ensure compliance with contractual obligations, and respond promptly to emerging risks.In developing economies, risk monitoring plays a vital role in addressing systemic challenges. For example, Essien *et al.* (2018) argue that transparent monitoring systems enhance accountability and reduce the influence of corruption in supplier management.

2.2.1.4 Risk assessment

Once risks are identified, they must be assessed based on two key parameters: likelihood and impact. Likelihood refers to the probability of the risk event occurring, while impact measures the potential consequences for the project if the risk materializes (Acebes*et al.*, 2024; Aloini*et al.,* 2021, as cited by Nyambo, 2023). By evaluating these parameters, procurement managers can categorize risks into high, medium, or low priority. High-priority risks require immediate attention and mitigation efforts, whereas lower-priority risks may be monitored and addressed as needed. Kuang *et al*. (2019) assert that early risk identification and assessment are critical for the successful implementation of risk management practices. When potential risks are recognized at the outset of a procurement process, it becomes easier to develop mitigation plans and integrate risk considerations into supplier selection criteria. This ensures that suppliers who have demonstrated strong risk management capabilities are favored in the selection process.In developing economies, like Nigeria, where procurement systems are often fraught with challenges such as corruption and political interference, robust risk assessment frameworks are indispensable. By incorporating tools such as risk matrices, probabilistic models, and sensitivity analyses, organizations can systematically evaluate risks and prioritize resources accordingly.

2.2.2 Supplier Selection

Supplier selection is widely regarded as a critical determinant of procurement success, particularly in the delivery of public infrastructure projects where uncertainty and complexity are prevalent. It goes beyond identifying capable vendors to systematically evaluating their ability to contribute to project objectives under risk-laden conditions. The integration of risk management into supplier selection ensures that decisions are not solely cost-driven but account for factors such as reliability, resilience, and long-term performance (Ho et al., 2010). Drawing from existing literature and contextual realities within LBRBDA, this study conceptualises supplier selection along three interrelated dimensions: supplier capacity, supplier past performance, and supplier risk exposure. These dimensions capture the multifaceted nature of supplier evaluation and provide an operational framework for assessing how risk management practices influence procurement outcomes. Supplier capacity represents the extent to which a supplier possesses the technical expertise, resources, and financial strength necessary to deliver on project requirements. It reflects both current capability and the ability to withstand unexpected challenges without jeopardizing delivery timelines or incurring additional costs. Within public procurement settings, evaluating capacity is essential, as underperformance often stems from inadequate technical and financial foundations. Risk assessment frameworks therefore emphasize capacity as a decisive factor in selecting suppliers who can sustain performance under uncertainty (Schramm et al., 2020). Past performance serves as a proxy for reliability, offering insights into how suppliers have managed quality, timeliness, and budget adherence in previous projects. Empirical studies indicate that suppliers with strong track records are better positioned to navigate risks in future engagements, as their demonstrated resilience provides confidence in their capacity to handle disruptions (Saha & Joshi, 2024). In this study, past performance is treated as a key evaluative dimension, ensuring that historical evidence informs the selection of suppliers most likely to deliver consistently within high-risk public infrastructure environments. Supplier risk exposure captures the degree of vulnerability that a supplier faces from both internal weaknesses and external threats, such as financial instability, regulatory non-compliance, or volatile supply chains. High exposure increases the probability of disruptions, while low exposure signals greater resilience and preparedness. Evaluating supplier risk exposure therefore enables procurement teams to favor suppliers who are structurally and operationally positioned to minimize potential project disruptions (Zimmer et al., 2016). In the context of LBRBDA, this dimension is particularly salient given the prevalence of systemic risks in Nigeria's public procurement landscape.

2.3 Review of related empirical studies

Empirical scholarship consistently demonstrates that integrating risk management practices enhances supplier selection, yet the evidence base reveals important limitations that warrant further investigation. Several studies affirm the significance of risk identification as the foundation of supplier evaluation. For example, Anozie et al. (2024) in Nigeria, Kraljic (2022) in Europe, and Smeltzer and Siferd (2020) in the U.S. all emphasize that early recognition of supplier-related vulnerabilities strengthens procurement outcomes. Similarly, Lesisa et al. (2018) highlight how the absence of systematic identification processes undermines supplier evaluation. Collectively, these findings converge on the importance of identifying risks early, but they largely stop at descriptive associations, with limited exploration of how identification interacts with other dimensions of risk management to influence supplier decisions.

The literature also affirms the role of risk assessment in refining supplier choice. Sanders (2023), Jones et al. (2022), and Trkman and McCormack (2021) show that advanced assessment tools—ranging from hybrid models to scenario testing—help align supplier capacity with project-specific risks. However, while these models demonstrate methodological sophistication, they are often tested in high-uncertainty or defense-related contexts, leaving questions about their transferability to routine public procurement environments. Moreover, the heavy reliance on quantitative modeling risks overlooking softer, qualitative aspects of supplier evaluation, such as ethical compliance or institutional accountability.

