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Multi-model credit rating system for SMEs

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Multi-model credit rating system for SMEs: a deterministic methodology of arrangements with repetitions

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Abstract

The present document analyzes a proprietary methodological framework, developed in 2015, based on a deterministic approach for the temporal normalization of the scores on a two-year basis and for the determination of the final rating. The distinctive element of the system lies in the use of "*arrangements with repetitions*" to codify the expert decision rules for multi-model aggregation, in a rigorous logical structure. The system integrates five distinct scoring methodologies (SME Z-Score, Bank Score, EM Score, MCC Rating, Company Performance Index) reaching a final rating through an aggregation process that combines quantitative, qualitative and performance components.

Supplementary materials*: the technical appendices and spreadsheets supporting the analyses presented in the present work are available upon request from the author / or from the following repository:*
https://drive.google.com/drive/folders/1Xug079LzCeq1bBYpLaN1FhdQ6PoiHvvS

Keywords: credit rating, SME, arrangements with repetitions, time normalization, multi-model system, risk assessment.

1) Introduction

1.1) Context and purpose of the document

The multi-model rating system presented here represents a methodological approach developed about ten years ago for assessing the creditworthiness of Italian SMEs.
This document aims to analytically illustrate the implemented methodology, focusing on three fundamental elements:

- the engine for determining the scores of the individual models.
- the system of temporal normalization on a two-year basis through arrangements with repetitions.
- the final aggregation process.

The system should be contextualized in its development period (2014-2015), a historical phase characterized by:

- Pre-IFRS 9: the accounting standard IFRS 9 (International Accounting Standards Board, 2014), with full implementation in 2018, subsequently introduced requirements for probabilistic estimation of the Probability of Default (PD); this system was developed when current legislation favored rule-based approaches for unlisted SMEs.
- Data scarcity in the SME segment: the absence of deep historical series for small and medium-sized Italian companies made the application of Machine Learning models problematic, which would have suffered from "overfitting" on limited datasets; in this scenario, the deterministic approach based on expert rules therefore represented the most methodologically reliable solution that adhered to the operational context.
- Best practices pre-European banking crisis: the system reflects the risk management logics prevalent in the Italian banking system before the full implementation of Basel III (Basel Committee on Banking Supervision, 2010) and the EBA Guidelines (European Banking Authority, 2020).

The introduction on the historical context and the methodological paradigm that guided the development of the model is fundamental to understand that the contribution of the present work does not lie in the specific calibration of the parameters (which would require updating) but rather in the proposed methodological vision: i.e., adopting the logical architecture of the arrangements with repetitions as a multi-model aggregation tool with temporal memory.
This methodology is based on the principle that the combination of different models, each with its own strengths and specificities, as well as weighting on a two-year basis, can produce better results and reduce the average error compared to using any individual model. The multi-model approach would be inherently more robust as all models are unlikely to be simultaneously affected by the same anomalies. The combination of these models also provides a more complete and multidimensional assessment of credit risk.

1 The opinions expressed in the present work are attributable exclusively to the author.
The methodological framework described is the result of independent research developed and formalized by the author in the period 2014–2015; it is not based on software, proprietary methodologies, confidential databases or confidential know-how belonging to third parties.

1.2) Structure of the document

The present document is organized to allow a complete understanding of the model through four main sections:

1. Score models: analytical description of the five models implemented.
2. Arrangements with repetitions: illustration of the "*original methodology*" for temporal normalization.
3. Final aggregation: description of the weighting system and determination of the final rating.
4. Operational considerations: analysis of the implementation and usage aspects.

Each section is analytically documented, reporting formulas, calculation logic and evaluation criteria exactly as implemented in the original system developed in a Microsoft Windows environment, using the Microsoft Excel software and the connected Visual Basic for Application (VBA).

2) Score models

2.1) SME Z-Score Model

The SME Z-Score represents the adaptation of the famous Altman model (1968) specifically calibrated for small and medium-sized enterprises. The model uses a linear formula that combines five standardized financial variables to produce a score that is then converted into a rating classification.

Formula: $Z = 1.981 \times X_1 + 9.841 \times X_2 + 1.951 \times X_3 + 3.206 \times X_4 + 4.037 \times X_5$

2.1.2) Specific variables and formulas

X_1 - Flexibility Assets Rate:

$$X_1 = \frac{\text{Net Working Capital}}{\text{Total Assets}}$$

X_2 - Return on Retained Earnings (RORE):

$$X_2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}$$

X_3 - Return on Investment (ROI):

$$X_3 = \frac{\text{EBIT}}{\text{Total Assets}}$$

X_4 - Financial Autonomy Ratio:

$$X_4 = \frac{\text{Equity}}{\text{Total Liabilities}}$$

X_5 - Return on Sales Activity (ROSA):

$$X_5 = \frac{\text{Revenues}}{\text{Total Assets}}$$

2.1.3) Classification system

The Z-Score is converted into a rating through predefined thresholds:

$$\text{Classification}_Z = \begin{cases} \text{Low Probability of Default} & \text{if } Z > 8{,}105 \\ \text{Medium Probability of Default} & \text{if } 4{,}846 < Z \leq 8{,}105 \\ \text{High Probability of Default} & \text{if } Z \leq 4{,}846 \end{cases}$$

2.2) Bank Score

2.2.1) Discrete score structure

The Bank Score uses a discrete scoring system organized into four macro-assessment areas that cover all aspects relevant to credit assessment.
Each macro-area includes four specific indicators, for a total of twelve variables, each evaluated on a discrete scale that assigns scores from 0 to 3.

2.2.2) Macro-Area 1: Profitability and dynamics indicators

Operating profitability (1.1):

$$\text{Profitability} = \frac{\text{EBITDA}}{\text{Turnover}}$$

Score:

$$S_{\text{Profitability}} = \begin{cases} 3 & \text{if Profitability} \geq 15\% \\ 2 & \text{if } 10\% \leq \text{Profitability} < 15\% \\ 1 & \text{if } 5\% \leq \text{Profitability} < 10\% \\ 0 & \text{if Profitability} < 5\% \end{cases}$$

Self-financing capacity (1.2):

$$\text{Self} - \text{financing capacity} = \frac{\text{Cashflow}}{\text{Turnover}}$$

Score:

$$S_{\text{Self-financing}} = \begin{cases} 3 & \text{if Self-financing} \geq 15\% \\ 2 & \text{if } 8\% \leq \text{Self-financing} < 15\% \\ 1 & \text{if } 4\% \leq \text{Self-financing} < 8\% \\ 0 & \text{if Self-financing} < 4\% \end{cases}$$

Incidence of financial charges (1.3):

$$\text{Incidence of Financial charges} = \frac{\text{Financial Charges}}{\text{EBITDA}}$$

Score:

$$S_{\textbf{IncidenceFinancialCharges}} = \begin{cases} 0 & \text{if Incidence of Financial charges} < 0 \\ 3 & \text{if } 0 \leq \text{Incidence of Financial charges} \leq 20\% \\ 2 & \text{if } 20\% < \text{Incidence of Financial charges} \leq 40\% \\ 1 & \text{if } 40\% < \text{Incidence of Financial charges} \leq 60\% \\ 0 & \text{if Incidence of Financial charges} > 60\% \end{cases}$$

ROE (1.4):

$$\text{ROE} = \frac{\text{Net Income}}{\text{Shareholder's equity}}$$

Score:

$$\text{ROE Score} = \begin{cases} 3 & \text{if } ROE \geq 10\% \\ 2 & \text{if } 7{,}5\% \leq ROE < 10\% \\ 1 & \text{if } 3\% \leq ROE < 7{,}5\% \\ 0 & \text{if } ROE < 3\% \end{cases}$$

2.2.3) Macro-Area 2: Debt indicators

Financial debt ratio or Debt to sales ratio (2.1):

$$\text{Debt to Sales Ratio} = \frac{\text{Financial Debt}}{\text{Turnover}}$$

Score:

$$S_{\text{Debt to Sales Ratio}} = \begin{cases} 3 & \text{if Debt to Sales Ratio} \leq 50\% \\ 2 & \text{if } 50\% < \text{Debt to Sales Ratio} \leq 75\% \\ 1 & \text{if } 75\% < \text{Debt to Sales Ratio} \leq 100\% \\ 0 & \text{if Debt to Sales Ratio} > 100\% \end{cases}$$

Financial debt ratio (Current operations) (2.2):

$$\text{Current Debt Ratio} = \frac{\text{Net Financial Debt}}{\text{EBITDA}}$$

Score:

$$S_{\text{Current Debt Ratio}} = \begin{cases} 3 & \text{if Current Debt Ratio} \leq 4 \\ 2 & \text{if } 4 < \text{Current Debt Ratio} \leq 5{,}5 \\ 1 & \text{if } 5{,}5 < \text{Current Debt Ratio} \leq 7 \\ 0 & \text{if Current Debt Ratio} > 7 \end{cases}$$

Financial debt to equity Ratio (2.3):

$$\text{Net Debt to Equity} = \frac{\text{Net Financial Debt}}{\text{Shareholder's equity}}$$

Score:

$$S_{\text{Net Debt to Equity}} = \begin{cases} 3 & \text{if Net Debt to Equity} \leq 25\% \\ 2 & \text{if } 25\% < \text{Net Debt to Equity} \leq 32{,}5\% \\ 1 & \text{if } 32{,}5\% < \text{Net Debt to Equity} \leq 40\% \\ 0 & \text{if Net Debt to Equity} > 40\% \end{cases}$$