In terms of risk mitigation, studies such as El-Diraby et al. (2022) and Schramm et al. (2020) reveal that proactive strategies—including supplier collaboration, diversification, and contractual safeguards—improve supplier performance. Yet, these works treat mitigation as a stand-alone mechanism, rarely examining how it complements identification, assessment, or monitoring in a holistic framework. This compartmentalization limits our understanding of risk management as an integrated process rather than a set of discrete activities. Finally, risk monitoring and control is increasingly recognized as vital for sustaining procurement performance. Li et al. (2021), Hernandez et al. (2019), and Zimmer et al. (2016) demonstrate that continuous monitoring enhances supplier compliance and reliability. While persuasive, much of this research emphasizes technological enablers such as digital platforms and real-time data analytics. Less attention has been paid to institutional and managerial practices that may enable—or constrain—the effectiveness of monitoring in different organizational settings. Taken together, existing studies provide strong evidence that risk management practices influence supplier selection, but they often suffer from three limitations. First, the literature tends to treat risk management dimensions in isolation, overlooking their interdependence and cumulative effects. Second, many studies emphasize methodological sophistication (e.g., modeling, hybrid approaches) without adequately addressing practical implementation challenges in public institutions. Third, while supplier outcomes such as reliability and performance are frequently examined, there is less focus on how structured risk management frameworks can enhance decision-making quality in supplier selection itself. These gaps highlight the need for empirical research that integrates risk identification, assessment, mitigation, and monitoring into a unified framework, and tests their combined influence on supplier selection outcomes in public procurement contexts.

3 Methods

3.1 Research design

This study adopted a descriptive quantitative survey design, considered appropriate for investigating relationships between constructs where quantifiable evidence can be collected from a defined population. The choice of this design was guided by the objective of examining how distinct risk management practices predict supplier selection outcomes in a public procurement setting.

3.2 Population and Sampling

The target population comprised 101 procurement stakeholders within the Lower Benue River Basin Development Authority (LBRBDA), Makurdi. This group included project managers, procurement officers, and contract administrators directly involved in supplier evaluation and decision-making. Given the relatively small population size, a census sampling approach was employed to minimize sampling bias and maximize representation. Of the 101 questionnaires distributed, 88 were returned fully completed, representing a valid response rate of 87.1%, which is considered sufficient for inferential analysis.

3.3 Research Instrument

Data were collected using a structured questionnaire developed from established instruments in procurement and risk management literature. Items measuring risk identification, risk assessment, risk mitigation, and risk monitoring and control were adapted from Kuang et al. (2019), Gurtu and Johny (2021), and Schramm et al. (2020), while supplier selection indicators were drawn from Ho et al. (2010) and Yazdani et al. (2020). A five-point Likert scale (ranging from 1 = “Strongly Disagree” to 5 = “Strongly Agree”) was used to capture respondent perceptions. Content validity was established through expert review by three procurement academics and two senior procurement officers. A pilot test with 15 respondents outside the study sample confirmed clarity and appropriateness of items, with Cronbach’s alpha coefficients for all constructs exceeding the 0.70 threshold, indicating internal consistency. The pilot test not only assessed internal consistency but also provided insights into item clarity and contextual relevance. Feedback from participants led to minor rewording of two questions to eliminate ambiguity. This step ensured that the constructs were clearly understood by procurement stakeholders within the Nigerian public-sector context. To avoid bias, the 15 respondents who participated in the pilot test were excluded from the main study sample.

3.4 Data Collection Procedure

Questionnaires were self-administered with the support of LBRBDA management between January and March 2025 to ensure high participation. Respondents were briefed on the study’s purpose, assured of confidentiality, and informed consent was obtained. Completed questionnaires were collected over a three-week period, with follow-ups made to improve response rates.

3.5 Data Analysis

Data were coded and analyzed using SPSS version 26. Descriptive statistics (means, standard deviations, and frequencies) were employed to summarize respondents' characteristics and perceptions. Inferential analysis was conducted using multiple regression to examine the predictive effects of the four independent variables—risk identification, risk assessment, risk mitigation, and risk monitoring and control—on the dependent variable, supplier selection. Before applying the regression model, the model assumptions of linearity, normality, multicollinearity, and independence of errors were tested, and after the model application, tests of robustness were also conducted. The regression model for the study is econometrically specified as follows:

$$SS=\alpha+\beta_1 RI+\beta_2 RM+\beta_3 RMS+\beta_4 RA+\varepsilon$$

Where:
SS = Supplier Selection (dependent variable)
RI = Risk Identification
RM = Risk Mitigation
RMC = Risk Monitoring and Control
RA = Risk Assessment
α = Constant term
$\beta_1 \ldots \beta_4$ = Coefficients measuring the effect of each independent variable
ε = Error term