Working capital intensity Ratio (on turnover) (2.4):

$$\text{Working capital intensity Ratio} = \frac{\text{Current Assets}}{\text{Turnover}}$$

Score:

$$S_{\text{Working capital intensity}} = \begin{cases} 3 & \text{if Working capital intensity} > 1{,}2 \\ 2 & \text{if } 0{,}8 \leq \text{Working capital intensity} \leq 1{,}2 \\ 1 & \text{if } 0{,}6 \leq \text{Working capital intensity} < 0{,}8 \\ 0 & \text{if Working capital intensity} < 0{,}6 \end{cases}$$

2.2.4) Macro-Area 3: Capital structure and financial stability Ratios

Fixed asset coverage ratio QCI or Long-Term Solvency Ratio (3.1):

$$\text{QCI} = \frac{\text{Shareholder's equity} + \text{MLT Debt}}{\text{Fixed Assets}}$$

Score:

$$\text{QCI Score} = \begin{cases} 3 & \text{if } QCI > 1{,}5 \\ 2 & \text{if } 1 < QCI \leq 1{,}5 \\ 1 & \text{if } 0{,}5 < QCI \leq 1 \\ 0 & \text{if } QCI \leq 0{,}5 \end{cases}$$

Current Ratio CTR (3.2):

$$CTR = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

Score:

$$S_{\text{CTR}} = \begin{cases} 3 & \text{if CTR} \geq 2 \\ 2 & \text{if } 1{,}5 \leq \text{CTR} < 2 \\ 1 & \text{if } 1 \leq \text{CTR} < 1{,}5 \\ 0 & \text{if CTR} < 1 \end{cases}$$

Days Sales Outstanding (3.3):

$$\textbf{Days Sales Outstanding} = \frac{\text{Trade Receivables} \times 365}{\text{Turnover}}$$

Score:

$$S_{\text{Days Sales Outstanding}} = \begin{cases} 3 & \text{if Days Sales Outstanding} \leq 120 \\ 2 & \text{if } 120 < \text{Days Sales Outstanding} \leq 150 \\ 1 & \text{if } 150 < \text{Days Sales Outstanding} \leq 180 \\ 0 & \text{if Days Sales Outstanding} > 180 \end{cases}$$

Days Inventory Outstanding (3.4):

$$\text{Days Inventory Outstanding} = \frac{\text{Inventory} \times 365}{\text{Turnover}}$$

Score:

$$S_{\text{Days Inventory Outstanding}} = \begin{cases} 3 & \text{if Days Inventory Outstanding} \leq 120 \\ 2 & \text{if } 120 < \text{Days Inventory Outstanding} \leq 150 \\ 1 & \text{if } 150 < \text{Days Inventory Outstanding} \leq 180 \\ 0 & \text{if Days Inventory Outstanding} > 180 \end{cases}$$

2.2.5) Final score calculation

The determination of the score with the attribution of the relative alphabetical score follows the logic of the arrangements with repetitions which are referred to in the dedicated section (chapter 3).

2.3) EM Score Model

2.3.1) Alphabetical classification system

The EM Score model refers to the methodology developed by Altman *et al* (2010) for emerging markets, adapted to the context of Italian small and medium-sized enterprises. The model uses an approach that combines four fundamental indicators calculated on a time basis to identify development trends and predictive patterns.

2.3.2) Calculation formulas

The model builds a composite score that is subsequently converted into a 19-level alphabetical rating, from AAA to D, offering greater evaluation granularity compared to traditional models. Formula:

$$EM = 3{,}25 + 6{,}56 \times X_1 + 3{,}26 \times X_2 + 6{,}72 \times X_3 + 1{,}05 \times X_4$$

***Specific variables and formulas**:*

The variables X_1, X_2, X_3, X_4, are variables identical to those of the SME Z-Score model, to which the reader should refer.

2.3.3) Classification system

The EM-Score is converted into alphabetical classification through predefined thresholds that identify 19 rating ranges:

$$Rating_{EM}^{(1)} = \begin{cases} D & \text{if } EM < 1{,}75 \\ CCC^- & \text{if } 1{,}75 \leq EM < 2{,}5 \\ CCC & \text{if } 2{,}5 \leq EM < 3{,}2 \\ CCC^+ & \text{if } 3{,}2 \leq EM < 3{,}75 \end{cases}$$

$$Rating_{EM}^{(2)} = \begin{cases} B^- & \text{if } 3{,}75 \leq EM < 4{,}15 \\ B & \text{if } 4{,}15 \leq EM < 4{,}5 \\ B^+ & \text{if } 4{,}5 \leq EM < 4{,}75 \\ BB^- & \text{if } 4{,}75 \leq EM < 4{,}95 \\ BB & \text{if } 4{,}95 \leq EM < 5{,}25 \\ BB^+ & \text{if } 5{,}25 \leq EM < 5{,}65 \\ BBB^- & \text{if } 5{,}65 \leq EM < 5{,}85 \\ BBB & \text{if } 5{,}85 \leq EM < 6{,}25 \\ BBB^+ & \text{if } 6{,}25 \leq EM < 6{,}4 \end{cases}$$

$$Rating_{EM}^{(3)} = \begin{cases} A^- & \text{if } 6{,}4 \leq EM < 6{,}65 \\ A & \text{if } 6{,}65 \leq EM < 6{,}85 \\ A^+ & \text{if } 6{,}85 \leq EM < 7 \\ AA^- & \text{if } 7 \leq EM < 7{,}3 \\ AA & \text{if } 7{,}3 \leq EM < 7{,}6 \\ AA^+ & \text{if } 7{,}6 \leq EM < 8{,}15 \\ AAA & \text{if } EM \geq 8{,}15 \end{cases}$$

2.3.4) Categorization by risk class

The 19 alphabetical ratings are further aggregated into four macro-classes which summarize the level of credit reliability and the risk profile of the company.

Score Class:

$$Class = \begin{cases} \text{HIGH} & \text{if Rating} \in \{AAA, AA^+, AA, AA^-, A^+, A, A^-\} \\ \text{MEDIUM HIGH} & \text{if Rating} \in \{BBB^+, BBB, BBB^-\} \\ \text{MEDIUM LOW} & \text{if Rating} \in \{BB^+, BB, BB^-\} \\ \text{LOW} & \text{if Rating} \in \{B^+, B, B^-, CCC^+, CCC, CCC^-, D\} \end{cases}$$

Risk level:

$$Risk = \begin{cases} \text{MINIMUM} & \text{if Rating} = AAA \\ \text{MODEST} & \text{if Rating} \in \{AA^+, AA, AA^-\} \\ \text{LOW} & \text{if Rating} \in \{A^+, A, A^-\} \\ \text{ACCEPTABLE} & \text{if Rating} \in \{BBB^+, BBB, BBB^-\} \\ \text{ACCEPTABLE WITH CAUTION} & \text{if Rating} \in \{BB^+, BB, BB^-\} \\ \text{SPECIFIC CAUTION} & \text{if Rating} \in \{B^+, B, B^-, CCC^+\} \\ \text{UNDER STRICT ATTENTION} & \text{if Rating} \in \{CCC, CCC^-, D\} \end{cases}$$

2.4) MCC Rating

2.4.1) Sector based differentiation

The MCC Rating model corresponds to the evaluation methodology of the Guarantee Fund for Small and Medium-Sized Enterprises (Fondo di Garanzia per le Piccole e Medie Imprese, Law 662/1996), as documented in the Fund's Operational Provisions, Part VI "Economic-Financial Evaluation Criteria", operational version until the reform of March 2019 (Mediocredito Centrale, MCC, 2012)[2].

The model includes three sectoral configurations:

- A1: Manufacturing industry, construction, hotel owners, fishing.
- A2: Trade, services, tenant hotels.
- A3: Special sectors with annual production cycles.

Methodological note*: the use of the pre-reform version (2012) of the MCC Rating responds to the need for historical consistency since the original backtesting of the model was performed using the parameters and sectoral thresholds in force at that time. To maintain these parameters, under the circumstances here, ensures the integrity of the retrospective empirical validation, avoiding methodological anachronisms.*

2.4.2) Calculation system

The system used four main ratios, each adapted to sector specificities:

[2] Since 2017, the Guarantee Fund has used a completely reformed rating system compared to the one proposed.

1. Fixed assets coverage ratio (A1) / Current ratio (A2, A3).
2. Financial autonomy (differentiated by sector).
3. Liquidity (A1, A2) / Current assets turnover ratio (A3).
4. Incidence of operating activities on turnover.

The specific thresholds for each sector configuration are reported in the official Operational Provisions of the Guarantee Fund[3].

2.4.3) Final score

The final score is the sum of the 4 indicators (maximum 12 points):

- 10-12 points: Rating A (HIGH quality).
- 7-9 points: Rating B (MEDIUM quality).
- 0-6 points: Rating C (LOW quality).

The two-year normalization of the MCC rating takes place based on the range assigned by the model: the first range translates into the assignment of rating A, the second range translates into rating B and the third range into rating C.

2.5) Company performance index

2.5.1) Origin and transformation of the M-Index

The Company Performance Index represents the operational transformation of the theoretical intuition of the M-Index developed by Mella *et al* (2011), its conceptual framework having evolved into a quantitative system with precise thresholds and automatic scoring algorithms.