Table 1: Variables Used in the Study

Variable Type	Variable Name	Description / Indicators	Source(s)
Dependent Variable	Supplier Selection (SS)	Evaluated in terms of supplier capacity, past performance, and risk exposure.	Ho et al. (2010); Yazdani et al. (2020)
Independent Variable	Risk Identification (RI)	Extent to which potential supplier-related risks (financial, technical, regulatory, logistical) are identified early in the procurement cycle.	Kuang et al. (2019); Huma et al. (2020)
Independent Variable	Risk Assessment (RA)	Evaluation of identified risks in terms of likelihood and impact, prioritizing high-risk areas.	Acebes et al. (2024); Aloini et al. (2021)
Independent Variable	Risk Mitigation (RM)	Strategies adopted to minimize risk likelihood or impact (e.g., diversification, guarantees, prequalification).	Schramm et al. (2020); Gurtu& Johny (2021)
Independent Variable	Risk Monitoring & Control (RMC)	Continuous tracking of supplier performance and risk indicators to ensure early response and compliance.	Li et al. (2021); Hernandez et al. (2019)
Control Variables	Demographic Characteristics	Respondent role (project manager, procurement officer, contract administrator), years of experience, and department. Included to account for heterogeneity in perceptions.	Authors' survey instrument (2025)

Note. All independent variables were measured using multiple Likert-type items on a 5-point scale (1 = Strongly Disagree to 5 = Strongly Agree). The responses reflect the risk management practices of LBRBDA as experienced by participants in their current roles over the past three years, ensuring that perceptions are both recent and institutionally grounded.

4 Results and Discussion

4.1 Pre-Diagnostic Tests

Before conducting the main regression analysis, pre-diagnostic tests were performed to verify that the data met the assumptions required for valid regression results. Descriptive statistics and correlations were first examined to summarize the data and identify preliminary relationships among variables.

Normality tests (Shapiro–Wilk) assessed whether the variables followed an approximately normal distribution, while multicollinearity diagnostics (VIF and Tolerance) ensured that the independent variables were not highly correlated. These checks provide confidence that the regression results would be statistically sound and interpretable.

Table 2: Descriptive Statistics

Variable	N	Minimum	Maximum	Mean	Std Deviation
Risk identification	88	1.000	5.000	3.87	0.812
Risk mitigation	88	1.000	5.000	3.76	0.792
Risk monitoring and control	88	1.000	5.000	3.68	0.821
Risk assessment	88	1.000	5.000	3.79	0.805
Supplier selection	88	1.000	5.000	3.85	0.833

Source: SPSS output of Researchers' Computations, 2025.

The descriptive statistics in Table 2 show that all the study variables recorded relatively high mean values above 3.5 on a 5-point Likert scale, indicating strong agreement among respondents on their importance in the operations of the Lower Benue River Basin Development Authority. Risk identification recorded the highest mean of 3.87 (SD = 0.812), suggesting that identifying risks is the most emphasized practice. This is followed closely by supplier selection with a mean of 3.85 (SD = 0.833), reflecting the Authority's strong focus on choosing suppliers carefully.

Risk assessment had a mean of 3.79 (SD = 0.805), while risk mitigation scored 3.76 (SD = 0.792), both showing that evaluating and reducing risks are integral to procurement practices. Risk monitoring and control had the lowest mean of 3.68 (SD = 0.821), though still relatively high, suggesting that while monitoring risks is practiced, it may not be as consistently emphasized as the other dimensions.

The closeness of these mean values and the relatively small standard deviations highlight a general consensus among respondents that risk management practices are actively applied and play a critical role in effective supplier selection within the organization.

Table 3: Correlation Result

	RI	RM	RMC	RA	SS
Risk identification (RI)	1				
Risk mitigation (RM)	.721**	1			
Risk monitoring and control (RMC)	.684**	.612**	1		
Risk assessment (RA)	.653**	.587**	.609**	1	
Supplier selection (SS)	.639**	.564**	.592**	.618**	1
N	88	88	88	88	

Correlation is significant at 0.01 level (2 tailed)

Source: SPSS Output of Researchers' Computations, 2025.

The correlation results in Table 3 provide further insight into the relationships between risk management practices and supplier selection. All the correlation coefficients are positive and statistically significant at the 0.01 level, confirming strong associations among the variables. Notably, risk identification ($r = 0.639$), risk mitigation ($r = 0.564$), risk monitoring and control ($r = 0.592$), and risk assessment ($r = 0.618$) all correlate positively with supplier selection. This implies that improvements in any of these practices are likely to enhance the supplier selection process.

Additionally, the strong inter-correlations among the risk management practices themselves (ranging from 0.587 to 0.721) suggest that these practices are complementary, and their combined application contributes to better procurement outcomes within the Authority.

Table 4: Normality of Variables (Sharpiro-Wilk)

Variable	Statistic (W)	Df	Sig.
Risk identification (RI)	0.981	88	0.164
Risk mitigation (RM)	0.978	88	0.118
Risk monitoring and control (RMC)	0.984	88	0.207
Risk assessment (RA)	0.987	88	0.288
Supplier selection (SS)	0.989	88	0.335

Source: SPSS Output of Researchers' Computations, 2025.

The Shapiro–Wilk test in Table 4 was used to examine the normality of the variables. The results showed that all the variables (Risk Identification, Risk Mitigation, Risk Monitoring and Control, Risk Assessment, and Supplier Selection Practices) have p-values greater than 0.05 (ranging from 0.118 to 0.335).

Since none of the p-values fall below the 0.05 threshold, the null hypothesis of normality cannot be rejected. This indicates that all the study variables are approximately normally distributed, satisfying the regression assumption of normality.

Table 5: Multicollinearity Diagnostic (Tolerance & VIF)

Variable	Tolerance	VIF
Risk identification (RI)	0.726	1.377
Risk mitigation (RM)	0.708	1.423
Risk monitoring and control (RMC)	0.752	1.330
Risk assessment (RA)	0.741	1.349

Source: SPSS Output of Researchers' Computations, 2025.

The Variance Inflation Factor (VIF) and Tolerance values in Table 5 were used to test for multicollinearity among the predictors. All tolerance values are above 0.70, while all VIF values are below 2 (ranging from 1.330 to 1.423).
Since the general rule is that tolerance values greater than 0.2 and VIF values less than 10 indicate no multicollinearity, these results confirm that there is no multicollinearity among the independent variables. This means the predictors contribute unique information to the model without excessive overlap.

4.2 Regression Analysis

Table 6: Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin Watson statistic
1.0	0.872	0.761	0.752	0.489	1.984

a. Predictors: (Constant), Risk assessment, Risk monitoring and control, Risk mitigation, Risk identification
b. Dependent: Variable: Supplier selection

Source: SPSS Output of Researchers' Computations, 2025.

The regression analysis as presented in Table II was conducted to examine the extent to which risk management practices influence supplier selection in the Lower Benue River Basin Development Authority (LBRBDA). From the model summary, the correlation coefficient R was found to be 0.872, indicating a strong positive relationship between the set of independent variables and supplier selection.
The R-squared (R^2) value of 0.761 suggests that approximately 76.1% of the variation in supplier selection can be explained by the combined effect of risk identification, risk assessment, risk mitigation, and risk monitoring and control.
The adjusted R-squared, which adjusts for the number of predictors in the model, stood at 0.752, further confirming the robustness of the model.
A Durbin-Watson statistic of 1.984 was observed, indicating that the residuals were not auto-correlated and that the assumptions of independence in the regression model were met. This result implies that the model, as a whole, is statistically significant, and at least one of the predictors has a meaningful effect on supplier selection within LBRBDA.

Table 7: Analysis of Variance (ANOVA)

Model		Sum of Squares	Df	Mean Square	F change	Sig.
1	Regression	54.331	4	13.583	91.417	0.000
	Residual	17.087630	83	0.206		
	Total	71.418	87			

a. Dependent Variable: Supplier selection
b. Predictors: (Constant), Risk assessment, Risk monitoring and control, Risk mitigation, Risk identification

Source: SPSS Output of Researchers' Computations, 2025.

The ANOVA result in Table 7 showed that the regression model is statistically significant (F = 91.417, p = 0.000), meaning that risk management practices collectively have a strong and meaningful effect on supplier selection in the Lower Benue River Basin Development Authority. The predictors (risk identification, risk mitigation, risk monitoring and control, and risk assessment) together explain a substantial proportion of the variation in supplier selection.

Table 8: Regression Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	T	Sig.
1	(Constant)	0.412	0.215	—	4.686	0.000
	Risk identification	0.725	0.077	0.628	9.467	0.000
	Risk mitigation	0.538	0.070	0.451	7.734	0.000
	Risk monitoring and control	0.384	0.060	0.301	6.361	0.001
	Risk assessment	0.292	0..069	0.187	4.231	0.000

Dependent Variable: Supplier selection
Source: SPSS Output of Researchers' Computations, 2025.