2.5.2) System structure: year score (static component)

The system evaluates company performance through five macro-areas, each with specific indicators.
The static component (year score) measures the exclusive performance of the year to which it refers (year n).
Methodological note*: a few indicators are evaluated solely through two-year variations (dynamic component) and therefore do not present a static score. The relevant formulas are described in Paragraph 2.5.3*

2.5.2.1) Macro-Area 1: Ratios on Balance in the capital structure

Fixed asset coverage ratio QCI (a):

$$\text{QCI} = \frac{\text{Shareholder's equity} + \text{MLT Debt}}{\text{Fixed Assets}}$$

Score:

$$\text{QCI Score} = \begin{cases} 15 & \text{if } QCI > 1{,}5 \\ 10 & \text{if } 1 < QCI \leq 1{,}5 \\ 5 & \text{if } 0{,}5 < QCI \leq 1 \\ 0 & \text{if } QCI \leq 0{,}5 \text{ or } QCI = 0 \end{cases}$$

Quick Ratio QTR (b)*:*

$$QTR = \frac{\text{Current Assets} - \text{Inventory}}{\text{Current Liabilities}}$$

Score:

$$\text{QTR Score} = \begin{cases} 15 & \text{if } QTR > 1{,}5 \\ 10 & \text{if } 1 < QTR \leq 1{,}5 \\ 5 & \text{if } 0{,}5 < QTR \leq 1 \\ 0 & \text{if } QTR \leq 0{,}5 \end{cases}$$

[3] The 2012 operational provisions are available in the historical archive of the Guarantee Fund (www.fondidigaranzia.it).

Current Ratio CTR (c):

$$CTR = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

Score:

$$\text{CTR Score} = \begin{cases} 15 & \text{if } CTR > 2{,}2 \\ 10 & \text{if } 2 < CTR \leq 2{,}2 \\ 5 & \text{if } 1{,}5 \leq CTR \leq 2 \\ 0 & \text{if } CTR < 1{,}5 \end{cases}$$

2.5.2.2) Macro-Area 2: Profitability indicators and structure of the Income Statement

1.1 Financial Leverage Relationship (RFF)

The assessment simultaneously integrates five components:

- ROI
- RODF
- SPREAD: difference between ROI-RODF
- DER: Debt-to-Equity Ratio (Financial Debts/Equity)
- RO: Operating income

Score:

$$\text{RFF Score} = \begin{cases} 15 & \text{if } ROI > 0 \wedge \text{SPREAD} > 0 \wedge DER < 1 \\ 10 & \text{if } ROI > 0 \wedge \text{SPREAD} > 0 \wedge DER > 1 \\ 5 & \text{if } ROI > 0 \wedge \text{SPREAD} < 0 \wedge DER < 1 \\ 0 & \text{if } ROI > 0 \wedge \text{SPREAD} < 0 \wedge DER > 1 \\ -5 & \text{if } ROI < 0 \wedge RO < 0 \end{cases}$$

Methodological note: *the additional Ratios of Macro-Area 2 (Operating Income, Incidence of Financial Charges on the Value of Production, Financial Debts/Value of Production Ratio) are assessed in Section 2.5.3.*

2.5.2.3) Macro-Area 3: EBITDA

Methodological note: *assessed exclusively in Section 2.5.3 (dynamic component).*

2.5.2.4) Macro-Area 4: relationship between net income and PONT

Components of the relationship:

- 4.1) PONT (Net Non-Recurring Items)
- 4.2) Net Income (RN)
- 4.3) Profit before taxes (RAI)
- 4.4) Net Operating Profit (EBIT)

Score:

$$\text{RN-PONT Score} = \begin{cases} 10 & \text{if } RN > 0 \wedge PONT < 0 \wedge EBIT > 0 \\ 5 & \text{if } PONT > 0 \wedge RN > 0 \wedge PONT < RN \wedge EBIT > PONT \\ 0 & \text{if } RN > 0 \wedge PONT > 0 \wedge PONT < RN \wedge PONT < RAI \\ -5 & \text{if } RN > 0 \wedge PONT > 0 \wedge PONT > RAI \\ 0 & \text{if } RN < 0 \wedge PONT < 0 \\ -5 & \text{if } RN < 0 \wedge PONT > 0 \\ -5 & \text{if } RN < 0 \wedge EBIT < 0 \wedge PONT \geq 0 \\ 5 & \text{if } RN > 0 \wedge EBIT > 0 \wedge PONT \geq 0 \end{cases}$$

Legend:

- **RN** = Net Income
- **PONT** = Net Non-Recurring Items (Non-Recurring income and expenses, or tax related items)
- **EBIT** = Net Operating Profit (Earnings Before Interest and Taxes)
- **RAI** = Profit before taxes

The formula assesses the quality of company income through: Net Income, Net Non-Recurring Items and EBIT, with a hierarchical logic. It attributes:

- 10 points for pure operating profitability (positive net income and positive EBIT, negative extraordinary balance).
- 5 points when the positive extraordinary components are limited compared to net income and EBIT, or for residual cases with positive income and positive EBIT.
- 0 points for positive income with controlled extraordinary components but lower than the pre-tax result, or for losses with negative extraordinary balances.
- -5 points when the extraordinary components exceed the pre-tax result, or for losses despite positive extraordinary balances, or for operating losses.

The system generates scores from -5 to +10, evaluating the sustainability and origin of the profitability.

2.5.2.5) Macro-Area 5: Incidence of operating cash flow on the value of production

Methodological note*: evaluated exclusively in Section 2.5.3 (Dynamic Component).*

2.5.3) System structure: two-year score (delta from year n to n-1)

The dynamic component evaluates the changes in the indicators between year n and year n-1, capturing the development trend of company performance.

The dynamic score is based on:

- P_n: Current year base score (section 2.5.2).
- D_{n,n-1}: Change in the indicator between year n and year n-1.

2.5.3.1 Fixed Assets Coverage Ratio (QCI) -Dynamic-

Score:

$$\mathrm{QCI}_{\mathrm{dynamic}}\mathrm{Score} = \begin{cases} 15 & \text{if } P_n = 15 \wedge D_{n,n-1} > 0 \\ 15 & \text{if } P_n = 15 \wedge D_{n,n-1} < 0 \wedge D_{n,n-1} > -0{,}025 \\ 10 & \text{if } P_n = 10 \wedge D_{n,n-1} > 0 \\ 10 & \text{if } P_n = 10 \wedge D_{n,n-1} < 0 \wedge D_{n,n-1} > -0{,}025 \\ 5 & \text{if } P_n = 5 \wedge D_{n,n-1} > 0 \\ 5 & \text{if } P_n = 5 \wedge D_{n,n-1} < 0 \wedge D_{n,n-1} > -0{,}025 \\ 0 & \text{if } P_n = 0 \wedge D_{n,n-1} > 0 \\ -5 & \text{if } P_n \in \{15,10,5,0\} \wedge D_{n,n-1} \leq -0{,}025 \end{cases}$$

Where:

$$P_n = \text{QCI Score year } n$$
$$D_{n,n-1} = \text{QCI Score change between year } n \text{ and } n-1$$

2.5.3.2 Quick Test Ratio (QTR) -Dynamic-

Score:

$$
\text{QTR}_{\text{dynamic}}\ \text{Score} = \begin{cases} 0 & \text{if } P_n = 0 \land D_{n,n-1} = 0 \\ 15 & \text{if } P_n = 15 \land D_{n,n-1} > 0 \\ 15 & \text{if } P_n = 15 \land D_{n,n-1} < 0 \land D_{n,n-1} > -0{,}025 \\ 10 & \text{if } P_n = 10 \land D_{n,n-1} > 0 \\ 10 & \text{if } P_n = 10 \land D_{n,n-1} < 0 \land D_{n,n-1} > -0{,}025 \\ 5 & \text{if } P_n = 5 \land D_{n,n-1} > 0 \\ 5 & \text{if } P_n = 5 \land D_{n,n-1} < 0 \land D_{n,n-1} > -0{,}025 \\ 0 & \text{if } P_n = 0 \land D_{n,n-1} > 0 \\ -5 & \text{if } P_n \in \{15,10,5,0\} \land D_{n,n-1} \leq -0{,}025 \end{cases}
$$

Where:

$$
\begin{aligned} P_n &= \text{QTR Score year } n \\ D_{n,n-1} &= \text{QTR Score change between year } n \text{ and } n-1 \end{aligned}
$$

2.5.3.3 SPREAD ROI-RODF -Dynamic-

Score:

$$
\text{SPREAD}_{\text{dynamic}}\ \text{Score} = \begin{cases} 15 & \text{if } P_n = 15 \land D_{n,n-1} > 0 \\ 10 & \text{if } P_n = 10 \land D_{n,n-1} > 0 \\ 5 & \text{if } P_n = 5 \land D_{n,n-1} > 0 \\ 0 & \text{if } P_n = 0 \land D_{n,n-1} > 0 \end{cases}
$$

Where:

$$
\begin{aligned} P_n &= \text{Base Score year } n \text{ assigned to the Financial Leverage Relationship} \\ D_{n,n-1} &= \text{Variation of the ROI-RODF spread from year } n \text{ to } n-1 \end{aligned}
$$

2.5.3.4 RO (Operating Income) –Dynamic-

Score:

$$
\text{RO Score} = \begin{cases} -5 & \text{if } RO_n < 0 \land RO_{n-1} < 0 \\ -2{,}5 & \text{if } RO_n < 0 \land RO_{n-1} > 0 \\ -2{,}5 & \text{if } RO_n > 0 \land RO_{n-1} < 0 \\ 0 & \text{if } RO_n > 0 \land RO_{n-1} > 0 \end{cases}
$$

Where:

$$
\begin{aligned} RO_n &= \text{Operating Income year } n \\ RO_{n-1} &= \text{Operating Income year } n-1 \end{aligned}
$$