Further insight was drawn from the regression coefficients in Table 8, where the individual contributions of each risk management practice were examined. The standardized beta coefficient for Risk Identification was 0.628, with a t-value of 9.467 and a p-value of 0.000. This indicates that risk identification has the most substantial effect on supplier selection. Interpreting this in practical terms, for every unit increase in effective risk identification practices, there is a corresponding 0.628-unit increase in the likelihood of achieving optimal supplier selection, all else being equal. This highlights the critical role of early and thorough identification of procurement risks in enhancing supplier decisions.

Risk Mitigation followed closely, with a standardized beta of 0.451, a t-value of 7.734, and a p-value of 0.000. This result suggests that improved risk mitigation efforts, such as contingency planning and preventive controls, are strongly associated with better supplier choices. Here, a unit improvement in risk mitigation practices is associated with a 0.451-unit increase in the effectiveness of supplier selection. The effect of Risk Monitoring and Control was also significant, though slightly lower, with a standardized beta of 0.301, a t-value of 6.361, and a p-value of 0.001. This indicates that consistent monitoring and control mechanisms contribute positively to supplier selection, with each unit increase in monitoring efforts resulting in a 0.301-unit improvement in supplier outcomes.

Finally, Risk Assessment, though the weakest among the predictors, still showed a meaningful impact. It had a standardized beta of 0.187, a t-value of 4.231, and a p-value of 0.003, which is statistically significant. This implies that, while risk assessment plays a comparatively smaller role, it remains a relevant factor in supplier selection. A unit improvement in risk assessment correlates with a 0.187-unit increase in supplier selection effectiveness.

4.3 Post-Diagnostic Tests of the Regression Model

To ensure the regression results are reliable, post-diagnostic tests were conducted. The Durbin–Watson test checked for autocorrelation, the Breusch–Pagan test assessed heteroscedasticity, and the Ramsey RESET test examined model specification. These tests confirmed that the assumptions of linear regression are met, supporting the robustness of the model linking risk management practices to supplier selection in LBRBDA.

i. Durbin-Watson Test for Autocorrelation

The Durbin–Watson statistic for the regression model as earlier presented in Table 4 is 1.984. This test examined whether residuals from the regression are independent or showed autocorrelation. The acceptable range for Durbin–Watson is roughly 1.5 to 2.5 for no serious autocorrelation concerns. Since 1.984 falls within this range, it indicates that residuals are independent, and there is no evidence of autocorrelation in the regression model predicting supplier selection from risk management practices in LBRBDA.

Table 9: Breusch-Pagan Test for Heteroschedasticity

Test statistics	Df	p-values	Decision
6.120	4	0.191	No evidence of autocorrelation

Source: SPSS Output of Researchers' Computations, 2025.

The Breusch–Pagan test in Table 9 evaluates whether the variance of residuals is constant (homoscedasticity). The test statistic here is $\chi^2 = 6.120$ with a p-value of 0.191.

The null hypothesis assumes homoscedasticity (constant variance). Since the p-value > 0.05, the null hypothesis is not rejected. This means there is no evidence of heteroscedasticity, confirming that the model residuals have constant variance across all levels of the predictors.

Table 10: Ramsey RESET Test (Model Specification Check)

Test statistics (F)	Df1	Df2	p-values	Decision
1.137	3	83	0.257	No model misrepresentation

Source: SPSS Output of Researchers' Computations, 2025.

The Ramsey RESET test assesses whether the model is correctly specified, particularly checking for omitted variables or incorrect functional form. The test statistic is F = 1.370 with df1 = 3 and df2 = 83, and a p-value of 0.257. The null hypothesis assumes no misspecification.

Since $p > 0.05$, we fail to reject the null, indicating that the model is correctly specified and no evidence of omitted variables or functional form issues exists. In a nutshell, since all three post-diagnostic tests confirm that the regression model is robust: residuals are independent, variance is constant, and the model specification is appropriate. This strengthens confidence in the regression results linking risk management practices to supplier selection in LBRBDA.

4.4 Test of hypotheses and discussion of findings

In line with the aim of investigating the effect of risk management practices on supplier selection within the Lower Benue River Basin Development Authority (LBRBDA), four null hypotheses were tested using regression analysis. The objective was to determine whether specific components of risk management (namely, risk identification, risk assessment, risk mitigation, and risk monitoring and control) exert any statistically significant influence on supplier selection decisions.

Ho_1: Risk identification has no significant effect on supplier selection in LBRBDA

The regression result revealed that risk identification demonstrated the strongest influence on supplier selection, with a standardized beta coefficient of 0.628, a t-value of 9.467, and a p-value of 0.000. This means that for every unit increase in the application of effective risk identification practices, there is a corresponding and substantial increase in the quality and appropriateness of supplier selection.