2.5.3.5 Incidence of financial charges on the value of production -Dynamic-

Score:

$$
\text{IOF}_{\text{dynamic}}\ \text{Score} = \begin{cases} 15 & \text{if } P_n = 15 \land D_{n,n-1} < 0 \\ 15 & \text{if } P_n = 15 \land 0 < D_{n,n-1} < 0{,}025 \\ 10 & \text{if } P_n = 10 \land D_{n,n-1} < 0 \\ 10 & \text{if } P_n = 10 \land 0 < D_{n,n-1} < 0{,}025 \\ 5 & \text{if } P_n = 5 \land D_{n,n-1} < 0 \\ 5 & \text{if } P_n = 5 \land 0 < D_{n,n-1} < 0{,}025 \\ 0 & \text{if } P_n = 0 \land D_{n,n-1} < 0 \\ -5 & \text{if } D_{n,n-1} \geq 0{,}025 \text{ or } (P_n = 0 \land D_{n,n-1} \geq 0) \end{cases}
$$

Where:

$$
\begin{aligned} P_n &= \text{Score Incidence of financial charges year } n \\ D_{n,n-1} &= \text{Score Variation of Incidence of financial charges from year } n \text{ to } n-1 \end{aligned}
$$

Ratio between financial debts and value of production -Dynamic-

Score:

$$
\mathrm{RDFVP}_{\mathrm{dynamic}}\ \mathrm{Score} = \begin{cases} 15 & \text{if } P_n = 15 \wedge D_{n,n-1} < 0 \\ 15 & \text{if } P_n = 15 \wedge 0 < D_{n,n-1} < 0{,}025 \\ 10 & \text{if } P_n = 10 \wedge D_{n,n-1} < 0 \\ 10 & \text{if } P_n = 10 \wedge 0 < D_{n,n-1} < 0{,}025 \\ 5 & \text{if } P_n = 5 \wedge D_{n,n-1} < 0 \\ 5 & \text{if } P_n = 5 \wedge 0 < D_{n,n-1} < 0{,}025 \\ 0 & \text{if } P_n = 0 \wedge D_{n,n-1} < 0 \\ -5 & \text{if } (P_n = 15 \wedge D_{n,n-1} \geq 0{,}025) \text{ or} \\ & (P_n = 10 \wedge D_{n,n-1} \geq 0{,}025) \text{ or} \\ & (P_n = 5 \wedge D_{n,n-1} \geq 0{,}025) \text{ or} \\ & (P_n = 0 \wedge D_{n,n-1} \geq 0) \end{cases}
$$

Where:

P_n = Score Ratio between financial debts and value of production year n

$D_{n,n-1}$ = Variation of Ratio between financial debts and value of production from year n to $n-1$

2.5.3.6 Gross operating margin (EBITDA) –Dynamic-

Score:

$$
\mathrm{EBITDA}_{\mathrm{dynamic}}\ \mathrm{Score} = \begin{cases} 15 & \text{if } P_n = 15 \wedge D_{n,n-1} > 0 \\ 10 & \text{if } P_n = 10 \wedge D_{n,n-1} > 0 \\ 5 & \text{if } P_n = 5 \wedge D_{n,n-1} > 0 \\ 0 & \text{if } P_n = 0 \wedge D_{n,n-1} > 0 \\ 5 & \text{if } P_n = -5 \wedge D_{n,n-1} > 0 \\ -5 & \text{if } P_n \in \{15,10,5,0\} \wedge D_{n,n-1} < 0 \\ -5 & \text{if } P_n = -5 \wedge D_{n,n-1} < 0 \end{cases}
$$

Where:

P_n = EBITDA Score year n

$D_{n,n-1}$ = Variation of EBITDA Score from year n to $n-1$

2.5.3.7 Relationship between net income and PONT –Dynamic-

Score:

$$
\text{RN-PONT}_{\mathrm{dynamic}}\mathrm{Score} = \begin{cases} 10 & \text{if } P_n = 15 \wedge \Delta PONT < 0 \wedge \Delta RN > 0 \\ 5 & \text{if } P_n = 5 \wedge \Delta PONT > 0 \wedge \Delta RN > 0 \wedge PONT_n > PONT_{n-1} \wedge RAI_n > RAI_{n-1} \\ 0 & \text{if } P_n = 5 \wedge \Delta PONT > 0 \wedge PONT_n < RAI_n \wedge PONT_{n-1} < RAI_{n-1} \\ 0 & \text{if } P_n = 0 \wedge \Delta RN > 0 \wedge \Delta PONT > 0 \wedge PONT_n > RAI_n \wedge PONT_{n-1} > RAI_{n-1} \\ 0 & \text{if } \Delta RN < 0 \wedge \Delta RN < 0 \\ -5 & \text{if } \Delta RN < 0 \wedge \Delta PONT > 0 \end{cases}
$$

Where:

P_n = Score year n attributed to the Relationship between Net Income and PONT

$\Delta PONT$ = Variation of Net Non-Recurring Items from year n to $n-1$

ΔRN = Variation of Net Income from year n to $n-1$

RN_n = Net Income year n

RN_{n-1} = Net Income year $n-1$

$PONT_n$ = Net Non-Recurring Items year n

RAI_n = Profit before taxes year n

RAI_{n-1} = Profit before taxes year $n-1$

2.5.3.8 Incidence of operating cash flow on the value of production -Dynamic-

Score:

$$\text{ICFOVP}_{\text{dynamic}}\ \text{Score} = \begin{cases} 10 & \text{if } P_{RN-PONT} = 10 \wedge \Delta ICFOVP > 0 \\ 10 & \text{if } P_{RN-PONT} = 10 \wedge \Delta ICFOVP < 0 \wedge \Delta ICFOVP > -0{,}025 \\ 5 & \text{if } P_{RN-PONT} = 5 \wedge \Delta ICFOVP > 0 \\ 5 & \text{if } P_{RN-PONT} = 5 \wedge \Delta ICFOVP < 0 \wedge \Delta ICFOVP > -0{,}025 \\ 0 & \text{if } P_{RN-PONT} = 0 \wedge \Delta ICFOVP > 0 \\ 0 & \text{if } P_{RN-PONT} = 10 \wedge \Delta ICFOVP \leq -0{,}025 \\ 0 & \text{if } P_{RN-PONT} = 5 \wedge \Delta ICFOVP \leq -0{,}025 \\ 0 & \text{if } P_{RN-PONT} = 0 \wedge \Delta ICFOVP < 0 \\ -5 & \text{if } P_{RN-PONT} = -5 \wedge \Delta ICFOVP < 0 \\ 5 & \text{if } P_{RN-PONT} = -5 \wedge \Delta ICFOVP > 0 \end{cases}$$

Where:

$P_{RN-PONT}$ = Score year n of the Relationship between Net Income and PONT
$\Delta ICFOVP$ = Variation of Incidence of operating cash flow on the value of production from year n to $n-1$

2.5.4) Classification system

The total score of the Company Performance Index is given by the sum of the scores on an annual basis and those on a two-year basis: $\text{TOTAL SCORE} = \sum \text{Base Score} + \sum \text{Delta Score}$

The final assessment or Total Score is assigned based on the total score according to the following classification:

$$\text{Total Score} = \begin{cases} \text{EXCEPTIONAL} & \text{if Score} \geq 140 \\ \text{VERY GOOD} & \text{if } 120 \leq \text{Score} < 140 \\ \text{GOOD} & \text{if } 90 \leq \text{Score} < 120 \\ \text{NORMAL} & \text{if } 50 \leq \text{Score} < 90 \\ \text{AT RISK} & \text{if } 30 \leq \text{Score} < 50 \\ \text{TO BE RESTRUCTURED} & \text{if } 0 \leq \text{Score} < 30 \\ \text{SEVERE DISTRESS} & \text{if Score} < 0 \end{cases}$$

The Score Class derives from the Total Score:

$$\text{Class} = \begin{cases} A & \text{if Total Score} \in \{\text{EXCEPTIONAL, VERY GOOD}\} \\ B & \text{if Total Score} \in \{\text{GOOD, NORMAL}\} \\ C & \text{if Total Score} \in \{\text{AT RISK, TO BE RESTRUCTURED}\} \\ D & \text{if Total Score} = \text{SEVERE DISTRESS} \end{cases}$$

3) System of arrangements with repetitions

3.1) Mathematical foundations of the system of arrangements with repetitions

The system of arrangements with repetitions constitutes the innovative methodological core of the model[4], operating on three levels:

1. determination of the annual score for the Bank Score.
2. two-year normalization for each of the score models, SME Z-Score, Bank Score, EM Score.
3. concatenation and final aggregation of the five two-year scores SME Z-Score, Bank Score, EM Score, MCC Rating, Company Performance Index, normalized into a single quantitative score.

Arrangements with repetitions of n elements, taken k at a time, are defined mathematically as: D(n,k)= n^k
This formula represents the number of ways to choose k elements from a set of n elements.

4 The application of the combinatorial formula D(n,k)=n^k to credit rating represents an application innovation, not an abstract mathematical discovery (the formula is a foundation of combinatorics). The methodological originality lies in the operational use of this structure to:
(a) manage multi-model configurations with two-year temporal memory.
(b) overcome the intrinsic limitations of simple weighted averages, unable to capture distinct combinatorial patterns that can converge to the same numerical value.
(c) encode expert decision rules in deterministically replicable algorithmic form.

In the context of the scoring system, the parameters assume specific meanings:

For the two-year normalization:

- n = 4 (possible score states: A, B, C, D).
- k= 2 (periods considered: year n-1, year n).
- Total combinations: 4^2 = 16.

For the final quantitative score:

- n = 4 (possible score states: A, B, C, D).
- k = 5 (two-year scores of the five models: Z-Score, EM-Score, MCC Rating, Bank Score, Performance Index).
- Total combinations: 4^5 = 1024.

For the annual Bank Score:

- n = 4 (possible score states: A, B, C, D).
- k = 4 (classes of indicators per section).
- Total combinations per section: 4^4 = 256.

The system uses numerical coding (1=A, 2=B, 3=C, 4=D) to facilitate automatic calculations and subsequent conversion into alphabetical classifications.

3.2) Two-year normalization system

The practical implementation of the system of arrangements with repetitions for the two-year normalization is achieved through a matrix structure that systematically enumerates all 16 possible score combinations between two consecutive years.
Operating logic: the system compares the score of year n-1 with the score of year n, attributing a final score based on the frequency distribution of the score classes in the two-year period.
Through the described approach, both the current position and the trend pattern of the company are captured.
The matrix of arrangements for the two-year system is organized as follows:

Item	Year N-1	Year N	Combination	Calculation A	Calculation B	Calculation C	Calculation D	Final Score
1	A	A	AA	2	0	0	0	A
2	A	B	AB	1	1	0	0	B
3	A	C	AC	1	0	1	0	C
4	A	D	AD	1	0	0	1	D
5	B	A	BA	1	1	0	0	B
6	B	B	BB	0	2	0	0	B
7	B	C	BC	0	1	1	0	B
8	B	D	BD	0	1	0	1	B
9	C	A	CA	1	0	1	0	C
10	C	B	CB	0	1	1	0	B
11	C	C	CC	0	0	2	0	C
12	C	D	CD	0	0	1	1	C
13	D	A	DA	1	0	0	1	D
14	D	B	DB	0	1	0	1	B
15	D	C	DC	0	0	1	1	C
16	D	D	DD	0	0	0	2	D

Table 1 Matrix of arrangements with repetitions for a two-year system

Legend:

- Columns 1-3: they identify the combination of scores between the two years.
- Columns 4-7: they count the occurrences of each class (A, B, C, D) in the combination.
- Column 8: final score attributed according to the classification algorithm (described in Section 3.3.4)

The procedural methodology of the system of arrangements with repetitions is therefore based on a sequential process that guarantees the systematic management of all possible combinations.

3.3) Exemplification of the sequential process

The functioning of the system is illustrated through the application of the Bank Score model, adopting a sequence structured in nine analytical phases; this example allows us to clarify the operational logic of the framework and the process of aggregating the results[5].

3.4) Temporal normalization process on a two-year basis

Based on the illustrated and formalized sequential process, it is possible to proceed to the phase of aggregation of the results, aimed at determining the overall rating class and evaluating the consistency between the applied models.
The process described also applies for the normalization of the score obtained in the last two years (n, n-1) for each model.
Once the normalized scores on a two-year basis have been obtained for each model, we proceed to determine a single alphascore (quantitative score), deriving from the five normalized two-year scores, through the system of arrangements with repetitions, following the same logic as the sections described above.

4) Final aggregation system: Quantitative Score

To determine the final quantitative score, the process of arrangements with repetitions is applied to aggregate the 5 two-year scores into a single quantitative score, managing the 1024 possible combinations (4^5).

$$\text{Quantitative Score} = \text{Arrangements}(S_{MCC}, S_{Z}, S_{EM}, S_{PS}, S_{Bank})$$

Where: S represents the normalized two-year score of each model.

4.1) Final rating structure

To process the final rating, the three fundamental dimensions of the credit assessment are considered: quantitative analysis (deriving from the system of arrangements with repetitions), qualitative analysis and trend analysis.

4.2) Components of the aggregation system

4.2.1) Qualitative analysis section

Qualitative analysis evaluates non-quantifiable aspects through numerical models based on the analyst's discretionary assessment.

Components of Qualitative Analysis
In a non-exhaustive form:

Governance and corporate structure
Competitive positioning: position in the reference market
Corporate strategy: consistency and sustainability of the strategies
Historical track record

Qualitative Scoring System:

- A: Excellent (3 points)
- B: Good (2 points)
- C: Sufficient (1 point)
- D: Insufficient (-1 point)

4.2.2) Trend Analysis

The source of the data for the performance score is the Central Credit Risk Register of the Bank of Italy, an information system managed by the Bank of Italy according to the provisions of Circular no. 139 (Bank of Italy, 1991). Based on the performance variables contained therein, appropriately reclassified, a series of indices are processed, aimed at analyzing, not only in a static manner, the past of the borrower's credit relationships but also at reprocessing them in a dynamic way, deriving important qualitative information from them.
By analyzing each relationship in detail and attributing a weight to a series of performance indices, a performance score based on qualitative and quantitative indicators is determined.
In particular, the following aspects are analysed:

[5] For a complete consultation of the sequential process and the matrix tables used (Base Matrix, Alfascore, Frequency Matrix and Classification Vector), please refer to the supplementary material available in the online repository link: https://drive.google.com/drive/folders/1Xug079LzCeq1bBYpLaN1FhdQ6PoiHvvS , including the extraction of the original spreadsheet used in the application case.

- the borrower's behavior in using credit lines.
- the level of appropriateness of the composition of the credit portfolio (balance between different technical forms).
- the perception of the borrower from the banking system (positioning of the borrower with respect to the banking system).
- the risk level detected on both direct and indirect positions.

Methodological note: *the detailed analysis of the performance indicators is reported in Paragraph 5.5.*

4.3) Final weighting system

The system produces a final rating which represents the synthesis of all the analyses conducted through:

- integration of the three dimensions of assessment.
- calibration through the weighting system.
- stability resulting from the two-year normalization.

4.3.1) Assignment of weights

The system attributes specific weights to the three components of the analysis: qualitative, quantitative and trend analysis. Given the alphabetic numerical conversion system, the relevant scoring scale is as follows:

- 3 if Score A (Excellent)
- 2 if Score B (Good)
- 1 if Score C (Sufficient)
- -1 if Score D (Insufficient)

The system weighs the overall score according to a creditworthiness logic, assigning the following weights:

- 35% for quantitative analysis (P_Q).
- 55% for trend analysis (P_A).
- 10% for qualitative analysis (P_Qual).

$$\text{Overall Score} = 0{,}35 \times P_Q + 0{,}55 \times P_A + 0{,}10 \times P_{Qual}$$

From the sum of the scores thus obtained, the final rating is determined using the following weighting formula:

$$\text{Overall Rating} = \begin{cases} A & \text{if Overall Score} > 2{,}5 \\ B & \text{if } 1{,}5 \leq \text{Overall Score} \leq 2{,}5 \\ C & \text{if } 0{,}5 \leq \text{Overall Score} < 1{,}5 \\ D & \text{if Overall Score} < 0{,}5 \end{cases}$$

Rationale for the predominant trend weight (55%): the choice, defined in the development phase (2015), to assign a predominant weight to the performance component, reflects the operational practice of Loan Origination, according to which negative behavioral evidence (for example, persistent overdrafts detected in the Central Credit Register) represent highly relevant elements in the assessment of creditworthiness. The 55% weight operates mathematically to align the final rating with this operational condition, preventing positive quantitative scores from masking behavioral warning signals; this methodological approach, based on the negative adjustment of anomalies detected in the Central Credit Register, proves now consistent with subsequent supervisory regulatory developments, in particular the EBA Guidelines on Loan Origination and Monitoring (European Banking Authority, 2020), strengthening the theoretical solidity of the weighting originally implemented.

4.4) Performance Score

The performance score is calculated on N banks (maximum 10 in the developed model) based on the reclassification of data from the Bank of Italy’s Central Credit Risk Register.
Below is the example of a calculation relating to a single bank which can be extended to N banks which can potentially be analyzed and calculated with the relevant algorithm for determining the final performance score.

4.4.1) Performance Score: detailed analysis

The model developed determines the score on the following parameters organized into eight macro-sections.

4.4.2) Usage pattern

Definition: breakdown by reporting intermediary of the assessment of usage pattern resulting from the analysis of cash credit risk recorded in the Central Credit Register.

The indices contained in this section are taken from statements reclassified monthly, subsequently reported on an annual basis, summarizing the debt exposure and behavior of the subject registered towards the credit system as a whole. In particular, the performance information (Operational Credit Line and Used Credit Amount) is reported, broken down by intermediary, aggregated according to the risk census category of the cash credit section (Self-liquidating, with Maturity, Revocable).

Performance variables used:

- Operational Credit Line granted by reporting intermediary: amount of monthly/annual credit granted.
- Used by reporting intermediary: amount actually used in the month/year.
- Total operational credit granted: amount of global credit granted by the banking system in the month/year.
- Total credit used: size of the financial debt incurred by the registered entity (monthly/annual exposure amount to the system).
- Granted by a Bank/Total Granted: share of the global credit granted by a single banking intermediary.
- Single Bank Used/Total Used: share of total use per single bank.

From the ratio between Used/Granted, for each single bank and for all reporting subjects, the *Overdraft Bank Rate* and the *Overdraft System Rate* performance indicators are processed, respectively, used to analyze the situation of credit relationships; specifically, these indicators provide the percentage of monthly/annual use of the credit lines with respect to each reporting intermediary and with respect to the total of reports, highlighting any potential or occurred financial tensions and offering indications regarding the monthly/global requirement with reference to each credit line.
The analysis of this type of data represents one of the "early warning" tools traditionally used by the banking system to identify the deterioration of creditworthiness early during the process of monitoring the credit risk associated with the credit lines granted.