The extremely low p-value confirms that this effect is statistically significant and not due to chance. Therefore, the null hypothesis which stated that risk identification has no significant effect on supplier selection is rejected. This finding aligns strongly with Anozie *et al*. (2024), who showed that early risk identification, particularly relating to financial and logistical weaknesses, helped procurement teams in Nigeria select more reliable suppliers.

Similarly, Kraljic (2022) emphasized that firms which proactively identified strategic and operational risks were more effective in supplier prioritization within high-risk procurement categories. Smeltzer and Siferd (2020) also affirmed that early identification of raw material and supply volatility risks enabled better supplier selection in U.S. infrastructure projects.

The current study expands on this by highlighting its applicability in the Nigerian context, where infrastructural bottlenecks and political instability make risk identification even more critical.

Ho_2: Risk mitigation has no significant effect on supplier selection in LBRBDA

The regression results showed that risk mitigation also had a meaningful impact on supplier selection. With a beta coefficient of 0.451, a t-value of 7.734, and a p-value of 0.000, the results indicate a significant and positive relationship. This suggests that improvements in risk mitigation strategies lead to better-informed and more resilient supplier choices.

The rejection of the null hypothesis in this case supports the idea that proactive mitigation of supply-related risks contributes considerably to the selection process.

This outcome is in line with Sanders (2023), who developed a hybrid framework combining bankruptcy models with multicriteria scoring, which successfully assessed financial risk at the plant level. The framework proved instrumental in enhancing supplier selection, particularly during crises.

Likewise, Jones *et al*. (2022) and Trkman and McCormack (2021) found that robust risk assessment techniques (e.g., scenario analysis and stress testing) enhanced procurement decisions by aligning suppliers' capabilities with specific project risks. The current study confirms that such alignment reduces future disruptions and promotes smoother project delivery.

Ho_3: Risk monitoring and control have no significant effect on supplier selection in LBRBDA

The Influence of risk monitoring and control was also statistically significant, though to a slightly lesser extent. This variable yielded a standardized beta of 0.301, with a t-value of 6.361 and a p-value of 0.001. This finding implies that consistent tracking and control of procurement-related risks help maintain supplier performance standards and ensure compliance throughout the contract cycle. The significance level again warrants the rejection of the corresponding null hypothesis.

This finding is strongly supported by Li *et al*. (2021), whose study on 30 infrastructure projects in China demonstrated that real-time risk monitoring contributed to early detection of issues and strategic supplier engagement.

Zimmer *et al*. (2016) also confirmed that regular risk monitoring significantly improved supplier performance in European supply chains. Additionally, El-Diraby*et al*. (2022) found that collaborative monitoring between suppliers and procurement officers fostered risk transparency, improved compliance, and reduced project disruptions in Canadian infrastructure projects.

H_{O4}: Risk assessment has no significant effect on supplier selection in LBRBDA

Risk assessment, though the weakest among the four variables, was still found to have a significant positive effect on supplier selection. The beta coefficient stood at 0.187, with a t-value of 4.231 and a p-value of 0.003. This indicates that even though its impact is more modest, proper evaluation and analysis of potential risks still play a valuable role in informing supplier decisions. The p-value being below the 0.05 threshold confirms the statistical relevance of this variable, and thus, the null hypothesis is also rejected. This finding is supported by Hernandez *et al*. (2019) who found that suppliers who actively responded to monitored risks, particularly those related to environmental and safety compliance, received more favorable evaluations during project delivery. Additionally, Trkman and McCormack (2021) supported this finding by stating that assessing to match supplier capacity to project-specific risks significantly reduced procurement delays and enhanced project efficiency.

5 Conclusion and recommendations

5.1 Conclusion

This study investigated the effect of risk management practices on supplier selection in the Lower Benue River Basin Development Authority (LBRBDA), Makurdi. The findings confirmed that all four dimensions of risk management (risk identification, risk assessment, risk mitigation, and risk monitoring and control) significantly influence supplier selection decisions within the organization. Among these, risk identification emerged as the most influential factor, followed by risk mitigation, then risk monitoring and control, with risk assessment having the least but still significant effect. The study concludes that the success of supplier selection in LBRBDA depends largely on the organization's ability to proactively identify potential risks, evaluate their implications, design mitigation strategies, and continuously monitor procurement processes. These practices ensure that suppliers selected are not only cost-effective but are also reliable, compliant, and aligned with long-term organizational goals.