4.4.2.1) Usage pattern: indicators for self-liquidating credit lines

Indicators for self-liquidating credit lines

Assessment algorithm:

$$\text{Usage pattern: self-liquidating credit lines}(r) = \begin{cases} 0 & \text{if } r = \emptyset \text{ or } r = 0 \\ \alpha_1 & \text{if } r \in (0; 0{,}25] \text{ (Partially Positive)} \\ \alpha_2 & \text{if } r \in (0{,}25; 0{,}85] \text{ (Positive)} \\ \alpha_3 & \text{if } r \in (0{,}85; 0{,}9999] \text{ (Partially Negative)} \\ \alpha_4 & \text{if } r \in [1{,}00; 1{,}50] \text{ (Negative)} \\ \alpha_5 & \text{if } r > 1{,}50 \text{ (Extremely Negative)} \end{cases}$$

$$\text{where: } r = \frac{\text{credit used}}{\text{credit granted}}, \quad \alpha_i \in \mathbb{R}^+, \quad i = 1, \dots, 5$$

Indicators for revocable credit lines

Assessment algorithm:

$$\text{Usage pattern: revocable credit lines}(r) = \begin{cases} 0 & \text{if } r = \emptyset \text{ or } r = 0 \\ \alpha_1 & \text{if } r \in (0; 0{,}25] \text{ (Partially Positive)} \\ \alpha_2 & \text{if } r \in (0{,}25; 0{,}85] \text{ (Positive)} \\ \alpha_3 & \text{if } r \in (0{,}85; 0{,}9999] \text{ (Partially Negative)} \\ \alpha_4 & \text{if } r \in [1{,}00; 1{,}50] \text{ (Negative)} \\ \alpha_5 & \text{if } r > 1{,}50 \text{ (Extremely Negative)} \end{cases}$$

$$\text{where: } r = \frac{\text{credit used}}{\text{credit granted}}, \quad \alpha_i \in \mathbb{R}^+, \quad i = 1, \dots, 5$$

Indicators for short-term credit lines

Assessment algorithm:

$$\text{Usage pattern: short-term credit lines}(r) = \begin{cases} \text{Partially Positive} & \text{if } 0\% \leq r < 100\% \\ \text{Positive} & \text{if } 100\% \leq r < 100{,}99\% \\ \text{Negative} & \text{if } 100{,}99\% \leq r < 150\% \\ \text{Extremely Negative} & \text{if } r \geq 150\% \end{cases}$$

$$\text{where: } r = \frac{\text{credit used}}{\text{Total operational credit granted}}$$

Indicators for medium-long term credit lines

Assessment algorithm:

$$\text{Usage pattern: medium-long-term lines MLT}(r) = \begin{cases} \text{Partially Positive} & \text{if } 0\% \leq r < 100\% \\ \text{Positive} & \text{if } 100\% \leq r < 100{,}99\% \\ \text{Negative} & \text{if } 100{,}99\% \leq r < 150\% \\ \text{Extremely Negative} & \text{if } r \geq 150\% \end{cases}$$

$$\text{where: } r = \frac{\text{credit used}}{\text{Total operational credit granted}}$$

4.4.3) Proper use

Trend indicators used:

- **overdraft by reporting intermediary**: i.e. the amount expressed in absolute values, resulting from the sum of the overdrafts/unpaid amounts reported by the intermediary; the data allows us to verify how much the registered entity has been negatively exposed compared to the credit granted to it, thus highlighting the extent of the overdraft/unpaid credit recorded.
- **total overdraft:** it shows the overall amount of the sum of overdrafts/unpaid payments in the system, in absolute terms.
- **total overdraft/total granted:** percentage of incidence of the total overdraft reported on the total granted by the banking system.

Indicators for self-liquidating credit lines

Assessment algorithm:

$$\text{self-liquidating overdrafts} = \begin{cases} \text{Partially Positive} & \text{if Months in overdraft } = 0 \\ \text{Partially Negative} & \text{if } 0 < \text{Months in overdraft} \leq 1 \text{ and Average UTR} \leq 101\% \\ \text{Negative} & \text{if } 0 < \text{Months in overdraft} \leq 3 \text{ and Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if } 0 < \text{Months in overdraft} \leq 12 \text{ and Average UTR} > 101\% \end{cases}$$

where:
$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$
$\text{Months in overdraft} = \text{Number of months with overdrafts}$

Indicators for revocable credit lines

Assessment algorithm:

$$\text{overdrafts on revocable lines} = \begin{cases} \text{Partially Positive} & \text{if Months in overdraft} = 0 \\ \text{Partially Negative} & \text{if } 0 < \text{Months in overdraft} \leq 1 \text{ and Average UTR} \leq 101\% \\ \text{Negative} & \text{if } 0 < \text{Months in overdraft} \leq 3 \text{ and Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if } 0 < \text{Months in overdraft} \leq 12 \text{ and Average UTR} > 101\% \end{cases}$$

Where:
$\text{Months in overdraft} = \text{Number of months with overdrafts}$
$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$

Indicators for short-term credit lines

Assessment algorithm:

$$\text{overdraft: short-term lines} = \begin{cases} \text{Partially Positive} & \text{if Months in overdraft} = 0 \\ \text{Negative} & \text{if Months in overdraft} > 0 \text{ and } 100\% < \text{Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if Months in overdraft} > 0 \text{ and Average UTR} > 101\% \\ \text{Partially Negative} & \text{if Months in overdraft} > 0 \text{ and Average UTR} \leq 100\% \end{cases}$$

Where:

$$\text{Months in overdraft} = \text{Number of months with overdrafts}$$
$$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$$

Indicators for medium-long term credit lines

Assessment algorithm:

$$\text{overdraft: medium-long-term lines} = \begin{cases} \text{Partially Positive} & \text{if Months in overdraft} = 0 \\ \text{Negative} & \text{if Months in overdraft} > 0 \text{ and } 100\% < \text{Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if Months in overdraft} > 0 \text{ and Average UTR} > 101\% \\ \text{Partially Negative} & \text{if Months in overdraft} > 0 \text{ and Average UTR} \leq 100\% \end{cases}$$

Where:

$$\text{Months in overdraft} = \text{Number of months with overdrafts}$$
$$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$$

4.4.4) Recurring overdrafts (last quarter)

The indicators used in this section correspond to the same performance variables used in the previous section but with a temporal logic limited to the last quarter rather than extended to a year.

Indicators for self-liquidating credit lines

Assessment algorithm:

$$\text{Self-liquidating Overdrafts: quarter} = \begin{cases} \text{Partially Positive} & \text{if Overdrafts} = 0 \\ \text{Negative} & \text{if } 0 < \text{Overdrafts} < 3 \\ \text{Extremely Negative} & \text{if Overdrafts} \geq 3 \end{cases}$$

Where:

$$\text{Overdrafts} = \text{Number of overdrafts in the past quarter}$$

Indicators for revocable credit lines

Assessment algorithm:

$$\text{Overdrafts Revocable credit lines: quarter} = \begin{cases} \text{Partially Positive} & \text{if Overdrafts} = 0 \\ \text{Negative} & \text{if } 0 < \text{Overdrafts} < 3 \\ \text{Extremely Negative} & \text{if Overdrafts} \geq 3 \end{cases}$$

Where:

$$\text{Overdrafts} = \text{Number of overdrafts in the past quarter}$$

Indicators for short-term credit lines

Assessment algorithm:

$$\text{Overdrafts: short-term lines} = \begin{cases} \text{Partially Positive} & \text{if Overdrafts} = 0 \\ \text{Negative} & \text{if Overdrafts} > 0 \text{ and } 100\% < \text{Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if Overdrafts} > 0 \text{ and Average UTR} > 101\% \\ \text{Partially Negative} & \text{if Overdrafts} > 0 \text{ and Average UTR} \leq 100\% \end{cases}$$

Where:

$$\text{Overdrafts} = \text{Number of overdrafts in the past quarter}$$
$$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$$

Indicators for medium/long-term credit lines

Assessment algorithm:

$$\text{Overdrafts: MLT lines} = \begin{cases} \text{Partially Positive} & \text{if Overdrafts} = 0 \\ \text{Negative} & \text{if Overdrafts} > 0 \text{ e } 100\% < \text{Average UTR} \leq 101\% \\ \text{Extremely Negative} & \text{if Overdrafts} > 0 \text{ and Average UTR} > 101\% \\ \text{Partially Negative} & \text{if Overdrafts} > 0 \text{ and Average UTR} \leq 100\% \end{cases}$$

Where:

$$\text{Overdrafts} = \text{Number of overdrafts in the past quarter}$$

$$\text{Average UTR} = \text{Average ratio credit Used/credit Granted}$$

4.4.5) Unapproved credit use

An **extremely negative** assessment is typically assigned when, with respect to the reference period, the specific case of "absolute overdraft" occurs, referring to cases of use in the absence of a granted credit line.

$$\text{Used credit not granted} = \{\text{Extremely Negative} \quad \text{if used credit} > 0 \text{ and credit granted} = \emptyset$$

4.4.6) Indirect risk positions

Indirect risk positions are constituted by reports relating to the class of self-liquidating risks.
They refer to past due loans that expired during the month preceding the one being surveyed, distinguishing between paid and unpaid, attributing them to each reporting bank and reported with the relevant monthly/annual total.
Variables and performance indicators used:

- **paid overdue receivables**: valued when payment is received after the contractual deadline but within the month of observation. They report delays in collection by final debtors (customers of the registered entity) without representing definitive default.
- **unpaid overdue receivables (unpaid):** valued when payment is not received within the month of observation, representing a default by the final debtor.
- **Unpaid Overdue Rates**: an indicator that measures the ratio between unpaid overdue receivables and total overdue receivables, highlighting the share of unpaid debts compared to the overall maturities of the commercial portfolio.

Paid overdue receivables

Assessment algorithm:

$$\text{Paid overdue receivables} = \begin{cases} \text{no negative adjustment} & \text{if } 0\% < R \leq 10\% \\ \text{Partially Negative} & \text{if } 10\% < R \leq 35\% \\ \text{Negative} & \text{if } 35\% < R \leq 75\% \\ \text{Extremely Negative} & \text{if } R > 75\% \end{cases}$$

Where:

$$R = \frac{\text{Total Paid overdue receivables}}{\text{Total credit Used}}$$

Unpaid overdue receivables

Assessment algorithm:

$$\text{Unpaid overdue receivables} = \begin{cases} \text{no negative adjustment} & \text{if } 0\% < \text{Unpaid overdue receivables} \leq 5\% \\ \text{Partially Negative} & \text{if } 5\% < \text{Unpaid overdue receivables} \leq 10\% \\ \text{Negative} & \text{if } 10\% < \text{Unpaid overdue receivables} \leq 50\% \\ \text{Extremely Negative} & \text{if Unpaid overdue receivables} > 50\% \end{cases}$$

Where:

$$\text{Unpaid overdue receivables} = \text{Ratio Unpaid overdue receivables on Total credit used}$$

Unpaid overdue rates

Assessment algorithm:

$$\text{Unpaid Overdue Rates} = \begin{cases} \text{no negative adjustment} & \text{if } 0\% < \text{Unpaid Rate} \leq 10\% \\ \text{Partially Negative} & \text{if } 10\% < \text{Unpaid Rate} \leq 25\% \\ \text{Negative} & \text{if } 25\% < \text{Unpaid Rate} \leq 50\% \\ \text{Extremely Negative} & \text{if Unpaid Rate} > 50\% \end{cases}$$

Where:

$$\text{Unpaid Rate} = \frac{\text{Average Unpaid Overdue}}{\text{Total Overdue}}$$

4.4.7) Direct risk positions

The direct risk positions represent impaired loans of the registered entity, registered in the Central Credit Register according to the classification in force at the time of the model development[6].

Variables used:

- **Past due:** reporting from an objective evaluation (delays beyond the time threshold).
- **Bad loans:** reporting from the intermediary's subjective assessment (state of insolvency of the debtor).
- **Restructured loans:** exposures with modified contractual conditions.
- **Credits passed to loss:** exposures removed from the balance sheet.

Evaluation criterion:

Any evaluation in the categories indicated above automatically determines the attribution of the **"Extremely *Negative" rating.***

4.4.8) Risk positions of the past month

This section traces the same evaluation methods as in paragraph 4.4.2 (usage pattern), with a temporal focus referring exclusively to the last month, in relation to the following loan categories:

- Self-liquidating Credit lines
- Revocable Credit Lines
- Short-Term Credit Lines
- Medium-Long Term Credit Lines

4.4.9) Overall bank positions

The overall bank position represents the summary assessment of the relationship between the registered entity and each reporting intermediary.

The central indicator is the *Overdraft Bank Rate* which measures the percentage ratio between the amount actually used and the total operating amount granted, for each individual bank.

Assessment algorithm:

$$\text{OSR} = \begin{cases} \text{Extremely Positive} & \text{if no Overdraft and } 100\% \leq \text{Overdraft Rate} \leq 100{,}99\% \\ \text{Partially Positive} & \text{if } 0\% < \text{Overdraft Rate} \leq 25\% \\ \text{Positive} & \text{if } 25\% < \text{Overdraft Rate} \leq 85\% \\ \text{Partially Negative} & \text{if } 85\% < \text{Overdraft Rate} \leq 99{,}9999\% \\ \text{Negative} & \text{if } 99{,}9999\% < \text{Overdraft Rate} \leq 150\% \\ \text{Extremely Negative} & \text{if Overdraft Rate} > 150\% \end{cases}$$

Where:

$$\text{Overdraft Rate} = \frac{\text{Total credit Used Bank}}{\text{Total credit Granted Bank}}$$

4.4.10) Discretionary component

Section reserved for the analyst to attribute a discretionary evaluation aimed at integrating or modulating the assessment deriving from the algorithmic processing.

4.4.11) Single bank score

It refers to the final score determined based on the weighting of the sections of the model determined through the following equation:

$$\text{Score} = 3N_{A+} + 2N_A + N_{B+} - N_{B-} - 2N_C - 3N_D$$

Where:

$$N_{A+} = \text{Extremely Positive}, N_A = \text{Positive}, N_{B+} = \text{Partially Positive}, N_{B-} = \text{Partially Negative}, N_C = \text{Negative}, N_D = \text{Extremely Negative}$$

Hence the determination of the final assessment on the single bank based on the following algorithm:

6 The classification refers to the legislation in force at the time of the model development; the 2015 reform modified a few definitions without altering the conceptual categories.

$$
\text{Single Bank Score} = \begin{cases} \text{Extremely Negative} & \text{if Past Due} \neq 0 \text{ or Bad loans} \neq 0 \\ \text{Extremely Positive} & \text{if Score} > 12 \\ \text{Positive} & \text{if } 9 < \text{Score} \leq 12 \\ \text{Partially Positive} & \text{if } 6 < \text{Score} \leq 9 \\ \text{Partially Negative} & \text{if } 3 < \text{Score} \leq 6 \\ \text{Negative} & \text{if } 0 \leq \text{Score} \leq 3 \\ \text{Extremely Negative} & \text{if Score} < 0 \end{cases}
$$

4.4.12) Overall assessment of the banking system

The overall rating aggregates the scores of the banks with which the registered entity has relationships, using a dynamic threshold proportional to the number of banks:

$$
\text{Overall Score} = \begin{cases} \text{Extremely Positive} & \text{if Score Sum} \geq \text{Target Threshold} \\ \text{Positive} & \text{if Target Threshold} - 2 < \text{Score Sum} \leq \text{Target Threshold} \\ \text{Partially Positive} & \text{if Target Threshold} - 4 < \text{Score Sum} \leq \text{Target Threshold} - 2 \\ \text{Partially Negative} & \text{if Target Threshold} - 6 < \text{Score Sum} \leq \text{Target Threshold} - 4 \\ \text{Negative} & \text{if Target Threshold} - 9 \leq \text{Score Sum} \leq \text{Target Threshold} - 6 \\ \text{Extremely Negative} & \text{if Score Sum} < \text{Target Threshold} - 9 \end{cases}
$$

Where:

$$
\text{Target Threshold} = \frac{20}{N_{\text{banks}}}, \ \text{Score Sum} = \sum_{i=1}^{N_{\text{banks}}} \text{Score}_{\ i}\,, N_{\text{banks}} = \text{Number of banks considered (max 10)}
$$

The model uses a proportional threshold (20/N_banks).

Connection Overall Score - Score Class:

$$
\text{Score Class} = \begin{cases} A+ & \text{if Overall Score} = \text{Extremely Positive} \\ A- & \text{if Overall Score} = \text{Positive} \\ B+ & \text{if Overall Score} = \text{Partially Positive} \\ B- & \text{if Overall Score} = \text{Partially Negative} \\ C & \text{if Overall Score} = \text{Negative} \\ D & \text{if Overall Score} = \text{Extremely Negative} \end{cases}
$$

Connection Score Class - Score Category:

$$
\text{Score Category} = \begin{cases} \text{HIGH} & \text{if Score} \in \{A+, A-\} \\ \text{MEDIUM HIGH} & \text{if Score} \in \{B+, B-\} \\ \text{MEDIUM LOW} & \text{if Score} = C \\ \text{LOW} & \text{if Score} = D \end{cases}
$$

Where:

$$
\text{Score} = \text{Score Class assigned}
$$

4.4.13) Complementary indicators

The model integrates three complementary indicators that provide additional perspectives on the position of the registered entity:

Financial Duration

It evaluates the appropriateness of the temporal composition of the credit structure:

$$
\text{Financial Structure } = \begin{cases} \text{Short term credit lines with no risk signals} & \text{if Short} = 0 \text{ and Others} > 0 \\ \text{Appropriate} & \text{if } 0\% < \text{Short Ratio} \leq 50\% \\ \text{Caution} & \text{if } 50\% < \text{Short Ratio} < 60\% \\ \text{Short term Unbalanced} & \text{if } 60\% \leq \text{Short Ratio} < 100\% \\ \text{Maturing credit lines with no risk signals} & \text{if Short Ratio} \geq 100\% \end{cases}
$$

Where:

$$\text{Short} = \text{Self liquidating} + \text{Revocable; Short Ratio} = \frac{\text{Self liquidating} + \text{Revocable}}{\text{Total Operational Credit}}$$

Overall Perception of the System

It evaluates the aggregate utilization pattern across the entire banking system:

$$\text{Perception of the System} = \begin{cases} \text{Regular utilization: Positive Perception} & \text{if } 25\% < \text{Total UTR} < 85\% \\ \text{Low utilization: Partially Positive Perception} & \text{if } 0\% < \text{Total UTR} < 25\% \\ \text{Tension: Partially Negative Perception} & \text{if } 85\% < \text{Total UTR} < 99{,}99\% \\ \text{Irregular utilization: Negative Perception} & \text{if } 100\% < \text{Total UTR} < 150\% \\ \text{Significant Overdraft: Extremely Negative Perception} & \text{if Total UTR} > 150\% \end{cases}$$

Where:

$$\text{Total UTR} = \frac{\sum \text{utilized Banks}}{\sum \text{granted Banks}}$$

5) Conclusions and future developments

5.1) Summary of methodological contributions

The present work documents a multi-model credit rating system for SMEs developed in 2014-2015, with two distinctive elements:

- two-year temporal normalization through arrangements with repetitions.
- aggregation based on configurational consistency rather than weighted average.