5.2 Recommendations

Based on the findings of this study, several recommendations are advanced to strengthen procurement practices at the Lower Benue River Basin Development Authority (LBRBDA) and similar public-sector organizations. First, since risk identification demonstrated the strongest influence on supplier selection, LBRBDA should invest in structured tools and training programs that enhance the early detection of potential procurement and supplier-related risks. Risk mitigation also emerged as a significant factor, and it is therefore recommended that the Authority formulate and institutionalize strategies such as risk-sharing contracts, supplier diversification, and insurance mechanisms to minimize vulnerabilities. Furthermore, effective risk monitoring and control are essential in ensuring supplier compliance with performance standards; hence, the adoption of automated monitoring systems supported by key performance indicators (KPIs) would allow for continuous evaluation of supplier reliability. Although risk assessment exhibited the weakest predictive effect among the dimensions, it remains a critical practice. Procurement officers should therefore be adequately trained to apply both quantitative and qualitative tools—such as risk matrices, impact-likelihood charts, and SWOT analyses—to support more robust and informed supplier decisions.

While these recommendations are tailored to the institutional realities of LBRBDA, their relevance extends to other public procurement agencies operating under similar conditions of uncertainty, regulatory oversight, and supplier performance variability. The central implication is that risk management should be embedded within supplier selection as a standard practice, regardless of organizational context. However, the extent to which each recommendation applies will depend on contextual factors such as institutional capacity, procurement maturity, and regulatory frameworks. Thus, the recommendations are both locally grounded and potentially generalisable, offering a practical framework that can be adapted by public-sector organizations across developing economies seeking to strengthen procurement resilience and supplier performance.

5.3 Implications of the Findings

The findings of this study carry important theoretical implications for both Risk Management and Supplier Selection theories. From a risk management perspective, the results lend empirical support to the Risk-Based Procurement Approach, which emphasizes proactive risk identification and mitigation as central to procurement success. The strong predictive effect of risk identification and mitigation demonstrates that risk management is not merely a supporting function but a determinant factor in procurement decision-making, thereby extending the theoretical argument that procurement outcomes are enhanced when risk considerations are integrated at the front end of supplier selection. This reinforces the proposition that risk management theory should be broadened beyond its traditional focus on financial or operational safeguards to include strategic supplier evaluation as a mechanism for minimizing procurement vulnerabilities.

In relation to supplier selection theory, the findings substantiate the claim that supplier evaluation is multidimensional, shaped not only by cost and technical competence but also by the supplier's ability to withstand and respond to risk. The results confirm that the dimensions of risk management—particularly risk identification and mitigation—directly inform supplier selection, thus validating supplier selection theory's emphasis on systematic evaluation of capacity, past performance, and risk exposure. Moreover, by showing that risk monitoring and assessment, though weaker, still significantly contribute to supplier decisions, this study advances supplier selection theory by embedding risk-awareness as an integral criterion in evaluating supplier suitability.

Taken together, the study bridges both theoretical domains by demonstrating that risk management practices are not external to supplier selection but are embedded within it, suggesting that future theoretical models should treat supplier selection as a risk-informed process rather than a neutral evaluation of supplier attributes. This theoretical integration provides a richer understanding of how procurement performance can be enhanced in uncertain public-sector environments.

5.4 Limitations of the Study

Despite offering valuable insights into the relationship between risk management practices and supplier selection in the public sector, this study is not without limitations. First, the research focused on a single institution—the Lower Benue River Basin Development Authority (LBRBDA)—which may limit the generalisability of the findings to other public procurement agencies with different institutional structures or regulatory environments. Second, the study relied on self-reported data collected through structured questionnaires, which may be subject to social desirability bias or perceptual differences among respondents. Although steps were taken to ensure validity and reliability, the use of perceptual measures rather than objective procurement performance data presents inherent limitations. Third, the cross-sectional design captures relationships at a single point in time and does not account for potential changes in risk management practices or supplier performance over time. Longitudinal or panel data could provide a more robust understanding of causal dynamics. Finally, while multiple regression analysis was applied to test the hypothesized relationships, other advanced modeling techniques, such as structural equation modeling (SEM), might have offered deeper insights into the mediating or moderating effects between variables.

Moreover, the use of a Likert scale, while providing a standardized measurement, is not without challenges. It assumes equidistance between response options, may be prone to ambiguity in interpretation, and is susceptible to social desirability or acquiescence bias. However, these risks were minimized by employing multiple items (ranging from four to six per construct) to capture each variable more comprehensively. Despite this precaution, future studies could complement Likert-based surveys with qualitative methods, such as interviews or procurement audits, to triangulate responses and reduce bias.

An additional observation is that the mean scores of all four independent variables were relatively close, ranging from 3.68 to 3.87. This clustering may partly reflect socially desirable or politically correct responses, given the sensitivity of procurement issues in public institutions. While the regression results are robust, this raises the need for future studies to integrate objective indicators such as Key Risk Indicators (KRIs), procurement performance audits, or archival supplier performance data. Such measures would provide a stronger evidence base to validate self-reported practices and offer deeper insights into procurement risk management dynamics.

These limitations do not undermine the study’s contributions but highlight areas where future research could build upon the current findings.

5.5 Suggested Areas for Further Studies

Building on the findings of this study, future research could extend the scope beyond the Lower Benue River Basin Development Authority (LBRBDA) to other Ministries, Departments, and Agencies (MDAs) in order to determine whether the influence of risk management practices on supplier selection is consistent across diverse institutional contexts. Comparative studies across multiple agencies would provide broader insights into how organizational structures, regulatory compliance levels, and procurement maturity shape the role of risk management in supplier evaluation. In addition, subsequent research could move beyond the selection phase to explore the longitudinal impact of risk management practices on post-selection outcomes, such as supplier performance, contract execution success, and long-term value for money. Such extensions would enrich the literature by linking risk-aware supplier selection decisions to the sustainability and effectiveness of public procurement outcomes.

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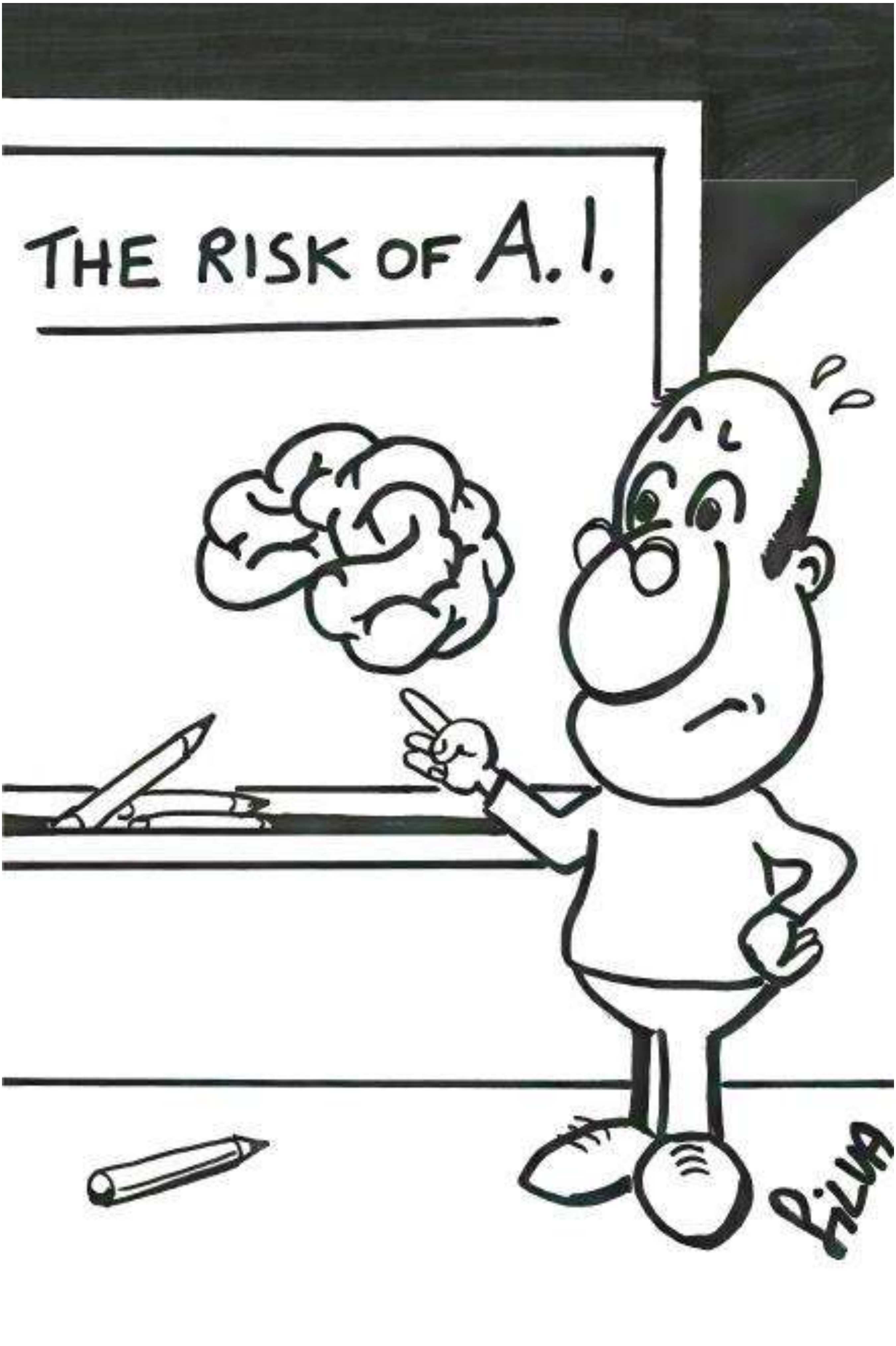
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