The supplementary material available in the online repository[7] fully documents a real case and five further cases in short, highlighting how the method was operationally applied and how the system aggregates five quantitative models (SME Z-Score, EM Score, MCC Rating, Bank Score, Performance Index) with performance analysis to produce a final rating consistent with the actual riskiness observed.

5.2) Nature of the model: a deterministic expert system and empirical robustness

The system presented is configured as a "*Deterministic Expert System*" (Rule-Based Expert System), a methodological category which is distinct from statistical Machine Learning models.

Focus: design choice

The determination of the weights (35%-55%-10%) and of the classification thresholds through expert judgment represents a mindful methodological choice, motivated by:

- privilege of logical stability: a rule-based system maintains decisional consistency over time, without being affected by the loss of validity of patterns learned from past data, which no longer represent current conditions.
- replicability of the human decision-making process (human in the loop): the system algorithmically codifies the rules that an expert credit analyst would apply, guaranteeing uniformity of evaluation between different operators.
- operation on individual SMEs without the need for Big Data: unlike ML models that require thousands of observations for training, the expert system effectively evaluates each company by applying a structured combination of consolidated indicators, rather than relying on emerging statistical patterns.

Focus: temporal contextual framing of the model

The deterministic approach constitutes the most solid solution from a methodological point of view and the most economically sustainable, considering the context in which it was developed, characterized by:

- absence of structured public datasets on Italian SMEs.
- limited computational power available for small financial structures.
- pre-IFRS 9 regulation which did not require statistical estimates of the Probability of Default (PD).

---

[7] The application cases are available as supplementary material in the online repository: https://drive.google.com/drive/folders/1Xug079LzCeq1bBYpLaN1FhdQ6PoiHvvS. The repository includes: (a) a fully documented real case with complete step-by-step application of the arrangements with repetitions methodology; (b) five additional cases presented in summary form; (c) the original Excel spreadsheet implementing the model, including the Base Matrix, Alfascore table, Frequency Matrix and Classification Vector used in the sequential process described in Section 3.3.

Focus: model limitations

With reference to the model presented, the following limitations should be specified:

- calibration not statistically optimized: the weights are not obtained by optimizing a loss function on historical data (for example by reducing false positives and false negatives in the training set), but they rather derive from an expert assessment of operational risks in the banking sector.
- empirical classification thresholds: the alphabetic score thresholds were mostly inherited from the original models without any recalibration (except for the Z-Score) on the Italian context.
- absence of large-scale backtesting: the documented empirical validation (see supplementary material at the online repository, footnote 7) is limited to 6 real cases, constituting an insufficient number for robust statistical inferences.

Focus: future developments of the model

The characteristics described above do not invalidate the system for its original purposes (internal rating for small portfolios), but they do make a methodological update necessary for contemporary applications, through:

- Bayesian optimization of weights on modern datasets.
- validation on larger panels.
- systematic comparison with external benchmarks.

5.3) Applicability and relevance of the framework in the context of Explainable AI (XAI)

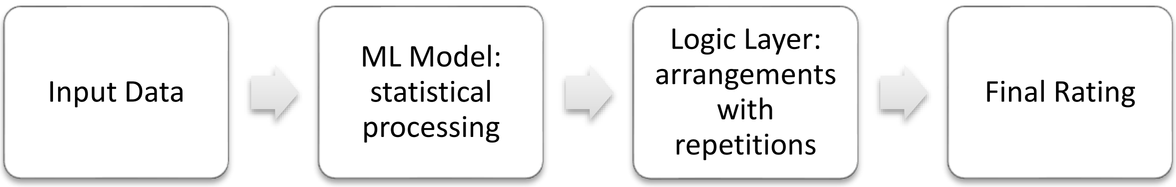
Today, the deterministic character of the system presents a renewed strategic value: the ability to operate at a logical verification level (Logic Layer) within complex Artificial Intelligence architectures.

5.3.1) The model as a guardrail mechanism for the Black Box

The documented framework qualifies as White Box architecture *par excellence*, while Machine Learning (Black Box) algorithms excel in the statistical processing of large volumes of data, although they often lack:

- decisional transparency: impossibility to analytically trace the path that leads from input to output, a critical problem in the credit sector considering the GDPR (EU Regulation 2016/679, art. 22), which recognizes the right of interested parties to the "*explanation*" of automated decisions.
- explicit regulatory controls: to ensure that predictions comply with hard-coded regulatory constraints.

This model is proposed as a potential complement to Hybrid AI architecture:



Guardrail Functioning:

The system intervenes downstream of the ML processing, applying deterministic logical constraints ('*hard stops*'), such as:

- if the ML predicts a "B" rating but the Central Credit Register detects bad loans, the Logic Layer automatically forces a downgrade to D.
- if the quantitative scores are volatile (A-D-B in the three-year period), the system of arrangements with repetitions applies codified smoothing rules, preventing potentially erroneous ratings.

The described architecture ensures:

- regulatory compliance: every decision is transparent and traceable through logging, aligned with the requirements of the EU Regulation on Artificial Intelligence (European Union, 2024, articles 12-13).
- robustness to bias: guardrails prevent statistical biases in the training set from propagating into the final rating.

5.3.2) Combinatorics as a methodological basis for Hybrid AI

The presented combinatorial architecture could form the structural basis for developing Neuro-Symbolic AI systems, in which:

- the neural component (neural networks, gradient boosting) dynamically optimizes weights and thresholds on contemporary data.
- the symbolic component (arrangements matrix) preserves the logical structure and the decision-making transparency.

Operational scenario:

1. An LSTM model predicts scores with confidence intervals.
2. The expert system codifies qualitative, quantitative and trend rules (for example if event "*X*" occurs: minimum penalty -2 classes).

3 The combinatorial matrix (for example: 4^5) aggregates the results ensuring complete traceability.

This integration allows you to:
- exploit the predictive power of modern ML.
- maintain the transparency required by GDPR, EU AI Act, banking regulations.
- allow deterministic audits of credit decisions.

Conclusion: The lasting value of the system does not lie in the (historically situated) 2015 parameters, but in the architectural intuition of being able to use discrete combinatorial structures to formalize expert knowledge in algorithmically replicable and verifiable form, which today is central to the debate on responsible and trustworthy AI.

5.4) Cross-sector applicability of the Combinatorial Framework

The logical architecture of the system, based on combinatorial aggregation, suggests a potential methodological transferability to other decision-making domains.

Requirements for the extension and applicability of the model in further areas

The system of arrangements with repetitions appears technically adaptable to contexts that present:
- heterogeneity of the information sources: hence the need to synthesize quantitative, qualitative and trend indicators of a different nature.
- transparency requirements: regulated contexts where the decision must be auditable and justifiable (no "*black box*").
- presence of hard stops: situations in which specific negative conditions must prevail over positive signals (a non-linear logic that cannot be represented with weighted averages).
- temporal memory: relevance of development trends as well as the instantaneous state.

Areas of future investigation (illustrative examples):

1) *ESG (Environmental, Social, Governance) assessment*

Scenario: an investment fund has to assign ESG ratings to companies by combining:
- Environmental score (CO_2 emissions, circular economy).
- Social score (job security, diversity).
- Governance score (transparency, anti-corruption).

Problem of weighted averages: a company with excellent environmental and governance scores could, with them, compensate for serious violations of human rights, producing an unacceptable "*averaged*" overall rating.
Combinatorial solution: the 4^3 matrix (3 ESG dimensions, 4 A-D classes) encodes hard stops:
- "A-A-D" configuration (D in governance for corruption) with a Final Rating D, regardless of E and S.
- "B-C-B" configuration with a Final Rating C (the algorithm makes social vulnerability prevail).

2) *Supply Chain Risk Management*

Scenario: A multinational company has to classify critical suppliers by combining:
- Financial score (capital solidity).
- Operational score (on-time delivery, quality rate).
- Reputational score (legal disputes, sanctions).

Problem: a financially sound supplier but with pending legal disputes on product safety should not obtain a high rating.
Combinatorial solution: a 4^3 matrix that imposes an automatic downgrade for negative reputational score, even in the presence of excellent financial performance.
Methodological note: *these examples are purely illustrative of potential research directions.*

Effective applicability requires:
- complete redefinition of weights, thresholds and indicators for the specific sector.
- empirical validation on datasets which represent the new domain.
- adaptation of the combinatorial structure ($k \neq 5$, or $n \neq 4$ classes could be needed).

The methodological contribution does not lie in the mere transfer of parameters specific to credit evaluation in other contexts, but in demonstrating that the adoption of discrete combinatorial structures is feasible in certain domains and, probably more appropriate compared to the use of weighted averages, which tend to generate potentially misleading and, generally, semantically improper aggregations.

5.5) Purpose of the document

The present document pursues objectives of:

- methodological documentation: making an approach developed and operationally tested available to the professional and academic community.
- knowledge transfer: providing all the necessary elements to understand, replicate or adapt the system.

The complete documentation (algorithms, formulas, implementation process) has been made public to facilitate replicability and future developments.

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